Public and Catholic District School Board Writing Partnerships

Course Profile

Functions

Grade 11
University/College Preparation
MCF3M

• *for teachers by teachers*

This sample course of study was prepared for teachers to use in meeting local classroom needs, as appropriate. This is not a mandated approach to the teaching of the course. It may be used in its entirety, in part, or adapted.

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Course Overview
Functions, Grade 11, University/College Preparation, MCF3M

Prerequisite: Principles of Mathematics, Grade 10, Academic

Course Description

This course extends student experiences with functions and trigonometry and introduces some financial applications of mathematics. Many of the expectations of this course are based on direct extensions of concepts introduced in Grades 9 and 10. Having previously explored linear and quadratic relationships, students study various polynomial and rational functions, and investigate the relationship of functions and their inverses. Students not only consolidate their previous study of trigonometry but also discover new properties and contexts to which they can be applied. Prior graphing and algebraic skills are consolidated and extended in this course. Identifying connections between the algebraic and graphical representations of functions continues to be an important skill.

Successful completion of MCF3M Functions will prepare students for the two Grade 12 University Preparation courses (Advanced Functions and Introductory Calculus, MCB4U and Mathematics of Data Management, MDM4U) and for the two Grade 12 College Preparation courses (Mathematics for College Technology, MCT4C and College and Apprenticeship Mathematics, MAP4C).

The majority of university-bound students and students planning to study technology or apprenticeship programs at college are expected to take the MCF3M Functions course. In the delivery of the program emphasis must be placed on helping the students to build solid foundations so that they will keep open doors to their own futures. Emphasis must also be placed on:

- practice and consolidation of skills;
- applications that address a broad scope of scenarios;
- diagnosis, identification, and remediation of skill weaknesses;
- reflection on and summary of new learning;
- explorations and activities that help students become more independent problem solvers.

Because of the intended destination of students enrolled in MCF3M, the contextual examples and activities should be drawn from a wide variety of areas with minimal emphasis on areas that are generally mathematically intensive (e.g., engineering, computer science, pure mathematics, etc.).

In the Financial Applications of Sequences and Series strand, students acquire the tools required to make sound personal financial decisions. Students investigate and solve problems involving applications of sequences related to compound interest, annuities, and financial decision-making. In the Trigonometric Functions strand, students investigate and apply properties of the primary trigonometric functions and develop a competency for the manipulation of these functions. This strand has been divided into two units: Trigonometry (including radian measure and the sine and cosine laws) and Applied Trigonometry (with emphasis on the study of sinusoidal functions). The Tools for Operating and Communicating with Functions strand, which has been divided into two units: Algebraic Manipulation of Functions and Function Notation, Inverses and Transformations. This allows students to develop skills in operating with various algebraic expressions and to develop facility in using function notation and in communicating reasoning.

An emphasis on technology allows students to investigate efficiently and effectively. Appropriate technology enables students to more easily visualize concepts and also allows for more time for consolidation, practice and the necessary remediation of skills.

How This Course Supports the Ontario Catholic School Graduate Expectations

This course encourages the Catholic learner to develop his/her God-given gifts and abilities to promote growth toward personal responsibility in preparation for a chosen career path. Throughout this course, emphasis should be placed on moral, ethical, and realistic decision-making in an effort to build responsible citizenship. The classroom environment should instill a spirit of cooperation, rather than competition amongst students, and foster a collaborative sense of community. This course provides many opportunities for students to work effectively as interdependent team members and to acknowledge and respect others for their opinions.

Course Notes

This course profile builds on previous mathematics course profiles written for Grades 9 and 10. The Grade 11 course profiles produced by the Catholic and Public systems represent a collaborative effort between the two writing teams. Due to the common core of learning expectations with the MCR3U course, a common unit breakdown has been suggested, and four different sample units have been developed. Thus, the MCR3U and MCF3M course profiles can also be used as complementary resources. With appropriate adjustments to the complexity of problems, timing, and the need for abstract thinking, a teacher of either course can use activities from either profile. It should be noted that at this level, it is appropriate in certain situations for mathematics itself to provide the context for new concepts, while in this MCF3M profile it would be more appropriate to provide other contexts as well. The sample units provided in this (MCF3M) profile are Function Notation, Inverses and Transformations and Financial Applications of Sequences and Series while the sample units provided in the MCR3U profile are Algebraic Manipulations of Functions and Applied Trigonometry. In addition to these four complete "model" units, a less detailed Unit Overview Chart offers a recommended clustering of expectations for the remaining units, which can provide a starting point from which teachers can develop their own, individualized units.

The recommended sequence of units in this course begins with Exploring Functions: Connecting Algebra and Geometry, which continues the investigative approach to relationships undertaken in Grades 9 and 10, and lays the groundwork for a deeper algebraic and geometric understanding of functions, which is the fundamental theme of the course. Activity 1.4 of the detailed unit in the MCR3U Course Profile has been removed from this profile since it is based on an expectation exclusive to the MCR3U course. The time allotted for this activity has been realigned to two other activities. In order to provide a more contextual framework and then a need for skill development, Activity 3.9 for the MPM2D Public Course Profile can be used as a springboard for Activity 1.1 of this Course Profile. Similarly, Activity 1.7 of the MPM1D Public Course Profile can be used to provide a contextual framework for Activity 1.6 of this Course Profile. This unit is followed by the Function Notation, Inverses and Transformations unit, which includes a number of important concepts consolidated and extended in the later two Trigonometry units. The Financial Applications of Sequences and Series unit, which follows, will also use concepts from the previous units.

For some students, mathematics is perceived to be a collection of isolated and complex topics, each requiring skills that may soon be forgotten. The mathematics teacher must address these perceptions by creating a context in which students can learn and connect concepts and skills. Students must be exposed to a variety of teaching, learning, and problem solving techniques to best synthesize the information presented by the curriculum, and should be provided applications and context to bring meaning to their learning.

The activities in this profile are designed to both introduce and consolidate skills necessary for success in this course and the appropriate destination math courses. These activities can be used in conjunction with or independently of one another. Alternate teaching strategies and suggestions for technological tools are included to help teachers present the lessons contained in the activities.

Since this course has been designed to prepare students for further studies at the university or college level, which requires a substantial level of expertise in mathematics, the specific nature of the learning activities should reflect these destinations. In particular, students in this course should routinely be challenged with investigations and problems that require a high degree of independent effort. Students destined for university or college should have the opportunity to develop and demonstrate their problem-solving abilities.

Students with learning disabilities will need specific guidance in order to benefit from the investigative approach presented in this profile. Review of prerequisite skills and instructions in the use of technology, and in particular graphing calculators, will be necessary before commencing activities in this profile. Clear and precise instructions with examples will need to be provided.

The Achievement Chart for Mathematics is the basis of assessment and evaluation for this course. In the Principles of Mathematics - Academic (Grade 10) Course Profile (p.11) includes charts suggesting strategies that can be used for the assessment and evaluation of all categories of the Achievement Chart. In addition, a chart outlining the component actions that are needed for successful inquiry and problem solving in particular is also included (p.12). These charts provide an excellent base with which to begin the implementation of these strategies and for teachers of this course to extend depending on their degree of readiness. Another excellent resource is the "Concerning Assessment and Reflective Evaluation (CARE)" package of materials, available for free download at - http://www.oame.on.ca. Among the resources included in the package are generic rubrics for Communication and Thinking, Inquiry and Problem Solving skills, along with suggested applications of these instruments.

Units: Titles and Time

Unit 1	Exploring Functions: Connecting Algebra and Geometry	20 hours
* Unit 2	Function Notation, Inverses and Transformations	20 hours
* Unit 3	Financial Applications of Sequences and Series	25 hours
Unit 4	Trigonometry	15 hours
Unit 5	Trigonometric Functions	20 hours
Unit 6	Final Summative Assessment	10 hours

* These units are fully developed in this Course Profile.

Unit Overviews

Unit 1: Exploring Functions: Connecting Algebra and Geometry

Time: 20 hours

Unit Description

Students investigate quadratic functions and related concepts from algebraic and geometric perspectives, in order to deepen their understanding and prepare them for further explorations of functions. A winter recreation theme is loosely woven throughout selected activities in the unit, providing a contextual framework for students to solve problems, both with and without the use of graphing technology. Students solve first-degree inequalities and graph their solutions on number lines. Skills involving operations with polynomials and rational expressions are consolidated, and then extended to the complex number system, which is introduced in this unit. Students apply the method of completing the square in order to solve maximum/minimum problems involving quadratic functions. Algebraic and graphical methods are used in to determine the roots of quadratic equations. The exponent laws are applied to expressions, which have powers containing integer and rational exponents. Students discover the nature of exponential functions and solve exponential equations.

Ontario Catholic School Graduate Expectations: CGE2b, CGE2c, CGE3c, CGE3e, CGE4a, CGE4b, CGE4f, CGE5a, CGE5g, CGE7b, CGE7j.

Unit Overview Chart

Activity	Time	Expectations	Assessment	Tasks
1.1 A Range of Possibilities	150 min	OCV.01, OCV.03, OC1.01, OC1.02, OC3.04, OC3.05	Knowledge Communication	Solve and graph inequalities; Perform operations with polynomials
1.2 Ski-Jumping to the Max!	150 min	OCV.01, OCV.03, OC1.03, OC1.05, OC3.01, OC3.05	Knowledge Inquiry Communication Application	Complete the square; Investigate the graphs of quadratic functions
1.3 Rooting Around the Parabola	150 min	OCV.01, OCV.03, OC1.04, OC1.05, OC3.02, OC3.05	Knowledge Inquiry Communication Application	Determine real/complex roots of quadratic equations; Relate roots to x-intercepts of quadratic functions
1.4 Can We Please be Rational?!	150 min	OCV.01, OCV.03, OC1.06, OC3.01, OC3.03, OC3.04, OC3.05	Knowledge Communication	Perform algebraic operations on rational expressions; State restrictions on the variables
1.5 Power Play	150 min	OCV.01, OCV.03, OC1.07, OC3.01, OC3.05	Inquiry Communication Application	Explore powers with rational exponents; Apply exponent laws to powers containing integer and rational exponents
1.6 It's Snowing Cats and Dogs!	150 min	OCV.01, OCV.03, OC1.08, OC.3.03, OC3.05	Knowledge Application	Explore exponential relations; Solve exponential equations
1.7 Summative Assessment	75 min	All Overall Expectations within unit All Specific Expectations within unit	Knowledge Inquiry Communication Application	Summative assessment

Note: Of the 20 hours in this unit, 3.75 hours has been allotted for remediation and consolidation of skills to be used at the discretion of the teacher for needs of the students.

Unit 2: Function Notation, Inverses and Transformations

Time: 20 hours

Unit Description

Students, through authentic models, are introduced to the definition of a function and the associated notation. Students use graphing technology and paper-and-pencil tasks to investigate the properties of functions, their inverses and transformations of functions. The investigations are used to introduce and extend the use of function notation to inverses and transformations. Students explore the domain and range of functions, inverses and transformations.

Unit Overview Chart

Activity	Time	Expectations	Assessment	Tasks
2.1. Wrap Around Follow up Skills	150 min 75 min	OCV.02, OCV.03, OC2.01, OC2.02, OC3.01, OC3.02, OC3.04, CGE2c, CGE3c, CGE4f, CGE5e, CGE5g	Knowledge Inquiry Application Communication	Investigate the concept of function; formalize the definition of function; apply function notation
2.2. Home on the Range Follow up Skills	150 min 75 min	OCV.02, OCV.03, OC2.02, OC2.03, OC3.03, OC3.05, CGE4f, CGE5a	Knowledge Communication Application	Explore through the use of graphing technology the properties of various functions. Investigate domain and range
2.3. Follow The Bouncing Ball	105 min	OCV.02, OCV.03, OC2.02, OC3.03, OC3.04, OC3.05, CGE2b, CGE3c, CGE5a	Application Knowledge Communication	Develop a model with the use of graphing technology; apply appropriate function notation with the model; investigate properties of the function/model
2.4. Let's Switch Seats! Follow up Skills	105 min 75 min	OCV.02, OCV.03, OC2.04, OC3.03, OC3.05, CGE3c, CGE4f	Application Inquiry	Investigate the properties of inverse functions; discover algebraic approaches for finding the inverse of functions.
2.5. On the Move	150 min	OCV.02, OCV.03, OC2.06, OC2.07, OC2.08, OC3.02, OC3.04, CGE2c, CGE5a	Knowledge Application Communication	Investigate the effect of transformations on mathematical functions; apply appropriate function notation to transformation(s) of functions
2.6. Be My Valentine	105 min	OCV.02, OCV.03, OC2.06, OC2.07, OC2.08, OC3.01, OC3.05, CGE2b, CGE2f	Knowledge Application Communication Inquiry	Apply and analyse transformations and functions
2.7. Consolidating and Connecting	105 min	OCV.01, OCV.02, OCV.03, OC1.03, OC2.02, OC2.06, OC2.07, OC3.02, OC3.03, CGE3c, CGE4f, CGE5a	Knowledge Communication Application Inquiry	Summative assessment

Note: Suggested Follow-up Skills and Extension Activities are listed in the activities that can be used to supplement the activities. Along with these, the teacher can use appropriate remediation and consolidation of skills to account for the total of 20 hours in the unit.

Unit 3: Financial Applications of Sequences and Series

Time: 25 hours

Ontario Catholic School Graduate Expectations: CGE2b, CGE2c, CGE3b, CGE3c, CGE3e, CGE4a, CGE4f, CGE5a, CGE5c, CGE7b, CGE7c.

Unit Description

Students investigate arithmetic and geometric sequences and series. This knowledge serves as the basis for applications of personal finance. Students develop the formula for compound interest and solve problems related to compound interest and annuities. As skills are developed, students use spreadsheets to investigate the cost of borrowing when interest rates, compound periods, lending terms, etc., are varied. The activities are designed to reflect the type of decisions that students are likely to face in the future. Students apply skills with linear and exponential functions.

Unit Overview Chart

Activity	Time	Expectations	Assessment	Tasks
3.1 Investigating Attributes of Sequences	150 min	FAV.01, FA1.01, FA1.02	Inquiry, Communication	Investigate sequences
3.2 Summing Up: Arithmetic Sequences and Series	150 min	FAV.01, FAV.02, FA1.04, FA1.05, FA2.04	Knowledge, Application, Inquiry	Develop formula for arithmetic sequences and series
3.3 Compound Interest: Exploring Geometric Sequences	150 min	FAV.01, FAV.02, FA1.03, FA1.04, FA2.01, FA2.02, FA2.05	Knowledge, Application, Communication	Use a financial application to investigate geometric sequences. Develop the compound interest formula
3.4 Applications: Finding the Amount and the Present Value of a Long Term Investment	75 min	FAV.02, FAV.03, FA2.02, FA3.01, FA3.02	Knowledge, Application	Extend knowledge of the compound interest formula and introduce financial applications of a graphing calculator
3.5 Introduction to Geometric Series	75 min	FAV.01, FA1.05	Knowledge, Application	Develop the formula for the sum of a geometric series
3.6 Applications: Finding the Amount and the Present Value of an Annuity	150 min	FAV.02, FA2.02, FA2.03, FA3.05	Inquiry, Communication	Apply geometric sequences and series to finding the amount and present value of an annuity
3.7 What Happens When? : Changing the Time, Rate and Amount	150 min	FAV.03, FA3.01, FA3.05	All	Use financial applications of a graphing calculator to investigate the effect of changing conditions when borrowing and saving
3.8 Mortgages: How They Work	150 min	FAV.03, FA3.02, FA3.03, FA3.04, FA3.05	Application, Communication	Use technology to generate amortization tables
3.9 Financial Decision Making: A Case Study	300 min	All	All	Summative Assessment Activity

Note: In order to account for the 25 hours in this unit, 2.5 hours has been allotted to practise essential skills, review and to do other assessments.

Unit 4: Trigonometry

Time: 15 hours

Ontario Catholic School Graduate Expectations: CGE2b, CGE2c, CGE3c, CGE3e, CGE4a, CGE4b, CGE4f, CGE5a, CGE5g, CGE7b, CGE7j.

Unit Description

Students consolidate and extend triganomic concepts first introduced in Grade 10. Students use the primary trigonometric ratios, the sine law, and the cosine law to model and solve two- and three-dimensional problems involving acute, right, and oblique triangles. Students investigate the relationship between degree and radian measure, and explore the use of the unit circle and special triangles to determine selected values of the primary trigonometric ratios. Methods of proof are introduced and applied to verify trigonometric identities. Students develop the skills to manipulate and solve trigonometric equations.

Unit Overview Chart

Cluster	Expectations	Assessment	Focus
1	TFV.02, TF2.01, TF2.02, TF2.03, TF2.07	Knowledge Inquiry Application	Define terms and concepts; Convert degrees to radians; Apply radian measure
2	TFV.01, TF1.02, TF2.03	Knowledge Inquiry Communication Application	Review the primary trigonometric ratios; Review the sine and cosine laws for acute triangles; Explore the sine and cosine laws for oblique triangles; Investigate the ambiguous case of the sine law
3	TFV.02, TF2.03, TF2.04	Knowledge Inquiry Communication Application	Investigate special triangles
4	TFV.02, TF1.01, TF2.05	Knowledge Inquiry Communication Application	Derive the unit circle; Use the Pythagorean theorem to prove identities; Discuss the use of proof
5	TFV.02, TF2.03, TF2.06, TF2.07	Knowledge Inquiry	Solve linear and quadratic trigonometric equations
6	All expectations within unit	Knowledge Inquiry Communication Application	Summative assessment

Unit 5: Trigonometric Functions

Time: 20 hours

Ontario Catholic School Graduate Expectations: CGE2a, CGE2c, CGE3c, CGE3e, CGE4b, CGE5a, CGE5e, CGE5f, CGE5g.

Unit Description

Students investigate the periodic nature and graphical properties of the primary trigonometric functions. Using technology, students explore the effects of simple transformations on their graphs and equations. Students apply these concepts to model authentic problems.

Unit Overview Chart

Activity	Time	Expectations	Assessment	Tasks
5.1 Surf's Up! Let's Catch the Sine Wave	75 min	TFV.02, TFV.03, TF2.07, TF3.01, CGE3c	Knowledge	Derive $y = \sin x$ and $y = \cos x$ from the unit circle
5.2 Transformations: More Than Meets the Eye	75 min	TFV.02, TFV.03, OCV.02, TF2.07, TF3.01, TF3.02, TF3.03, TF3.04, TF3.05, OC2.06, OC2.07, OC2.08, CGE3c, CGE5a, CGE5e, CGE5f	Knowledge Inquiry Communication	Investigate the properties of various transformations of $y = \sin x$ and $y = \cos x$
Follow-up Skills	30 min			
Give Me a Sine	75 min			
Follow-up Skills	45 min			
5.3 Don't Go Off On A Tangent	75 min	TFV.03, TF3.01, TF3.06, CGE3c, CGE 4b, CGE 4f	Knowledge Inquiry Communication	Investigate the properties of $y = \tan x$
Follow-up Skills	30 min			
5.4a It's a Spring Thing	150 min	TFV.03, TFV.04, OCV.02, TF3.02, TF3.03, TF3.04, TF3.05, TF4.01, TF4.02, OC2.06, OC2.08, CGE2c, CGE3c, CGE4b, CGE5a, CGE5g	Knowledge Inquiry Communication Application	Model the motion of a mass on a spring
5.4b Ferris Fair	150 min			Model the height of a gondola on a Ferris wheel
Follow-up Skills	75 min			
5.4c Let the Sine Shine In	150 min			Model an hours of daylight function
Follow-up Skills	60 min			
5.5 Summative Assessment	150 min	All expectations within unit CGE2b, CGE3c, CGE3e	Knowledge Inquiry Communication Application	Apply concepts in familiar and unfamiliar contexts

Note: Additional time can be allocated for remediation and consolidation of skills (to be used at the discretion of the teacher), depending on the needs of the students, to account for the total of 20 hours in the unit.

Unit 6: Final Summative Assessment

Time: 10 hours

Ontario Catholic School Graduate Expectations: CGE2b, CGE2c, CGE3c, CGE3e, CGE4a, CGE4b, CGE4f, CGE5a, CGE5g, CGE7b, CGE7j.

Overall Expectations: All

Unit Description

Summative assessment should be designed to provide the opportunity for students to demonstrate comprehensive learning in each of the four achievement categories. Some ideas are suggested in the chart that follows, however any of the various assessment tools mentioned in the Assessment Strategies section could be used. A short paper-and-pencil task would review key terms and concepts. Investigations comparing the buying and leasing of a car yield a wide variety of applications pertaining to both personal finance and the modeling of functions. An assignment exploring trigonometric inverses (for example, the arcsine function) would serve to review the concepts introduced in the functions and trigonometry units. This topic also provides students with an exposure to a subject further explored in Grade 12. These topics are suggested as one possible way to revisit the expectations in a new mathematical context. Accordingly, students are to be assessed solely on the expectations of this course, and not on the extension topics themselves. Due to the particular emphasis of cumulative tests and examinations in university and college programs, a formal examination should be a prominent role in the final summative assessment of the student.

Unit Overview Chart

Cluster	Expectations (by Strand)	Assessment	Focus
1	All Strands	Knowledge Application	Review Key Concepts and Terms
2	• Financial Applications of Sequences and Series • Tools for Operating and Communicating with Functions	Knowledge Inquiry Communication Application	Examine Financial Commitments of Owning a Car: Buying vs. Leasing
3	• Trigonometric Functions • Tools for Operating and Communicating with Functions	Knowledge Inquiry Communication Application	Explore Trigonometric Inverses
4	All Strands	Knowledge Inquiry Communication Application	Final Examination

Note: Time should also be spent helping students prepare for the final assessment tasks and especially for the formal final examination (i.e., identify areas of weakness and provide remediation in these areas).

Teaching/Learning Strategies

In order to address the wide range of expectations in this course, a variety of teaching, learning, and assessment strategies and tools need to be used. Teachers should assume a variety of roles (including guide, facilitator, consultant, and instructor), and should employ a variety of strategies including:

- a balance of whole-class, small group, mixed-ability structured group, and individual instruction through student-centred and teacher-directed activities (group work should be carefully structured along cooperative learning principles to be effective);
- the use of rich contextual problems which engage students and provide them with opportunities to demonstrate learning;

- the prompting, supporting, and challenging of individual students as well as the class as a whole;
- approaches that will accommodate multiple learning styles (e.g., the provision of verbal and written instructions, the inclusion of hands-on activities, etc.);
- the use of technological tools and software (e.g., graphing software, dynamic geometry software, the Internet, spreadsheets, and multimedia) in activities, demonstrations, and investigations to facilitate the exploration and understanding of mathematical concepts;
- the use of learning/performance tasks that are designed to link several expectations and give the students occasion to demonstrate their optimal levels of achievement through the demonstration of skill acquisition, the communication of results, the ability to pose extending questions following an inquiry, and the determination of a solution to unfamiliar problems;
- the use of accommodations, remediation, and/or extension activities, where necessary, to meet the needs of exceptional students;
- the provision of opportunities for students to practise and extend their skills and knowledge outside of the classroom.

In addition to the contribution of the teacher, students themselves should play an active role in their own learning. In order to successfully complete the requirements of this course, students are expected to:

- develop an increased responsibility for their own learning;
- be accountable for pre-requisite skills;
- participate as active learners;
- engage in explorations using technology;
- apply individual and group learning skills;
- describe the mathematical patterns that emerge verbally, algebraically, and visually in the course of learning.

Assessment Strategies

An effective assessment program in mathematics must include a balance of diagnostic, formative and summative assessment instruments that incorporate the categories of learning as defined in The Achievement Chart for Mathematics. One approach is shown below:

	Knowledge/ Understanding	**Thinking/Inquiry Problem Solving**	**Communication**	**Application**
final examinations	•	•	•	•
journals	•		•	•
observations		•	•	•
oral presentations	•		•	
performance tasks	•	•	•	•
portfolios	•	•	•	•
quizzes	•			
reports/assignments	•		•	•
student-teacher conferences	•		•	
unit tests	•	•	•	•

Note: Assessment tools such as observational checklists, performance criteria, rubrics, The Achievement Chart for Mathematics, marking schemes, rating scales, peer evaluation, and self-evaluation can and should be used to assist in developing objective and consistent evaluations of student achievement.

Assessment & Evaluation of Student Achievement

Assessment as defined in the document *Ontario Secondary Schools, Grades 9-12*: *Program and Diploma Requirements, 1999,* is "the process of gathering information from a variety of sources (including assignments, demonstrations, projects, performances, and tests) that accurately reflects how well students are achieving the curriculum expectations" (p. 31). Assessment tools should be designed to allow students to demonstrate the full extent of their learning across the four categories. As teachers will use a variety of assessment tools, it is necessary to ensure that a consistent standard is maintained. Thus, these tools should be developed with the learning expectations of the course as the criteria for this standard. That is, a grade of 70-79% using an objective marking scheme should be equivalent to a Level 3 performance, as defined by the Achievement Chart. Teachers may find it more appropriate to use rubrics to assess Thinking, Inquiry, Problem Solving, and Communication skills, but to use objective scales for Knowledge, Understanding, and Application skills. High quality assessment can measure individual and group performance, and individual performance within a group.

The students' effective demonstration of communication skills is an essential component of this course when evaluating achievement. Students are required to display and convey their knowledge and understanding of concepts, share their process of thought and inquiry, and justify their application of concepts in an unfamiliar situation. In addition, their *ability* to communicate these skills is also assessed.

It should also be noted that teachers must continue to expand their own understanding of application skills to include non-routine problems. This view requires a shift from the *specific* application of concepts (i.e., familiar situations), to the *general* application of concepts (i.e., unfamiliar situations).

Assessment strategies and tools must address a wide variety of teaching and learning styles in addition to the criteria established by the learning expectations. Tests consisting only of questions that ask students to perform algorithms and apply their knowledge do not necessarily offer an opportunity for students to demonstrate Level 4 performance. Also, it is understood that students will meet course expectations at a variety of performance levels. An effective and well-balanced assessment program will provide students with several opportunities to demonstrate growth and improvement over time, across all of the knowledge and skill categories.

Evaluation, as defined by *Ontario Secondary Schools, Grades 9-12: Program and Diploma Requirements, 1999,* is "the process of judging the quality of a student's work on the basis of established achievement criteria, and assigning a value to represent that quality" (p. 31). Whereas assessment is the collecting of information about student performance in a variety of methods, evaluation is the determination of a quantitative value describing the student's overall level of achievement. Effective assessment, evaluation, and reporting require the teacher to do more than just average marks. While averaging may be more useful in some Knowledge and Application skill categories, it is not comprehensive enough for accurate reporting in the Inquiry and Communication skill categories. The use of rubrics is a suggested technique for these categories. As students can be expected to improve their performances over time, particular emphasis should be placed on their most recent and most consistent level of achievement.

Students who receive a final performance evaluation of Level 3 or better are believed to be well prepared for work in any of the following Grade 12 mathematics courses: *Advanced Functions and Introductory Calculus* (MCB4U*), Mathematics of Data Management* (MDM4U), *Mathematics for College Technology* (MCT4C), and *College and Apprenticeship Mathematics* (MAP4C). Hence students need to be prepared for university programs that require some degree of mathematical expertise and for college programs that require a level of mathematics necessary for success in technology based and apprenticeship programs. Accordingly, in order to prepare students for the academic reality of university and college programs, proper attention should be placed on the effective preparation for a comprehensive final examination. While other rich, performance-based activities can and should be part of the Final Summative Assessment unit a formal examination should play a more significant role in this course.

Seventy per cent of the grade will be based on assessments and evaluations conducted throughout the course. Thirty per cent of the grade will be based on a final evaluation in the form of an examination, performance, essay, and/or other method of evaluation.

Accommodations

For exceptional students teachers should refer to the student's Individual Education Plan (IEP) and use the recommendations to make any necessary accommodations. Teachers should work in consultation with resource teachers, ESL/ELD teachers, and parents or guardians to determine appropriate accommodations as they work through the course.

Accommodations for ESL/ELD Students

- Have students work in pairs, with peer tutors, or with classmates that have the same linguistic background.
- Use peer conferencing to reinforce instructions or information.
- Ask an ESL/ELD teacher to review questions, assignments, or assessment instruments.
- Provide sets of reference notes, outlines, or critical information, as well as models of charts, timelines or diagrams.
- Reinforce main ideas by using the think/pair/share peer-assessment strategy.
- Pair written instructions with verbal instructions.
- Use visuals to illustrate definitions for the students' dictionary of terms.
- Simplify instructions.
- Highlight key words or phrases.
- Brainstorm in groups using the students' first language if English is limited.
- Provide opportunities for students to practice oral presentation skills.
- Provide visual or auditory cues.

Accommodations for Students with Learning Disabilities

- Provide extensive student-teacher conferencing;
- Pair students. Due care must be given in the pairing to provide support, not solutions, for the identified student;
- Provide a list of terms (possibly simplified) before an activity begins;
- Modify handouts in terms of the terminology and content used, as well as the size and typeface of the selected font. Allow plenty of space for written responses;
- Allow assignments to be completed in alternate formats or using longer timelines;
- Keep manipulatives, grid paper, formula sheets, and other aids available for needs that arise;
- Contact parents or guardians for support and suggestions;
- Provide the students with oral pre-planning of activities.

Resources

This Course Profile has been provided as a resource to aid the teacher in delivering the curriculum. Through the discretionary use of other materials, the teacher can enrich, remediate, or otherwise supplement their students' education. The following is a partial list of resources widely available to the industrious teacher.

Software (Ministry-Licensed)

Geometer's Sketchpad (dynamic geometry)

Maple (word processor/programming)

Mastering Calculus (concept and skill development)

Math Trek (concept and skill development)
Virtual Tiles (algebraic concept and skill development)
Zap-a-Graph (graphing)

Websites

Note: The URLs for the websites have been verified by the writer prior to publication. Given the frequency with which these designations change, teachers should always verify the websites prior to assigning them for student use.

Canadian Education on the Web – http://www.oise.on.ca/~mpress/eduweb.html A compendium of Canadian education-related resources maintained by Marian Press at the Ontario Institute for Studies in Education/University of Toronto.

Education Network of Ontario – http://www.enoreo.on.ca/ ENO is a computer communications network for everyone who works in elementary and secondary education in Ontario. Members have private accounts, which entitle them to participate in moderated newsgroups on education topics and training.

Hewlett-Packard – http://www.hp.com/calculators/

National Council of Teachers of Mathematics – http://www.nctm.org

Ontario Association of Mathematics Educators – http://www.oame.on.ca

Ontario Curriculum Centre – http://www.curriculum.org

Texas Instruments – http://www.ti.com/calc/docs

Books, Periodicals, etc.

CARE (Concerning Assessment and Reflective Evaluation) Package (download from – http://www.oame.on.ca)

Stiggins, R. *Classroom Assessment for Student Success*. National Washington, D.C.: Education Association of the United States, 1998.

Gadanidis, G. *MathMania: Adventures in Mathematics*. London, ON: ISSN 0843-851X

The Mathematics Teacher. Reston, VA: National Council of Teachers of Mathematics (NCTM). ISSN 0025-5769

Connecting Mathematics: Addenda Series, Grades 9-12. Reston, VA: National Council of Teachers of Mathematics (NCTM), 1991. ISBN 0-87353-327-5

Burtz, H.L and K. Marshall. *Performance-Based Learning and Assessment.* California: Sage, 1996.

O.S.S.T.F. *Quality Assessment.* Toronto, ON: Educational Services Committee, 1999.

Taggart, G. *Rubrics – A Handbook for Construction and Use.* Lancaster, PA: 1998.

OSS Considerations

The following list of resources will support many of the Ontario Secondary School Policies as well as the Ontario Catholic Secondary School Graduate Expectations:

Ministry of Education Policy and Reference Documents

- *Choices into Action: Guidance and Career Education Program Policy*
- *Cooperative Education: Policies and Procedures for Ontario Secondary Schools*
- *Individual Education Plans: Standards for Development, Program Planning, and Implementation, 2000*
- *Mathematics, Grades 9-10*
- *Mathematics, Grades 11-12*
- *Ontario Schools Code of Conduct*

- *Ontario Secondary Schools, Grades 9 to 12: Program and Diploma Requirements*
- *Program Planning and Assessment, Grades 9-12*
- *Violence-Free Schools Policy*

The Ministry of Education has also published several resource documents, brochures, and policy/program memoranda in support of its OSS policies. They are available online at the Ministry of Education website, - http://www.edu.gov.on.ca/eng/document/document.html.

Publications Concerning Faith Development

- *Blueprints (Catholic Curriculum Cooperative - Central Ontario Region)*
- *Catholicity Across The Curriculum (Ontario Catholic School Trustees' Association)*
- *Educating the Soul (Institute for Catholic Education)*
- *Ontario Catholic Secondary School Graduate Expectations (Institute for Catholic Education)*
- *This Moment of Promise (Ontario Conference of Catholic Bishops)*

Career Goals/Cooperative Education Programs

- *Ontario Youth Apprenticeship Program*
- *Youth Employment Skills Program*

Community Partnerships

Refer to local board policies (*e.g., Relations with Business - Corporate Donations, Sponsorships and Agreements*).

Coded Expectations, Functions, Grade 11, University/College Preparation, MCF3M

Financial Applications of Sequences and Series

Overall Expectations

FAV.01 · solve problems involving arithmetic and geometric sequences and series;
FAV.02 · solve problems involving compound interest and annuities;
FAV.03 · solve problems involving financial decision making, using spreadsheets or other appropriate technology.

Specific Expectations

Solving Problems Involving Arithmetic and Geometric Sequences and Series

FA1.01 – write terms of a sequence, given the formula for the *n*th term;
FA1.02 – determine a formula for the *n*th term of a given sequence (e.g., the *n*th term of the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots$ is $\frac{n}{n+1}$);
FA1.03 – identify sequences as arithmetic or geometric, or neither;
FA1.04 – determine the value of any term in an arithmetic or a geometric sequence, using the formula for the *n*th term of the sequence;
FA1.05 – determine the sum of the terms of an arithmetic or a geometric series, using appropriate formulas and techniques.

Solving Problems Involving Compound Interest and Annuities

FA2.01 – derive the formulas for compound interest and present value, the amount of an ordinary annuity, and the present value of an ordinary annuity, using the formulas for the *n*th term of a geometric sequence and the sum of the first *n* terms of a geometric series;
FA2.02 – solve problems involving compound interest and present value;
FA2.03 – solve problems involving the amount and the present value of an ordinary annuity;
FA2.04 – demonstrate an understanding of the relationships between simple interest, arithmetic sequences, and linear growth;
FA2.05 – demonstrate an understanding of the relationships between compound interest, geometric sequences, and exponential growth.

Solving Problems Involving Financial Decision Making

FA3.01 – analyse the effects of changing the conditions in long-term savings plans (e.g., altering the frequency of deposits, the amount of deposit, the interest rate, the compounding period, or a combination of these) (*Sample problem*: Compare the results of making an annual deposit of $1000 to an RRSP, beginning at age 20, with the results of making an annual deposit of $3000, beginning at age 50);
FA3.02 – describe the manner in which interest is calculated on a mortgage (i.e., compounded semi-annually but calculated monthly) and compare this with the method of interest compounded monthly and calculated monthly;
FA3.03 – generate amortization tables for mortgages, using spreadsheets or other appropriate software;
FA3.04 – analyse the effects of changing the conditions of a mortgage (e.g., the effect on the length of time needed to pay off the mortgage of changing the payment frequency or the interest rate);
FA3.05 – communicate the solutions to problems and the findings of investigations with clarity and justification.

Trigonometric Functions

Overall Expectations

TFV.01 · solve problems involving the sine law and the cosine law in oblique triangles;
TFV.02 · demonstrate an understanding of the meaning and application of radian measure;
TFV.03 · determine, through investigation, the relationships between the graphs and the equations of sinusoidal functions;
TFV.04 · solve problems involving models of sinusoidal functions drawn from a variety of applications.

Specific Expectations

Solving Problems Involving the Sine Law and the Cosine Law in Oblique Triangles

TF1.01 – determine the sine, cosine, and tangent of angles greater than 90°, using a suitable technique (e.g., related angles, the unit circle), and determine two angles that correspond to a given single trigonometric function value;
TF1.02 – solve problems in two dimensions and three dimensions involving right triangles and oblique triangles, using the primary trigonometric ratios, the cosine law, and the sine law (including the ambiguous case).

Understanding the Meaning and Application of Radian Measure

TF2.01 – define the term *radian measure*;
TF2.02 – describe the relationship between radian measure and degree measure;
TF2.03 – represent, in applications, radian measure in exact form as an expression involving π (e.g., $\frac{\pi}{3}$, 2π) and in approximate form as a real number (e.g., 1.05);
TF2.04 – determine the exact values of the sine, cosine, and tangent of the special angles 0, $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$, and their multiples less than or equal to 2π;
TF2.05 – prove simple identities, using the Pythagorean identity, $\sin^2 x + \cos^2 x = 1$, and the quotient relation, $\tan x = \frac{\sin x}{\cos x}$;
TF2.06 – solve linear and quadratic trigonometric equations (e.g., $6\cos^2 x - \sin x - 4 = 0$) on the interval $0 \le x \le 2\pi$;
TF2.07 – demonstrate facility in the use of radian measure in solving equations and in graphing.

Investigating the Relationships Between the Graphs and the Equations of Sinusoidal Functions

TF3.01 – sketch the graphs of $y = \sin x$ and $y = \cos x$, and describe their periodic properties;
TF3.02 – determine, through investigation, using graphing calculators or graphing software, the effect of simple transformations (e.g., translations, reflections, stretches) on the graphs and equations of $y = \sin x$ and $y = \cos x$;
TF3.03 – determine the amplitude, period, phase shift, domain, and range of sinusoidal functions whose equations are given in the form $y = a\sin(kx + d) + c$ or $y = a\cos(kx + d) + c$;
TF3.04 – sketch the graphs of simple sinusoidal functions [e.g., $y = a\sin x$, $y = \cos kx$, $y = \sin(x + d)$, $y = a\cos kx + c$];
TF3.05 – write the equation of a sinusoidal function, given its graph and given its properties;
TF3.06 – sketch the graph of $y = \tan x$; identify the period, domain, and range of the function; and explain the occurrence of asymptotes.

Solving Problems Involving Models of Sinusoidal Functions

TF4.01 – determine, through investigation, the periodic properties of various models (e.g., the table of values, the graph, the equation) of sinusoidal functions drawn from a variety of applications;

TF4.02 – explain the relationship between the properties of a sinusoidal function and the parameters of its equation, within the context of an application, and over a restricted domain;

TF4.03 – predict the effects on the mathematical model of an application involving sinusoidal functions when the conditions in the application are varied;

TF4.04 – pose and solve problems related to models of sinusoidal functions drawn from a variety of applications, and communicate the solutions with clarity and justification, using appropriate mathematical forms.

Tools for Operating and Communicating with Functions

Overall Expectations

OCV.01 · demonstrate facility in manipulating polynomials, rational expressions, and exponential expressions;

OCV.02 · demonstrate an understanding of inverses and transformations of functions and facility in the use of function notation;

OCV.03 · communicate mathematical reasoning with precision and clarity throughout the course.

Specific Expectations

Manipulating Polynomials, Rational Expressions, and Exponential Expressions

OC1.01 – solve first-degree inequalities and represent the solutions on number lines;

OC1.02 – add, subtract, and multiply polynomials;

OC1.03 – determine the maximum or minimum value of a quadratic function whose equation is given in the form $y = ax^2 + bx + c$, using the algebraic method of completing the square;

OC1.04 – identify the structure of the complex number system and express complex numbers in the form $a + bi$, where $i^2 = -1$ (e.g., $4i$, $3 - 2i$);

OC1.05 – determine the real or complex roots of quadratic equations, using an appropriate method (e.g., factoring, the quadratic formula, completing the square), and relate the roots to the x-intercepts of the graph of the corresponding function;

OC1.06 – add, subtract, multiply, and divide rational expressions, and state the restrictions on the variable values;

OC1.07 – simplify and evaluate expressions containing integer and rational exponents, using the laws of exponents;

OC1.08 – solve exponential equations (e.g., $4^x = 8^{x+3}$, $2^{2x} - 2^x = 12$).

Understanding Inverses and Transformations and Using Function Notation

OC2.01 – define the term *function*;

OC2.02 – demonstrate facility in the use of function notation for substituting into and evaluating functions;

OC2.03 – determine, through investigation, the properties of the functions defined by $f(x) = \sqrt{x}$ [e.g., domain, range, relationship to $f(x) = x^2$] and $f(x) = \frac{1}{x}$ [e.g., domain, range, relationship to $f(x) = x$];

OC2.04 – explain the relationship between a function and its inverse (i.e., symmetry of their graphs in the line $y = x$; the interchange of x and y in the equation of the function; the interchanges of the domain and range), using examples drawn from linear and quadratic functions, and from the functions

$f(x) = \sqrt{x}$ and $f(x) = \frac{1}{x}$;

OC2.05 – represent inverse functions, using function notation, where appropriate;

OC2.06 – represent transformations (e.g., translations, reflections, stretches) of the functions defined by $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, $f(x) = \sin x$, and $f(x) = \cos x$, using function notation;

OC2.07 – describe, by interpreting function notation, the relationship between the graph of a function and its image under one or more transformations;

OC2.08 – state the domain and range of transformations of the functions defined by $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, $f(x) = \sin x$, and $f(x) = \cos x$.

Communicating Mathematical Reasoning

OC3.01 – explain mathematical processes, methods of solution, and concepts clearly to others;

OC3.02 – present problems and their solutions to a group, and answer questions about the problems and the solutions;

OC3.03 – communicate solutions to problems and to findings of investigations clearly and concisely, orally and in writing, using an effective integration of essay and mathematical forms;

OC3.04 – demonstrate the correct use of mathematical language, symbols, visuals (e.g., diagrams, graphs), and conventions;

OC3.05 – use graphing technology effectively (e.g., use appropriate menus and algorithms; set the graph window to display the appropriate section of a curve).

Ontario Catholic School Graduate Expectations

The graduate is expected to be:

A Discerning Believer Formed in the Catholic Faith Community who

CGE1a -illustrates a basic understanding of the **saving story** of our Christian faith;
CGE1b -participates in the **sacramental life** of the church and demonstrates an understanding of the centrality of the Eucharist to our Catholic story;
CGE1c -actively reflects on **God's Word** as communicated through the Hebrew and Christian scriptures;
CGE1d -develops attitudes and values founded on Catholic **social teaching** and acts to promote social responsibility, human solidarity and the common good;
CGE1e -speaks the **language of life**... "recognizing that life is an unearned gift and that a person entrusted with life does not own it but that one is called to protect and cherish it." (Witnesses to Faith)
CGE1f -seeks intimacy with God and celebrates **communion** with God, others and creation through prayer and worship;
CGE1g -understands that one's purpose or **call in life** comes from God and strives to discern and live out this call throughout life's journey;
CGE1h -respects the **faith traditions**, world religions and the life-journeys **of all people of good will**;
CGE1i -integrates faith with life;
CGE1j -recognizes that "sin, human weakness, conflict and forgiveness are part of the human journey" and that the cross, the ultimate sign of forgiveness is at the heart of **redemption**. (Witnesses to Faith)

An Effective Communicator who
CGE2a -listens actively and critically to understand and learn in light of gospel values;
CGE2b -reads, understands and uses written materials effectively;
CGE2c -presents information and ideas clearly and honestly and with sensitivity to others;
CGE2d -writes and speaks fluently one or both of Canada's official languages;
CGE2e -uses and integrates the Catholic faith tradition, in the critical analysis of the arts, media, technology and information systems to enhance the quality of life.

A Reflective and Creative Thinker who
CGE3a -recognizes there is more grace in our world than sin and that hope is essential in facing all challenges;
CGE3b -creates, adapts, evaluates new ideas in light of the common good;
CGE3c -thinks reflectively and creatively to evaluate situations and solve problems;
CGE3d -makes decisions in light of gospel values with an informed moral conscience;
CGE3e -adopts a holistic approach to life by integrating learning from various subject areas and experience;
CGE3f -examines, evaluates and applies knowledge of interdependent systems (physical, political, ethical, socio-economic and ecological) for the development of a just and compassionate society.

A Self-Directed, Responsible, Life Long Learner who

CGE4a -demonstrates a confident and positive sense of self and respect for the dignity and welfare of others;
CGE4b -demonstrates flexibility and adaptability;
CGE4c -takes initiative and demonstrates Christian leadership;
CGE4d -responds to, manages and constructively influences change in a discerning manner;
CGE4e -sets appropriate goals and priorities in school, work and personal life;
CGE4f -applies effective communication, decision-making, problem-solving, time and resource management skills;
CGE4g -examines and reflects on one's personal values, abilities and aspirations influencing life's choices and opportunities;
CGE4h -participates in leisure and fitness activities for a balanced and healthy lifestyle.

A Collaborative Contributor who

CGE5a -works effectively as an interdependent team member;
CGE5b -thinks critically about the meaning and purpose of work;
CGE5c -develops one's God-given potential and makes a meaningful contribution to society;
CGE5d -finds meaning, dignity, fulfillment and vocation in work which contributes to the common good;
CGE5e -respects the rights, responsibilities and contributions of self and others;
CGE5f -exercises Christian leadership in the achievement of individual and group goals;
CGE5g -achieves excellence, originality, and integrity in one's own work and supports these qualities in the work of others;
CGE5h -applies skills for employability, self-employment and entrepreneurship relative to Christian vocation.

A Caring Family Member who

CGE6a -relates to family members in a loving, compassionate and respectful manner;
CGE6b -recognizes human intimacy and sexuality as God given gifts, to be used as the creator intended;
CGE6c -values and honours the important role of the family in society;
CGE6d -values and nurtures opportunities for family prayer;
CGE6e -ministers to the family, school, parish, and wider community through service.

A Responsible Citizen who

CGE7a -acts morally and legally as a person formed in Catholic traditions;
CGE7b -accepts accountability for one's own actions;
CGE7c -seeks and grants forgiveness;
CGE7d -promotes the sacredness of life;
CGE7e -witnesses Catholic social teaching by promoting equality, democracy, and solidarity for a just, peaceful and compassionate society;
CGE7f -respects and affirms the diversity and interdependence of the world's peoples and cultures;
CGE7g -respects and understands the history, cultural heritage and pluralism of today's contemporary society;
CGE7h -exercises the rights and responsibilities of Canadian citizenship;
CGE7i -respects the environment and uses resources wisely;
CGE7j -contributes to the common good.

Unit 2: Function Notation, Inverses and Transformations

Time: 20 hours

Unit Description

Students, through authentic models, are introduced to the definition of a function and the notations associated with it. Students use graphing technology and paper-and-pencil tasks to investigate the properties of functions, their inverses and transformations of functions. The investigations are used to introduce and extend the use of function notation to inverses, and transformations. Students explore the domain and range of functions, inverses, and transformations.

Activity 2.1: Wrap Around

Time: 150 minutes

Description

Students investigate the concept of functions and formalize the definition of a function. By extending and consolidating previously studied skills and concepts, students also investigate domain and range both graphically and algebraically.

Strand(s) & Learning Expectations

Ontario Catholic School Graduate Expectations

CGE2c - an effective communicator who presents information and ideas clearly and honestly and with sensitivity to others;
CGE3c - a reflective and creative thinker who thinks reflectively and creatively to evaluate situations and solve problems;
CGE4f - a self-directed, responsible, life long learner who applies effective communication, decision-making, problem-solving, time and resource management skills;
CGE5e - a collaborative contributor who respects the rights, responsibilities and contributions of self and others;
CGE5g - a collaborative contributor who achieves excellence, originality, and integrity in one's own work and supports these qualities in the work of others.

Strand(s): Tools for Operating and Communicating with Functions

Overall Expectations

OCV.02 - demonstrate an understanding of inverses and transformations of functions and facility in the use of function notation;
OCV.03 - communicate mathematical reasoning with precision and clarify throughout the course.

Specific Expectations

OC2.01 - define the term function;
OC2.02 - demonstrate facility in the use of function notation for substituting into and evaluating functions;
OC3.01 - explain mathematical processes, methods of solution, and concepts clearly to others;
OC3.02 - present problems and their solutions to a group, and answer questions about the problems and the solutions;
OC3.04 - demonstrate the correct use of mathematical language, symbols, visuals (e.g., diagrams, graphs), and conventions.

Prior Knowledge & Skills

- construct tables of values and graph linear and quadratic relationships
- construct and manipulate relations
- translate verbal relationships into algebraic terms (and vice versa)

Planning Notes

- students are to be placed in groups of two or three
- pipe cleaners of various lengths are needed (10 cm – 20 cm)
- an overhead acetate for each group is needed (and water soluble markers)
- graph paper is needed

Teaching/Learning Strategies

Teacher Facilitation

To help set a context and provide a motivation for functions the following statement can be provided to the class: "The average daily temperature and time of year are related to each other."
This can prompt a class discussion as to how this can be represented and/or illustrated (i.e., graphically and/or algebraically). Students should be informed that this will be revisited later in this activity.
Even though the word "function" is not used in the first part of the student activity, it is designed for students to consolidate and extend their understanding of relationships between two variables. This will provide a platform for the teacher to define the term function and introduce the appropriate notations and definitions associated with it.
The teacher will give each group a pipe cleaner. The lengths can all be the same or different (suggested lengths are 10 cm, 12 cm, 14 cm, 16 cm, 18 cm, 20 cm).
The use of pipe cleaners is not essential but it does make the activity more tactile for the students.
Each group is to put their answers to questions 4, 7.c), 8.a), and 8.c) on their overhead acetates.
The teacher can choose selected groups (or all groups) to present their overhead solutions.

Student Activity

Every member of the group is to complete the following questions.

1. What is the length of the pipe cleaner?
   Bend 1 cm on each end of the pipe cleaner so that these ends will be parallel to each other and each perpendicular to the remaining piece of pipe cleaner.

2. a) The bottom part of the pipe cleaner is called the **base**. What is the length of the base in the diagram?
   b) What is the area if the bent ends are joined to form a rectangle?

3. Straighten out the pipe cleaner. Now do the same as above, bending each end 2 cm this time. Find the height, base, and area.

4. Continue straightening the pipe cleaner and bending it to complete the following table.

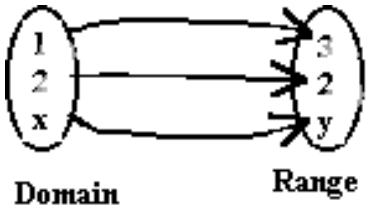
Height (h)	Base (b)	Area (A)
1		
2		
3		
4		

5. a) Why is the height considered a **variable** in the activity?
   b) Are there any other variables? If so, what are they?
6. Are there any **constants** in this activity? If so, what are they?
7. a) Is there a relationship between height (h) and base (b)?
   b) Describe this relationship?
   c) Write an algebraic equation for this relationship.
8. a) Describe the relationship between area (A) and height (h)
   b) Write an algebraic relationship.
   c) If the original length of the pipe cleaner was unknown (let it be k), write an algebraic relationship between area (A) and height (h)

Teacher Facilitation

Teacher uses the results of the above activity to define **function** (special correspondence between the elements of two sets; value of one quantity depends on or is related to the value of another quantity), **domain** and **range.**

Teacher can use the input/output idea to illustrate results of Student Activity 1 in diagram form (mapping/arrow notation). e.g.,



In the assignment below, the teacher will give each group an opportunity to come up with three other examples of functions to present to the class.

The teacher must ensure that students are aware that functions can be described by algebraic formulas, written statements, as a table (or diagram) of input/output data or by a graph.

The teacher should also facilitate students' understanding of the connection between the use of the "*Vertical Line Test*" for functions and the definition of a function. A variety of graphs could be given for the students to see the role of this excellent type of visual cue for functions.

The teacher may need to prompt some groups (or entire class) with examples (e.g., position of a speedometer needle depends on the value of the speed; square root key on a standard scientific calculator; linear and quadratic relationships; etc.).

It should be noted that task 5 (Question 5 on the Student Assignment – Part A) is the statement that was presented at the beginning of this activity. The teacher may need to prompt students to make the "proper" choice for the independent and dependent variables (and hence axes).

Student Assignment (Part A)

1. State three examples of functions.
2. Discuss why these examples are functions.
3. What are the domain and range of each?
4. Draw a diagram to illustrate each.
5. "The average daily temperature and time of year are related to each other."
   a) Sketch a reasonable graph that represents the above statement.
   b) Describe the graph/relationship.

Teacher Facilitation

Teacher uses the results from one group's Activity 1 to introduce function notation.
Example: $A = h(20 - 2h)$ is equivalent to $f(h) = h(20 - 2h)$
Teacher may want to illustrate and explain specific cases (e.g., $f(1), f(2.5), f(0), f(-3)$, etc.). Particular attention must be given to the common misconception by students that the brackets in $f(1)$ mean multiplication.
For the questions listed below, the students are to work in their groups, using the results from the pipe cleaner activity. Each student is to complete all questions.

Student Assignment (Part B)

(use the results from your pipe cleaner activity to answer the following)

1. Evaluate the following:
   a) $f(1)$ b) $f(2)$ c) $f(2.5)$ d) $f(5)$ e) $f(10)$ f) $f(0)$
   g) $f(-1)$ h) $f(f(1))$
2. Give a practical interpretation (using the context of the pipe cleaner activity) to each answer in #1.
3. Sketch a graph of the function.
4. Pose 3 different questions about the pipe cleaner activity that can be answered by referencing to the graph.
5. Explain how to answer your questions using your graph.

Assessment & Evaluation of Student Achievement

Learning skills, specifically teamwork, independence and initiative can be assessed in all the activities, especially in the group work components.
Conferencing can assess Knowledge/Understanding of groups and individuals during all activities.
Knowledge/Understanding can also be assessed by a quiz (or quizzes) that can be given after all activities or after any one of the activities. An objective marking scheme can be used.
Communication skills can be assessed using criteria such as; degree of clarity of explanations, appropriate use of mathematical language, use and clarity of mathematical diagrams or graphs, correct use of mathematical notations during any of the oral presentations recommended in the activities.
Individual student written reports (Student Assignment (Part A) question 5 and Student Assignment (Part B) can be assessed using a rubric.

Category	Level 1 (50 – 59%)	Level 2 (60 – 69%)	Level 3 (70 – 79%)	Level 4 (80 – 100%)
Inquiry - graph and describe a relationship	- limited understanding of a relationship	- some understanding of a relationship	- general understanding of a relationship	- thorough understanding of a relationship
Communication - use of clear and concise mathematical language for explanations	- limited precision and clarity	- sometimes uses appropriate mathematical terminology clearly and precisely	- most often uses appropriate mathematical language clearly and precisely	- consistently uses appropriate mathematical language clearly, precisely

Application - degree of independence for practical interpretation of function notation	- able to make connections with considerable assistance	- able to make connections with some assistance	- able to make connections with limited assistance	- able to make connections independently

Note: A student whose achievement is below level 1 (50%) has not met the expectations for this assignment or activity.

Follow-up Skills: 75 minutes
Teacher should supplement these activities with textbook exercises (include a wide variety of paper-and-pencil type questions):

- evaluating functions expressed in function notation
- find a given $f(a)$
- finding domain and range (discrete and continuous types)
- graphing functions expressed in function notation
- interpreting function notation using the graph of a function (e.g., find $f(1)$ from a given graph)
- interpreting function notation from diagrams and other visual forms
- use of "vertical line test" for functions (when is it a function? When is it a relation?)

Short quiz can be given after the follow-up skills to assess Knowledge and Application using an objective marking scheme.

Activity 2.2: Home on the Range!

Time: 150 minutes

Description

Students graph various relations with and without technology on the Cartesian Plane. Using their results students investigate properties of functions (such as shape, domain, range, intercepts, asymptotes, etc.).

Strand(s) & Learning Expectations

Ontario Catholic School Graduate Expectations
CGE4f - a self-directed, responsible, life long learner who applies effective communication decision-making, problem-solving, time and resource management skills;
CGE5a - a collaborative contributor who works effectively as an interdependent team member.

Strand(s): Functions and Relations
Overall Expectations
OCV.02 - demonstrate an understanding of inverses and transformation of functions and facility in the use of function notation**;**
OCV.03 - communicate mathematical reasoning with precision and clarity throughout the course.
Specific Expectations
OC2.02 - demonstrate facility in the use of function notation for substituting into and evaluating functions;
OC2.03 - demonstrate, through investigation, the properties of functions defined by $f(x) = \sqrt{x}$, [e.g., domain, range, relationship to $f(x) = x^2$] and $f(x) = 1/x$ [e.g., domain, range, relationship to $f(x) = x$];
OC3.03 - communicate solutions to problems and to findings of investigations clearly and concisely, orally and in writing, using an effective integration of essay and mathematical forms;
OC3.05 - use graphing technology effectively (e.g., use appropriate menus and algorithms; set the graph window to display the appropriate section of a curve).

Prior Knowledge & Skills

- use a graphing calculator or graphing software
- construct table of values and graph functions

Planning Notes

- The teacher may want to have the students use graphing calculators to verify their manually graphed functions on their graph paper. (Dynamic software may be used as an alternative to graphing calculators.)
- The teacher may need to book computer time, if dynamic software is to be used.
- Use an overhead projection tablet, if available.

Teaching/Learning Strategies

Teacher Facilitation

Divide the class into five groups. Every group is to do all graphs in question 1. If there is a concern that graphs could become messy doing all five graphs on the same axes, the teacher can have students do them on separate axes or group them in whatever fashion may be appropriate.

Graphs could be verified using technology. The teacher must ensure that students are aware the "$f(x)$ =" is the same as "y =", since graphing technology has defining equations in the letter. Each group will be designated a specific function to present to the class using technology. Group presentations will be used to address question 1 and 2 in the assignment below.

The teacher should use the results from question (1.iv)) and question (1.v)) to introduce and define the concept of asymptotes.

Students are to do questions 3, 4, and 5 individually.

In 3.ii), students should be analysing the nature of the asymptotes and the conditions for increasing and decreasing functions.

In question 5, students use $f(0)$ to analyse conditions for y-intercepts vs. asymptotes

Student Activity

Graphing Assignment: (verify your graphs with graphing technology)

1. Construct a table of values and graph each function on the same axes. (use different colours)
   i) $f(x) = x^2$ ii) $f(x) = x^3$ iii) $f(x) = \sqrt{x}$ iv) $f(x) = 1/x$ v) $f(x) = 2^x$
2. Find the domain, range, and any other special characteristics of each function.
3. Compare/contrast each pair of graphs (analyse shape, domain, range, asymptotes, etc.)
   i) $y = \sqrt{x}$ and $y = x^2$
   ii) $y = x$ and $y = 1/x$
   iii) $y = x^2$ and $y = x^3$
4. For parts 3. i), ii) and iii) find $f(0), f(1), f(2), f(3), f(4)$ and note similarities or differences.
5. Are there any pattern(s) for functions that have similar $f(0)$ and different $f(0)$ values?

Assessment & Evaluation Student Achievement

By ensuring that all members of each group are involved in the presentation, the teacher can assess Knowledge (accuracy of graphing, properties of functions and their graphs as criteria) and Communication (clarity of presentation, use of mathematical language and notations as criteria)

The teacher should observe and conference with individual students while they are doing questions 3, 4, and 5. A class discussion can be used to consolidate concepts.

Upon completion of the assignment, a quiz with an objective marking scheme can be used to assess Knowledge and Application.

Accommodations
Partner students who may be experiencing difficulties with the use of graphing technology with a team/group that has at least one member who is proficient in the use of graphing technology.

Follow-up Skills*:* 75 minutes
Teacher should supplement this activity with textbook exercises that should include finding domain, range, asymptotes and other characteristics of a wide variety of functions given algebraic equations or other forms of functions (e.g., written form, table of values, diagrams, graphs, etc.).
An open ended task in which a domain and/or range is given, then require students to give a function that would fit the domain and/or range would serve as an excellent assessment task.

Activity 2.3: Follow the Bouncing Ball
Time: 105 minutes

Description
Students use a graphing calculator and motion sensor (e.g., Calculator-Based Ranger; CBR) to gather data from a bouncing ball. The students fit an equation to the data (and graph) that is generated. Students investigate the properties of the function that best fits the data.

Strand(s) & Learning Expectations
Ontario Catholic School Graduate Expectations
CGE2b - an effective communicator who reads, understands and uses written materials effectively;
CGE3c - a reflective and creative thinker who thinks reflectively and creatively to evaluate situations and solve problems;
CGE5a - a collaborative contributor who works effectively as an interdependent team member.

Strand(s): Functions and Relations

Overall Expectations
OCV.02 - demonstrate an understanding of inverses and transformation of functions and facility in the use of function notation;
OCV.03 - communicate mathematical reasoning with precision and clarity throughout the course.

Specific Expectations
OC2.02 - demonstrate facility in the use of function notation for substituting into and evaluating functions;
OC3.03 - communicate solutions to problems and to findings of investigations clearly and concisely, orally and in writing, using an effective integration of essay and mathematical forms;
OC3.04 - demonstrate the correct use of mathematical language, symbols, visuals (e.g., diagrams, graphs), and conventions;
OC3.05 - use graphing technology effectively (e.g., use appropriate menus and algorithms; set the graph window to display the appropriate section of a curve).

Prior Knowledge & Skills
- Use a graphing calculator and a device to measure motion.
- Retrieve data from the graphing calculator and plot the data.
- Understanding and use of function notation ($y = f(x)$).

Planning Notes

- The students are placed in groups of two or three to do the activity
- Basketball (or similar size bouncing ball), graphing calculator and motion sensor supplied to every group
- The teacher will review the skills needed to use the graphing calculators and motion sensor

Teaching/Learning Strategies

Teacher Facilitation

Convey to the students that they will be gathering data of a uniformly accelerated basketball (gravity ensures uniform acceleration). Prior to this activity students have been introduced to the notation $y = f(x)$. The teacher should take the opportunity to discuss the context for this activity since this data represents a function that can be written in the form $d = f(t)$ (i.e., distance is a function of time).
Groups should gather their data by dropping the ball from an original height > 3 m (e.g., top of a staircase, bleachers, etc.).
The teacher may do a short demonstration of the calculator and the motion sensor set up, if necessary.
All students are to complete the student assignment component individually.

Student Activity

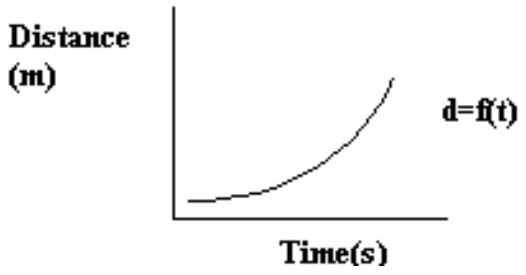
One member of the group holds the motion sensor at the top of an open staircase (height > 3 m) and another member holds the ball approximately 0.5 m below the motion sensor.
Set the measuring time of the motion sensor to the approximate time it takes the basketball to drop. (This may require a few test runs.)

When the CBR is ready, release the ball.
Observe the corresponding distance-time graph for the ball.
Repeat the activity at least once to verify the results.
Retrieve the data from the graphing calculator, construct a table of values and plot it on graph paper (to be done by every student).

Only the data gathered until the ball touches the floor (or ground) is to be plotted. Do **not** include any data after the first bounce.

Teacher Facilitation

When the activity is done, students can see the actual results on the calculator (it should resemble a half parabola opening upward) Distance on the graph represents the distance the basketball is away from the motion sensor at any point in time.



Student Assignment

1. State the dependent and independent variables.
2. a) Is distance a function of time or time a function of distance? Explain your answer.
   b) Using a graphing calculator, find the equation that corresponds to the function. (*quadratic regression*)
3. Find the domain and range.
4. What does the domain represent in this activity?
5. What does the range represent in this activity?
6. Find $f(1), f(0.5), f(0)$. What do they represent in this activity?

9. Find $f(-1)$. What does it represent in this activity? (*time cannot be negative*)
10. List as many properties as possible of the graph.

Assessment & Evaluation of Student Achievement

The student assignment can be collected as a written report.

- Knowledge can be assessed by asking students to find domain and range and to use function notation in context.
- Application can be assessed by asking students to interpret $f(1), f(0.5), f(0)$ on the $d = f(t)$ graph.
- Communication skills can be assessed by using criteria: the degree of clarity of explanations and the use of appropriate mathematical language.

Accommodations

Partner students in teams/groups to provide support for students who require peer assistance.

Activity 2.4: Let's Switch Seats

Time: 105 minutes

Description

Through the investigation of the graphs of various relations students discover properties of inverse functions. Students also discover the algebraic approach to finding the inverse of a function.

Strand(s) & Learning Expectations

Ontario Catholic Graduate Expectations

CGE3c - a reflective and creative thinker who thinks reflectively and creatively to evaluate situations and solve problems;
CGE4f - a self-directed, responsible, life long learner who applies effective communication, decision-making, problem-solving, time and resource management skills.

Strand(s): Functions and Relations

Overall Expectations

OCV.02 - demonstrate an understanding of inverses and transformations of functions and facility in the use of function notation;
OCV.03 - communicate mathematical reasoning with precision and clarity throughout the course.

Specific Expectations

OC2.04 - explain the relationship between a function and its inverse (i.e., symmetry of their graphs in the line $y = x$; the interchange of x and y in the equation of the function; the interchanges of the domain and range), using examples drawn from linear and quadratic functions, and form the functions $f(x) = \sqrt{x}$ and $f(x) = 1/x$);
OC3.03 - communicates solutions to problems into findings of investigations clearly and concisely, orally and in writing, using an effective integration of essay and mathematical forms;
OC3.05 - use graphing technology effectively (e.g., use appropriate menus and algorithms; set the graph window to display the appropriate section of a curve).

Prior Knowledge & Skills

Graphing Relations

- graph a relation using a table of values
- operate a graphing calculator or graphing software

Planning Notes

The use of a graphing calculator or graphing software may be used to allow the students to verify the graphs of the relations and the accompanying tables of values.

Teaching/Learning Strategies

Teacher Facilitation

The activity is to be done as an individual assignment with the teacher assisting with any difficulties. The teacher can introduce inverse function by having the students observe "inverses" on their scientific calculators (i.e., what is on top of the x^2 key? and consolidating previous concepts and characteristics of $y = x^2$ and $y = \sqrt{x}$,). As part of the class discussion the teacher can emphasize inverses in other aspects of mathematics and life.

For Example: What is the inverse of: i) multiplication ii) squared iii) up
iv) backwards v) right vi) subtraction

While working through the activity students may need some direction to realize the following about inverses:

1. they can be found by interchanging x and y in the table of values (i.e., in the ordered pairs);
2. they can be found algebraically by interchanging x and y in the defining equation;
3. they can be found graphically by reflection in the line $y = x$ (i.e., .symmetry in the line $y = x$);

Students should be informed, before starting the assignment, that the given pair of graphs in Part 1 are inverses and they will be looking for some special properties of inverses by using them.

Particular attention must be given to the introduction of the notation $f^{1}(x)$ in Part 2 and that the students understand this notation.

Student Assignment

Part 1

Graph each relation using a table of values.

Graph 1 $f(x) = 3x + 2$ Graph 2 $f = \{(x,y)/\ y = x^2, x>0\}$

$g(x) = \dfrac{x-2}{3}$ $g = \{(x,y)/\ y = \sqrt{x}\ , x>0\}100$

Part A

For Graph 1: (graph $y = x$, using a different colour, on your axes)

1. Find the domain and range of the functions f and g.
2. Compare the table of values for f and g, listing all similarities and differences.
3. Compare the graphs of f and g, listing all similarities and differences.
4. How are the graphs of f and g related with respect to the line $y = x$?
5. By putting the equations for f and g in the form "y = …" and "x = …," explain how the equation of one function can be used to find the other?
6. Determine the intercepts of f and g. Describe any special characteristics.
7. Determine the slopes of f and g. Describe any unique characteristics.

Part B

Repeat questions 1-6 for Graph 2.

Part 2

Given: $f(x) = 2x + 4$.

Interchange x and y in the defining equation of $f(x)$.

Solve for y in this new equation. The result will be called $f^{-1}(x)$.

1. Graph $y = f(x)$, $y = f^{-1}(x)$ and $y = x$ on the same Cartesian Plane.
2. Compare the slopes of $y = f(x)$ and $y = f^{-1}(x)$.
3. Compare the x and y intercepts of $y = f(x)$ and $y = f^{-1}(x)$.

4. Find any point [P(x,y)] on $y = f(x)$ and find the distance from point P to the line $y = x$.
5. Interchange the x and y coordinates of P (let this new point be Q). Find the distance from point Q to the line $y = x$.
6. How do the results in question 4 and 5 compare? Prove this result.
7. Find the slope of line $y = x$ and the slope of line PQ. How do they compare? Prove this result.

Assessment & Evaluation of Student Achievement

The student assignment can be used to assess Application (appropriate algebraic manipulations and calculations; validity of interpretations; quality of connections that are made between a function and its inverse).

The teacher should use a class discussion to consolidate all the algebraic and graphical relationships of a function and its inverse and well as the appropriate notations used for inverses.

Follow-up Skills: 75 minutes

The teacher should supplement this activity with textbook exercise questions (include a wide variety of pencil-and-paper type questions) that should include the following:

- given defining equation of a function, find its inverse algebraically;
- given a written definition of a function, describe its inverse;
- given a graphical model of a function, find its inverse graphically given a table of values (or parts of one), find the inverse;
- multi-step questions: e.g., given $f^{-1}(1) = 3$ and $f^{-1}(4) = 6$ and f is a linear function, find its equation. Find the inverse;
- find $f^{-1}(f^{-1}(x))$ for a variety of functions. What do you notice?

Activity 2.5: On the Move

Time: 150 minutes

Description

Students investigate the effect of transformations on the mathematical functions $f(x) = x, f(x) = x^2, f(x) = x^3, f(x) = , f(x) = \sqrt{x}$, $1/x$ using graphing technology. The student also formalize the algebraic approach to the transformation of functions using appropriate function notation.

Strand(s) & Learning Expectations

Ontario Catholic School Graduate Expectations

CGE2c - be an effective communicator who presents information and ideas clearly and honestly and with sensitivity to others;

CGE5a - be a collaborative contributor who respects the rights, responsibilities and contributions of self and others.

Strand(s): Functions and Relations

Overall Expectations

OCV.02 - demonstrate an understanding of inverses and transformations of functions and facility in the use of function notation;

OCV.03 - communicate mathematical reasoning with precision and clarity throughout the course.

Specific Expectations

OC2.06 - represent transformations (e.g., translations, reflections, stretches) of the functions defined by $f(x) = x, f(x) = x^2, f(x) = \sqrt{x}$, $f(x) = \sin x, f(x) = \cos x$, using function notation;

OC2.07 - describe, by interpreting function notation, the relationship between the graph of a function and its image under one or more transformations;

OC2.08 - state the domain and range of transformations of the functions defined by $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, $f(x) = \sin(x)$, and $f(x) = \cos(x)$;
OC3.02 - present problems and their solutions to a group, and answer questions about the problems and the solutions;
OC3.04 - demonstrate the correct use of mathematical language, symbols, visuals (e.g., diagrams, graphs), and conventions.

Prior Knowledge & Skills

Graphing Relations

- graph a relation using a table of values and operate a graphing calculator or graphing software
- understanding of the shape the basic functions used in the activity

Planning Notes

- Provide a graphing calculator available for every student (or at least one for each pair) or book computer time for the use of appropriate graphing software
- The teacher can use an overhead projection tablet for demonstration purposes
- The activity can be done without the use of graphing technology but the students will spend a significantly longer period of time graphing using a table of values.

Teaching/Learning Strategies

Teacher Facilitation

Students are to be placed into six groups. Each group presents to the class the result of its transformation(s) using (and answering) the questions assigned to each group incorporating the graphing calculator and the overhead tablet

The teacher should take the opportunity for class discussion to formalize the appropriate function notations associated with the various transformations (e.g., for a vertical stretch the defining equation for the function $y = f(x)$ becomes $y = af(x)$.

Student Activity

Part 1: Group Presentations

For the functions:

i) $f(x) = x$ ii) $f(x) = x^2$ iii) $f(x) = \sqrt{x}$ iv) $f(x) = 1/x$ v) $f(x) = x^3$

Groups 1 to 4:

1. Find and simplify the defining equation for all the graphs assigned to your group (look below). {Example: for the function $f(x) = x^3$, $f(x - 2) = (x - 2)^3 = x^3 - 6x^2 + 12x - 8$}
2. Graph each of the five functions listed above along with the group of transformations defined for your group on a separate Cartesian Plane.
   Group 1 $y = f(x)$, $y = f(x) + 5$, $y = f(x) - 6$
   Group 2 $y = f(x)$, $y = f(x - 4)$, $y = f(x + 2)$
   Group 3 $y = f(x)$, $y = 2f(x)$, $y = \frac{1}{2} f(x)$, $y = -3f(x)$
   Group 4 $y = f(x)$, $y = f(2x)$, $y = f\left(\frac{1}{3}x\right)$, $y = \left(-\frac{1}{2}x\right)$
3. How is the original function affected? (shape, orientation, key points, position, etc.) Describe the transformation that is applied.

Groups 5 and 6:

1. Find and simplify the defining equation for all the functions assigned to your group (look below) {Example: for the function $f(x) = x^2$, $3f(x + 1) = 3(x + 1)^2$ then expand and simplify)
2. Graph the functions listed above according to the transformations defined for your group on a separate Cartesian Plane.

Group 5 $y = f(x)$, $y = f(x + 4)$, $y = f(x) + 4$, $y = f(x - 3)$, $y = f(x) - 3$
Group 6 $y = f(x)$, $y = 2f(x)$, $y = f(2x)$, $y = -0.5f(x)$, $y = f(-0.5x)$

3. Describe how each transformation affects the function. Describe all similarities and differences.

Part 2: Individual Assignment

Teacher Facilitation

Each student is to be assigned two of the five functions listed at the beginning of Part 1. This part is to be completed and submitted by each student individually.

1. Find the defining equation for and graph f(x) according to the following transformations.
   $y = f(x)$ $y = f(2x) - 6$ $y = 1/2f(x + 4)$ $y = f(x - 4) + 3$
   $y = 3f(2x)$ $y = -0.5f(x - 7) + 2$ $y = 2f(0.5x)$ $y = -2f(3x - 1) - 5$
2. Describe how each transformation affects the function. Describe all similarities and differences.

Assessment & Evaluation of Student Achievement

By conferencing with the students during their group work, Knowledge can be assessed using the following criteria: i) accuracy of content, ii) correct algebraic manipulation, iii) appropriate use of function notation for transformations.

Communication can be assessed using the following criteria: i) clarity of presentation, ii) proper use of mathematical language, iii) proper use of technology.

The teacher can use Part 2 to assess Application (algebraic manipulation, use of function notation, interpretation of appropriate transformations) using an objective marking scheme.

Accommodations

Partner students in teams/groups to provide support for students who require peer assistance.

Follow-up Activities

1. An interesting and fun activity is to have the students draw a simple picture or shape (a box, face, house, logo, etc.) on a Cartesian Plane and state the key coordinates of that shape. Students can then perform one of each type of transformation (and/or multiple transformations) to their picture and show the results (which are often quite funny and interesting) to the class.
2. Draw a cartoon character on a Cartesian Plane. Students must apply (and describe) a series of transformations that will allow the character to be "animated."

Follow-up Skills: 75 minutes

The teacher should supplement these activities with textbook exercises (include a wide variety of paper-and-pencil type questions):

- Find the defining equation of a function with a given equation under the transformations.
- Find the defining equation of a function with a given equation under more than one transformation.
- Find the transformation(s) that would be necessary to transform the equation of a given function to another given function.
- Find the transformation(s) that would be necessary to transform the graph of a given function to another given function.

Activity 2.6: Be My Valentine

Time: 105 minutes

Description

Students, with the aid of graphing technology, construct a greeting card using appropriate functions and transformations. Students also analyse the functions and transformations both algebraically and graphically.

Strand(s) & Learning Expectations

Ontario Catholic School Graduate Expectations
CGE2b - an effective communicator who reads, understands and uses written materials effectively;
CGE4f - a self-directed, responsible, life long learner who applies effective communication, decision-making, problem-solving, time and resource management skills.

Strand(s): Tools for Operating and Communicating with Functions

Overall Expectations
OCV.02 - demonstrate an understanding of inverses and transformations of functions and facility in the use of function notation;
OCV.03 - communicate mathematical reasoning with precision and clarity throughout the course.

Specific Expectations
OC2.06 - represent transformations (e.g., translations, reflections, stretches) of the functions defined by $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, $f(x) = \sin x$, $f(x) = \cos x$, using function notation;
OC2.07 - describe, by interpreting function notation, the relationship between the graph of a function and its image under one or more transformations;
OC2.08 - state the domain and range of transformations of the functions defined by $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, $f(x) = \sin(x)$, and $f(x) = \cos(x)$;
OC3.01 - explain mathematical processes, methods of solution, and concepts clearly to others;
OC3.05 - use graphing technology effectively (e.g., use appropriate menus and algorithms; set the graph window to display the appropriate section of a curve).

Prior Knowledge & Skills

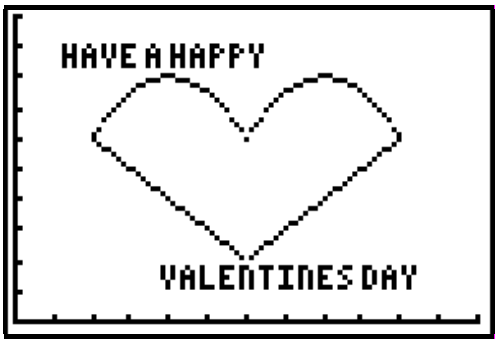
- use a graphing calculator (and a graph link cable for computer interfacing)
- analyse and graph algebraic relationships

Planning Notes

- need a graphing calculator for every student
- need an overhead projection tablet for demonstration purposes
- do the class demonstration in advance
- provide or ask students to bring graph paper

Teacher/Learning Strategies

Teacher Facilitation
Teacher will demonstrate a "Have A Happy Valentines Day" greeting card using a graphing calculator (and overhead projection tablet).
Discuss with the class the instructions necessary to get the picture.
Discuss with the class the analysis (appropriate functions and transformations) necessary to obtain the defining equations used to generate the picture.
If the teacher wants students to print out their cards, a demonstration must be given on the process of downloading calculator images to a computer.



Instructions for generating a Valentine's Day card:
(These generic calculator instructions; depending on the model adjustments may be necessary)

[**WINDOW**]	[**MODE**]
Xmin=0	Dot
Xmax=12	[**Y**=]setting
Xscl=1	$y_1=(-x+8)(x\$2)(x\#6)$
Ymin=0	$y_2=(x-4)(x\$6)(x\#10)$
Ymax=10	$y_3=((-0.5)(x-4)^2+8)(x\$2)(x\#6)$
Yscl=1	$y_4=((-0.5)(x-8)^2+8)(x\$6)(x\#10)$
	the $ and # are under [2nd**][**TEST**]

[2nd][**QUIT**]Home Screen
Text (5,10,"HAVE A HAPPY")
Text (50,30,"VALENTINES DAY")
****Text** is under [2nd][**DRAW**]
to type letters, LOCK the ALPHA key with [2nd][A-LOCK**]
*to change position of the words recall that **Text** (row#,column#, ……)
where the corners of the screen are: (0,0) (0,94)
(57,0) (57,94)

Student Activity
Construct a Greeting Card that includes an appropriate picture and saying, using a graphing calculator. As part of a journal assignment include a discussion and analysis on the process that was used to arrive at the picture. Be sure to include both a graphical and algebraic analysis, along with appropriate transformations and any adjustments that you made throughout the process.

Assessment & Evaluation of Student Achievement

If students are able to download their greeting card and instructions to a computer, a formal written report (including computer downloads and journal) can be submitted and assessed by the teacher. The graphical and algebraic analysis of the functions can be used as a basis for assessing Knowledge and Application using an objective marking scheme. If computer downloads are not used, students can submit those parts of the written report without technology (i.e., paper-and-pencil).
Upon completion of the activity, each student can be assigned a partner that they would have to present their greeting card and journal. Communication and Inquiry can be assessed using a self-and peer assessment checklist. This should be developed prior to the activity as part of a class discussion. Criteria that can be used include: the depth and clarity of the written and oral explanations, appropriate use of notations, symbols and diagrams, incorporation of the appropriate inquiry actions. Students can use column headings: Needs Improvement, Satisfactory, Good and Excellent.

Accommodations
Since student skill in using the graphing calculator may vary, it may be appropriate to allow students who need assistance to seek some help from peers who are proficient in the use of graphing technology.

Activity 2.7: Consolidating and Connecting
Time: 105 minutes

Description
In this activity students will consolidate and connect their understanding of functions with emphasis on the quadratic function. The activity can be used as either a teaching/learning opportunity or an assessment opportunity, depending on the needs and readiness of the students.

Strand(s) & Learning Expectations
Ontario Catholic School Graduate Expectations
CGE3c - thinks reflectively and creatively to evaluate situations and solve problems;
CGE4f - applies effective communication, decision-making, problem-solving, time and resource management skills;
CGE5a - works effectively as an interdependent team member.

Strand(s): Functions and Relations

Overall Expectations
OCV.01 - demonstrate facility in manipulating polynomials, rational expressions, and exponential expressions;
OCV.02 - demonstrate an understanding of inverses and transformations of functions and facility in the use of function notation;
OCV.03 - communicate mathematical reasoning with precision and clarity throughout the course.

Specific Expectations
OC1.03 - determine the maximum or minimum value of a quadratic function whose equation is given in the form $y = ax^2 + bx + c$, using the algebraic method of completing the square;
OC2.02 - demonstrate facility in the use of function notation for substituting into and evaluating functions;
OC2.06 - represent transformations (e.g., translations, reflections, stretches) of the functions defined by $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, $f(x) = \sin x$, $f(x) = \cos x$, using function notation;
OC2.07 - describe, by interpreting function notation, the relationship between the graph of a function and its image under one or more transformations;
OC3.02 - present problems and their solutions to a group, and answer questions about the problems and the solutions;
OC3.03 - communicate solutions to problems and to findings of investigations clearly and concisely, orally and in writing, using an effective integration of essay and mathematical forms.

Prior Knowledge & Skills
Quadratic Equation
- the properties of quadratic functions represented in algebraic, graphical, numerical, or contextual form.
- function notation
- domain and range
- transformations of functions
- difference tables

Planning Notes

- If this activity is used as a teaching/learning activity, form students placed into heterogeneous groups of 2 or 3.
- If the activity is used as an assessment activity, organize the students into small groups to discuss the problem set for approximately 10 minutes, without taking notes. Then require students to complete the activity independently
- Provide toothpicks
- graph paper and graphing calculators, if requested by the students

Teaching/Learning Strategies

Teacher Facilitation

Circulate around the classroom as students clarify their understanding of the problems. If any group is in need of extra support, provide toothpicks for them to create concrete models before moving to pictorial form. Students experiencing difficulties may need assistance in constructing appropriate categories (i.e., vertical toothpicks, horizontal toothpicks, inside (interior) toothpicks, outside (exterior) toothpick.

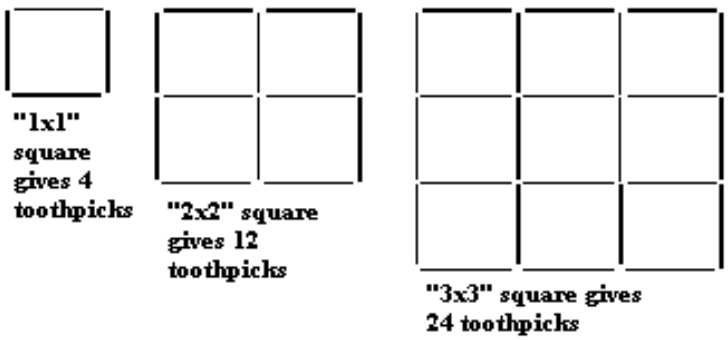
If this activity is being used as a teaching/learning activity, ensure that students are aware of the inquiry/problem solving stages they are engaged in for each part of question 1. The intent of question 2 is to help students connect their understanding of discrete vs. continuous functions, introduced in Grade 9, to the quadratic functions studied in Grade 10 and Units 1 and 2 of this course, and to set the stage for sequences to be introduced later in this course.

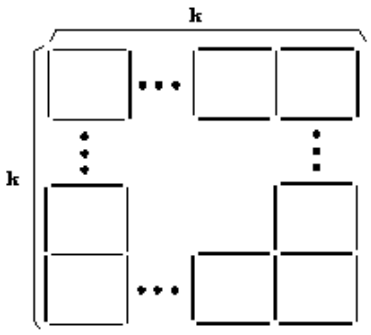
Ensure that students understand how they could be expected to deduce that the required equation in question 3 is 2(x2 – 2) despite the fact that they cannot read off the exact x-intercepts.

The intent of question 4 is to help students connect their understanding of difference tables to the coefficients of the corresponding equation. It also sets the stage for derivatives, by leading students to say that the "rate of change is increasing at a constant rate of 8".

More questions of a skill-based nature can be added at the discretion of the teacher.

Student Activity

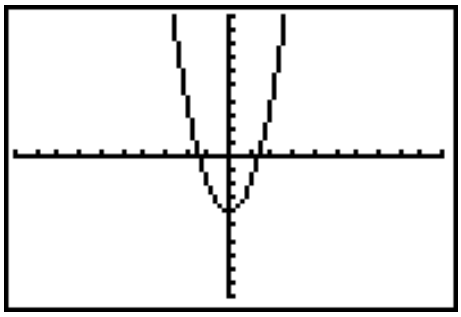




1. Use the toothpick diagrams above to answer the following questions:
   a) Predict the number of toothpicks needed to complete the 100th such toothpick diagram, providing detailed evidence of your analysis.
   b) Using function notation, write a formula for the number of toothpicks needed to complete the k^{th} toothpick diagram. Name the type of function and define its domain and range.
   c) Describe a function that, with appropriate domain restrictions, could be modeled by $f(k) = 4k$, where k is the number of toothpicks along one outside edge of the square.
   d) Describe a function that, with appropriate domain restrictions, could be modeled by $f(k) = k2$, where k is the number of toothpicks along one outside edge of the square.

e) Form a hypothesis about the number of interior toothpicks (not including those toothpicks on the outside perimeter). Confirm or refute your hypothesis.

2. The formula from question 1b) should be related to the function $y = 2x^2 + 2x$
   a) Describe the transformations that relate $y = 2x^2 + 2x$ to $y = x^2$
   b) Illustrate, on a graph, differences between the entire function $y = 2x^2 + 2x$ and the part of the function used to model the number of toothpicks in the k^{th} diagram. Explain your reasoning.
3. Find the equation of the function shown above, given that it is a polynomial of degree 4 or less and has integral numerical coefficients between –5 and 5.



4. a) Construct difference tables for the functions $y = x^2$, $y = 2x^2$, $y = 2x^2 + 2x$, $y = 2x^2 – 4x$
   b) What do all the difference tables have in common?
   c) Describe how the values in the different difference tables relate to the coefficients in the equation of the function.
   d) If a function has constant 2^{nd} differences of 8, what would the equation of the function look like?
   e) Describe what 8 measures in question 4d), using the phrase "rate of change."

Extension

1. Construct rectangles of the form $k \times (k + 1)$ and find the corresponding *f(x)*
2. Allow students to manipulate dimensions and find corresponding relationships

Assessment & Evaluation of Student Achievement

By conferencing with students during the teaching/learning activity the teacher can assess Knowledge (use of function notation, transformations and understanding of domain and range) and oral Communication (correct use of mathematical language and notations).

If the activity is being used to gather assessment data, question 1 could be assessed for Thinking/Inquiry/Problem Solving and Communication, question 2 for Application and Communication, question 3 for Thinking/Inquiry/Problem Solving and question 4 for Knowledge/Understanding. It would be appropriate to add more questions to gather further Knowledge/Understanding and Application assessment data.

Accommodations

Partner students in teams/groups to provide support for students who require peer assistance.
Allow extra time for those students who have it identified on their IEP.

Unit 3: Financial Applications of Sequences and Series

Time: 25 hours

Unit Description

Students work with arithmetic and geometric sequences and series. This knowledge serves as the basis for applications in the field of personal finance. Students develop the formula for compound interest and solve problems related to compound interest and annuities. As skills are developed, students use spreadsheets to investigate the cost of borrowing when interest rates, compound periods, lending terms, etc., are varied. The activities are designed to reflect the type of decisions that students are likely to face in the future. Students apply skills with linear and exponential functions. Students are expected to summarize the results of their investigations in a clear and concise manner.

Unit Synopsis Chart

Activity	Time	Expectations	Assessment	Tasks
3.1 Investigating Attributes of Sequences	150 min	FAV.01, FA1.01, FA1.02	Inquiry, Communication	Work with sequences
3.2 Summing Up: Arithmetic Sequences and Series	150 min	FAV.01, FAV.02, FA1.04, FA1.05, FA2.04	Knowledge, Application, Inquiry	Develop formula for arithmetic sequences and series
3.3 Compound Interest: Exploring Geometric Sequences	150 min	FAV.01, FAV.02, FA1.03, FA1.04, FA2.01, FA2.02, FA2.05	Knowledge, Application, Communication	Use a financial application to investigate geometric sequences. Develop the compound interest formula
3.4 Applications: Finding the Amount and the Present Value of a Long Term Investment	75 min	FAV.02, FAV.03, FA2.02, FA3.01, FA3.02	Knowledge, Application	Extend knowledge of the compound interest formula and introduce financial applications of a graphing calculator
3.5 Introduction to Geometric Series	75 min	FAV.01, FA1.05	Knowledge, Application	Develop the formula for the sum of a geometric series
3.6 Applications: Finding the Amount and the Present Value of an Annuity	150 min	FAV.02, FAV.03, FA2.02, FA2.03, FA3.05	Inquiry, Communication	Apply geometric sequences and series to finding the amount and present value of an annuity
3.7 What Happens When?: Changing the Time, Rate and Amount	150 min	FAV.03, FA3.01, FA3.05	All	Use financial applications of a graphing calculator to investigate the effect of changing conditions when borrowing and saving
3.8 Mortgages: How They Work	150 min	FAV.03, FA3.02, FA3.03, FA3.04, FA3.05	Application, Communication	Use technology to generate amortization tables
3.9 Financial Decision Making: A Case Study	300 min	All	All	Summative Assessment Activity

Note: 150 minutes has been allotted to practice essential skills, review and to do other assessments.

Assessment & Evaluation of Student Achievement

Some of the problems contained in the activities can be used to assess the inquiry process. Short quizzes can be used to assess Knowledge/Understanding and Application. Teachers might choose to give a short quiz on Arithmetic Sequences and Series, another on Geometric Sequences and Series and a third on the Compound Interest Formula and applications of the formula. In addition (or alternatively) a test on Activities 1 to 5 might be given. In Activities 6 to 8, students will apply skills from Activities 1 to 5 to investigate finance. Activity 9 is designed as a culminating assessment tool for the entire unit, with emphasis on financial applications. Communication skills should be assessed continuously throughout the unit. Students should use proper mathematical form, appropriate notation and explain their answers in a clear and concise manner. The Inquiry Rubric and the Communication Rubric produced by OAME are resources to use to assess student achievement.

Activity 3.1: Attributes of Sequences

Time: 150 minutes

Description

Students work with a variety of sequences. Arithmetic and geometric sequences provide a background for future financial applications and the Fibonacci sequence, as well as the sequence created by the Tower of Hanoi problem are included for interest and to emphasize that there are a variety of sequences that can be studied. Students are also introduced to notation used for sequences to be applied later with financial applications.

Strand(s) & Learning Expectations

Strand(s): Financial Applications of Sequences and Series

Overall Expectations

FAV.01 - solve problems involving arithmetic and geometric sequences and series.

Specific Expectations

FA1.01 - write terms of a sequence, given the formula for the nth term;
FA1.02 - determine a formula for the *n*th term of a given sequence.

Prior Knowledge & Skills

Understanding linear and exponential relations may help in transferring knowledge to relations studied in this activity.

Planning Notes

- Provide the students with a copy of the student activity.
- Use of a graphing calculator may be helpful, but is not essential. By graphing the relationships students can determine if the sequences are linear, quadratic, exponential, or other.

Teaching/Learning Strategies

Students should work individually or in pairs to work with the sequences. Discussion of the Fibonacci sequence (introduced in question 3) and the Tower of Hanoi problem (introduced in question 4) will ensure that students do not assume that the only types of sequences are arithmetic and geometric.

Student Activity

Investigate the following situations.

1. Fill in the chart below. The triangle is an equilateral triangle.

Side Length	1 cm	2 cm	3 cm	4 cm	5 cm
Perimeter					

   a) Predict the perimeter of the triangle if the side length is 20 cm.
   b) Identify the type of relationship between the perimeter and the side length.
   c) Create a formula that can be used to find the perimeter of an equilateral triangle with side length n.
   d) Verify that your formula works.

2. The cells in a culture divide every hour.
   a) If there are 500 cells now, how many cells will there be in 1 hour?
   b) Create a table showing the number of cells at the end of each hour for 5 hours.
   c) What type of relationship exists between the number of cells and the time?
   d) Predict the number of cells after 8 hours.
   e) Create an equation that relates the number of cells to the time.
   f) Verify that your formula works.

3. You have 12 sticks of different whole number lengths in a bag. What must their lengths be if, when you pull out any 3 of them, you cannot make a triangle? What is the shortest length of the longest stick?

4. Tameez has a board with 3 pegs. The first peg has 5 flat blocks, all of different sizes, arranged by size with the smallest on top and the largest on the bottom. He wants to move these blocks to the third peg in a minimum number of moves. He has set up rules for himself which include:
   i) only 1 block can be moved at a time.
   ii) A larger block cannot be put on a smaller one.
   a) What is the least number of moves Tameez can take to move the blocks? Explain your reasoning.
   b) Predict how many moves would be required if there were 10 blocks on the first peg.
   c) What is the general relationship between the number of blocks and the number of moves required?

5. In each of these examples a pattern can be found. Try to find the pattern and determine the next two terms for each of the following.
   a) 1, 3, 5, 7, __, __
   b) 12, 24, 48, __, __
   c) $\frac{3}{4}, \frac{5}{9}, \frac{7}{16}$, —, —

   These are examples of sequences of numbers. A sequence is a set of numbers, shapes, letters, etc. that are in a distinct or recognizable pattern.

6. In the sequence 3, 6, 9, 12, 15, 18… the fourth term is 12. We write this as $t_4 = 12$. State the values of t_2 and t_6. The terms of this sequence represent the perimeters found in question 1 above. We found that P = 3n where n represents the side length. For the sequence 3, 6, 9, 12, …
   $t_n = 3n$. Find t_{30} and t_{50}

Teacher Facilitation

The sequence developed in question 3 of the student activity is the Fibonacci Sequence. Two of the follow-up questions also result in the generation of this very interesting sequence. After completing the student activity present the students with some examples where, given a general term, they must find the first few terms (see follow-up questions). Students should also be presented with examples where, given the general term and the value of the term, they can determine where the term is located in the sequence.

Follow-up Questions

1. You have 7 steps to climb. You can go up 1 step or 2 steps at a time. In how many different ways can you climb the steps. Use the chart below to help organize your work.

Number of steps	1	2	3	4	5	6	7
Number of possible ways							

2. Using nickels and dimes only, in how many different ways can you make up various sums of money? (i.e., \$0.05, \$0.10, \$0.15, \$0.20, \$0.25, \$0.30); etc.
3. Given the general term, state the first 4 terms of each sequence:
   a) $t_n = 3n + 1$ b) $t_n = 2^n$
4. Find t_n for each of the sequences in question in the previous activity.

Assessment & Evaluation of Student Achievement

Students' ability to read and interpret directions could be informally assessed as they work. Students could be asked to write a journal entry describing the results of their investigation. Student work on questions 3 and 4 could be collected and assessed using a rubric which focuses on inquiry. Where verification or explanation is required, communication could be assessed.

Accommodations

Use concrete materials for students to better understand the activity.

Activity 3.2: Summing Up: Arithmetic Sequences and Series

Time: 150 minutes

Description

Students investigate, through financial applications and other contexts, arithmetic sequences and series and develop the formulas for the general term of an arithmetic series and the formula for the sum of the terms of an arithmetic sequence. During the investigation, students make a connection between arithmetic sequences and linear growth.

Strand(s) & Learning Expectations

Strand(s): Financial Applications of Sequences and Series

Overall Expectations

FAV.01 - solve problems involving arithmetic and geometric sequences and series;
FAV.02 - solve problems involving compound interest and annuities.

Specific Expectations

FA1.04 - determine the value of any term in an arithmetic or a geometric sequence, using the formula for the nth term of the sequence;
FA1.05 - determine the sum of the terms of an arithmetic or a geometric series using appropriate formulas and techniques;
FA2.04 - demonstrate an understanding of the relationships between simple interest, arithmetic sequences and linear growth.

Prior Knowledge & Skills

- first and second differences to identify whether a relationship is linear or quadratic
- notation associated with sequences and series
- use of a graphing calculator

Planning Notes

- A brief review of simple interest may be necessary before beginning this activity.
- Part A: Students work independently on the investigation.
- Part B: Prepare an overhead with the problem only. Have the students work in groups of four or fewer to investigate the problem.
- It would be helpful if students had access to graphing calculators.

Teaching/Learning Strategies

Part A: Teachers may need to emphasize the relationship between arithmetic sequences and linear growth. Ensure that students have obtained the correct formula for the general term of an arithmetic sequence.

Part B: Encourage students to develop a strategy to find the number of handshakes accurately. After the students have worked in groups, discuss the various strategies. Be sure to discuss the strategy of introducing the people to the room one at a time in order to develop the concept of an arithmetic series.

Student Activity

Part A

Carolyn received a gift of $1000 from her grandmother when she graduated from Grade 8. The money is to be invested to help Carolyn pay for university when she graduates from high school. She buys an investment that pays 4% per year simple interest. Complete the chart below to determine the amount that Carolyn will have at the end of each year for the next five years.

End of Year	Annual Interest	Total Amount
1	40	1040
2	40	1080
3		
4		
5		

1. Predict the amount that Carolyn would have if she kept the investment for ten years. On what are you basing your prediction?
2. What is the type of relationship between the number of years she has the investment and the total amount? *Hint:* Examine the first differences.
3. Determine the equation that best models the relationship between the year and the total amount.
4. Use the equation to predict the year that Carolyn will double her investment.
5. The total amounts that she has at the end of each year can be written as a sequence similar to the sequence 7, 10, 13, 16, 19, 22, …
   This sequence can be written 7, 7 + 3, 7 + 3 + 3, 7 + 3 + 3 + 3, 7 + 3 + 3 + 3 + 3, … or using the notation developed in Activity 3.1.
   $t_1 = 7 + 0(3)$
   $t_2 = 7 + 1(3)$
   $t_3 = 7 + 2(3)$
   $t_4 = 7 + 3(3)$
   $t_{20} = 7 + \_(3)$
   $t_n = 7 + (\quad)(3)$
   Simplify the last term of this sequence. This is called the general or "*n*th" term of the sequence.
6. Write the total amounts that you found in the chart above as the terms of a sequence.
7. Find the "*n*th" term using the method used in question 5. Compare this to the equation found in question 3.

8. Use this method to find t_n for each of the following sequences.
   a) -10, -6, -2, 2, 6, …
   b) 12, 8, 4, 0, -4, …
   c) $\frac{1}{2}$, 1, $\frac{3}{2}$, 2, …

The sequences in this section are called **arithmetic**. Describe the properties of any arithmetic sequence.

9. Give an example of an arithmetic sequence. Explain why you think it is arithmetic.
10. In general, every arithmetic sequence has a first term, a, and the difference between successive terms is a constant, d. Complete the missing information below.

    $t_1 = a$
    $t_2 = a + d$
    $t_3 = a + 2d$
    $t_4 = a + 3d$
    $t_{25} = a + (\quad)d$
    $t_n = a + (\quad)d$

The last term is the general term of any arithmetic sequence.

Part B

1. How Many Handshakes?
   a) Determine the number of handshakes required in a room that contains 5 people if each person shakes hands with every other person once?
   b) If one more person enters the room, how many more handshakes will occur? What is the new total number of handshakes? Predict the number of handshakes that will occur if there are 10 people in the room.
      You may find the following chart helpful to organize your work

Number of People	Number of Handshakes	Total
1	0	0
2	0+1	1
3	0+1+2	3
4		
5		

   c) Identify the type of relationship that exists between the number of people in the room and the total number of handshakes.
   d) Find an equation that best models this relationship. The number of handshakes if there are 5 people in the room can be found by adding the following 0+1+2+3+4. The numbers 0,1,2,3,4 are the terms of an arithmetic sequence. Explain why. When the terms of an arithmetic sequence are added, the result is called an arithmetic series.
   e) The number of handshakes required if there are 10 people in the room can be found be adding the terms of the arithmetic series 0+1+2+3+4+5+6+7+8+9.

$S_{10} = 0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9$
$S_{10} = 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 + 0$
Adding these two rows, we get
$2S_{10} = 9 + 9 + 9 + 9 + 9 + 9 + 9 + 9 + 9 + 9$
$2S_{10} = 10(9)$
$S_{10} = \frac{10(9)}{2}$
$= 45$

f) Use a similar process to find the total number of handshakes required if there were 20 people in the room, 50 people in the room. Find the total number of handshakes required if there are n people in the room.

2. The same method can be used to find the sum of the terms of any arithmetic series.
   a) Use this method to find the sum of the terms of the following arithmetic series:
      i) Find the sum of the first 9 terms of the series 9 + 11 + 13 + 15 + 17 + 19 +…
      ii) Find the sum of the first 10 terms of the series 10 + 12 + 14 + 16 + 18 + 20+…
   b) Note that the sum of the first 5 terms can be found by the following $S_5 = \frac{5(t_1 + t_5)}{2}$ Create a formula to find the sum of the first 8 terms.
   c) Create a formula to find the sum of the first n terms.
   d) Recall that $t_1 = a$, and $t_n = a + (n\text{-}1)d$ for an arithmetic sequence. Verify that the following formula represents the sum of n terms of an arithmetic series.
   $S = \frac{n[2a + (n-1)d]}{2}$
   e) Verify that S_{10} in the previous example is 45 by letting a = 0, and d = 1.
3. Friedrich Gauss was a famous Mathematician. When he was 10 years old his teacher gave him the problem of adding the whole numbers from 1 to 100. The problem was meant to keep him busy for several minutes, but he immediately said that the sum was 50(101). Explain how he might have used the S_n formula to get his result.

Teacher Facilitation

Students need to develop skills with arithmetic sequences and series. At the end of each part of the activity the concepts should be summarized and additional problems assigned from the students' text.

Extension

$S_{10} = 0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9$ can also be written $\sum_{i=0}^{9} i$. This notation is called Sigma or Summation Notation.

1. Write the series represented by
   a) $\sum_{k=1}^{5} (2k + 3)$ b) $\sum_{n=3}^{7} n^2$
2. Write the following series using summation notation
   a) 1 + 3 + 5 + 7 + 9 + 11 b) 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18

Assessment & Evaluation of Student Achievement

Learning skills, such as teamwork, could be informally assessed while students work in small groups. After students have had an opportunity to consolidate skills on arithmetic sequences and series, teachers could give a short quiz to assess Knowledge/Understanding. Teachers might collect solutions to questions 1 and 3 and assess Thinking/Inquiry and Problem Solving. Questions 2a and 2e could be used to assess Application.

Accommodations

- Handouts should be provided for students who have difficulty reading from the overhead screen.
- Partner student within teams to provide support for students who require peer assistance

Activity 3.3: Compound Interest: Exploring Geometric Sequences

Time: 150 minutes

Description

Students develop the concept of a geometric sequence through the use of compound interest. They then develop the formula for the general term of a geometric sequence using patterns discovered. The formula will be applied to develop the compound interest formula.

Strand(s) & Learning Expectations

Strand(s): Financial Applications of Sequences and Series

Overall Expectations

FAV.01 - solve problems involving arithmetic and geometric sequences and series;
FAV.02 - solve problems involving compound interest and annuities.

Specific Expectations

FA1.03 - identify sequences as arithmetic, geometric or neither;
FA1.04 - determine the value of any term in an arithmetic or a geometric sequence, using the formula for the nth term of the sequence;
FA2.01 - derive the formulas for compound interest and present value, the amount of an ordinary annuity, and the present value of an ordinary annuity, using the formulas for the nth term of a geometric sequence and the sum of the first n terms of a geometric series;
FA2.02 - solve problems involving compound interest and present value;
FA2.05 - demonstrate an understanding of the relationships between compound interest, geometric sequences, and exponential growth.

Prior Knowledge & Skills

- simple interest formula
- use of a graphing calculator and/or spreadsheet software

Planning Notes

- Students need access to a graphing calculator or spreadsheet software.
- Students can work individually or in pairs on the activity.
- Spreadsheets might be used to generate tables.
- The rule of 72 might be used where doubling applies.

Teaching/Learning Strategies

When students are completing the chart in the activity, suggest that they look for efficient ways of calculating the new principal. This will lead to the development of the geometric sequence. After students have completed Part A, the teacher should assign some follow-up questions from the text, or other resources. Before beginning Part B, discuss the vocabulary related to compound interest. Define conversion or compound period and discuss the meaning of statements like "interest is compounded semi-annually". Students may need some help developing the sequences in Part B. Make sure that they have the correct meanings for the variables in the compound interest formula.

Student Activity

Part A

In Activity 3.2, Part A, Carolyn placed the money her grandmother had given her in an investment that paid 4% per year simple interest. Understanding simple interest is very important to understanding how interest is actually calculated. It is very rare that an investment like the one in Activity 3.2 would be used. Usually interest is calculated on a regular basis and deposited in the account. The next time that interest is calculated the principal has increased. This is called **compound interest** because interest is paid on the interest that has been deposited. Complete the chart below to determine the amount that Carolyn will have at the end of four years if interest is 4% per year, compounded annually, and the interest earned is deposited at the end of each year.

Time (years)	Principal at the Beginning of the Year	Interest for the year	New Principal at the End of the Year
Now	-	-	$1000.00
1	$1000.00	1000(0.04)(1) = 40	1040.00
2	1040.00	1040(0.04)(1) = 41.60	1081.60
3			
4			

1. Compare the amount she has at the end of four years to the amount you found in Activity 3.2, Part A.
2. Determine the type of relationship that exists between the number of years that have elapsed and the total amount of money that Carolyn has accumulated.
3. Use a graphing calculator to find an equation that best models this relationship.
4. Predict the amount that Carolyn will have at the end of 10 years.
5. Determine approximately how long it will take for Carolyn to double her money. Can you find more than one method of doing this?
   Consider the sequence below:
   $1000, \quad 1000(1.04) \quad 1000(1.04)^2, \quad 1000(1.04)^3, \quad 1000(1.04)^4$
6. Calculate the value of the 5 terms of this sequence.
7. Compare the values to the "New Principal" in the chart above. What do you notice? Explain.
8. Use the pattern in the original sequence to calculate the amount that Carolyn will have at the end of 7 years.
9. Use the pattern to develop a formula for t_n. Compare this to the regression equation found above. Explain what you found.
   The sequence above is called a **geometric sequence**. Describe the properties of a geometric sequence.
10. Give an example of a sequence that is neither arithmetic nor geometric. Explain why they are not.
11. The geometric sequence 3, 6, 12, 24, … can be written in the form $3, 3(2), 3(2)^2, 3(2)^3, \ldots$ Find the value of the 5th term of the sequence. Find an expression for t_n.
12. Any geometric sequence can be written in the form $a, ar, ar^2, ar^3, ar^4, \ldots$ a is the first term and r is the ratio of successive terms. Find an expression for t_n.

Part B

In the first part of this activity you developed the formula for the nth term of a geometric sequence by looking at a problem involving compound interest. You discovered that the first term of the sequence $1000, 1000(1.04), 1000(1.04)^2, 1000(1.04)^3, 1000(1.04)^4$, is the principal deposited and the remaining terms represent the amount of money that has accumulated at the end of each year for 4 years.

Write a sequence that represents the amount of money that will accumulate if $5000 is invested at 6% per year, compounded semi-annually. Determine t_n. In this case, what does n represent?

Write a sequence that represents the amount of money that will accumulate if $10 000 is invested at 8% per year, compounded quarterly. Determine t_n. What does n represent?

Write a sequence that represents the amount of money that will accumulate if the principal is P and the interest rate per year is r. Interest is compounded monthly. Determine t_n. What does n represent?
The compound interest formula is $A = P(1 + I)^n$. Explain what *A, P, I,* and *n* represent.

Follow up Questions
At the end of each part of the activity, assign related questions. Some additional examples may be necessary. At the end of Part A, present the students with several different sequences (many examples will be available in the texts) and have them identify the type of sequence. Discuss how the formula for compound interest can be rearranged to find the present value of an investment or loan. Assign related questions for the students to develop skills.

Assessment & Evaluation of Student Achievement
Communication skills might be assessed after having the students write a journal entry summarizing their findings. Informal assessment of Learning Skills should be ongoing. Teachers could assign and collect solutions to a problem that requires the use of the compound interest formula to determine whether students are able to apply their knowledge to problem solving. Solutions to question 5 could be completed in a variety of ways. Thinking/Inquiry and Problem Solving might be assessed here.

Activity 3.4: Applications: Finding the Amount and the Present Value of a Long Term Investment

Time: 75 minutes

Description
Students extend their knowledge of compound interest and present value in order to make informed decisions about investments. Students learn to use the Financial Applications feature on a graphing calculator as well as the spreadsheet capabilities of software programs.

Strand(s) & Learning Expectations
Strand(s): Financial Applications of Sequences and Series

Overall Expectations
FAV.02 - solve problems involving compound interest and annuities;
FAV.03 - solve problems involving financial decision making, using spreadsheets or other appropriate technology.

Specific Expectations
FA2.02 - solve problems involving compound interest and present value;
FA3.01 - analyse the effects of changing the conditions in long-term savings plans;
FA3.02 - describe the manner in which interest is calculated on a mortgage (i.e., compounded semi-annually but calculated monthly) and compare this with the method of interest compounded monthly and calculated monthly.

Prior Knowledge & Skills
Understanding of the compound interest formula

Planning Notes

- Students need access to a graphing calculator or spreadsheet software.
- Prepare a reference sheet that can be distributed to the students outlining keystrokes on the graphing calculator.
- In the example provided, the steps required on a TI-83+ graphing calculator are outlined. If other technology is used, the instructions will require adjustment.

Teaching/Learning Strategies

Students require the use of a graphing calculator to complete this activity as it is written. Teachers may wish to lead the students using the overhead calculator, if one is available, so that the students can see the screen that they should have on their own calculator.

Student Activity

In the previous activity you calculated the amount that Carolyn would accumulate if $1000 was deposited for 4 years at 4%/a compounded annually. In this activity you will learn how to use the "Finance" feature of the TI83+ graphing calculator to evaluate this amount and investigate other related problems.

1. Press [MODE], cursor down to FLOAT 0, 1, 2, etc. Select [2]. The calculator will now automatically round all values to two decimal places. All values will be rounded to the nearest cent.
2. Press [APPS], [ENTER] to select Finance, [ENTER] to select TVM Solver. On the screen you should see
   N= (number of interest payments)
   I%= (annual interest rate)
   PV= (present value of the investment or loan)
   PMT= (periodic payment)
   FV= (future value of the investment or loan)
   P/Y= (payments per year)
   C/Y= (compound periods per year)
   PMT: END BEGIN
   Press 4 [ENTER] to store 4 years. Press 4 [ENTER] to store 4%. Press –1000 [ENTER] to indicate a cash outflow of $1000. Press 0 [ENTER] to show a payment of 0 (Periodic payments will be considered in future sections. In this activity this value will always be 0.) Press 0 [ENTER] to show a future value of 0. Press 1 [ENTER] to show 1 payment per year. Press 1 [ENTER] to show 1 compound period per year. Select PMT: END. This indicates the payments are due at the end of each period. Cursor up to FV. Press [ALPHA] [Solve]. The value of the investment at the end of 4 years will appear.
3. Carolyn discovers she can invest the $1000 at 4%/a compounded semi-annually. Adjust the values to calculate how much she will have at the end of 4 years if she chooses this option.
   Cursor to N=. Press 8 [ENTER] to indicate 8 interest payments. Cursor to FV=. Press 0 [ENTER]. Cursor to P/Y=. Press 2 [ENTER] to indicate 2 payments per year. Cursor to C/Y=. Press 2 [ENTER] to indicate 2 compound period per year. Cursor to FV=. Press [ALPHA] [Solve]. The value of the investment after 4 years will appear.
4. Find the amount that will accumulate if Carolyn finds an investment with interest at 4%/a compounded monthly.
   Compare the following investments.
   $8000 is invested at 5.25%/a compounded semi-annually for 5 years.
   $8000 is invested at 5%/a compounded monthly for 5 years.
   Which is the better investment? Why?

5. Carolyn will need about $15 000 for her first year of university. (This includes tuition, books, residence and spending money.) How much would she have to invest now at 4%/a compounded monthly to ensure she has enough 4 years from now?
   Go to the Financial Applications program on the TI-83+ graphing calculator. Enter the following: N=48, I=4%, PV=0, PMT=0, FV=15000, P/Y=12, C/Y=12. Cursor to PV and press [ALPHA] [Solve]. Explain what each of the values entered means.

Follow up Questions
Provide the students with opportunities to practise skills with the finance program on the calculator. There will be many examples available in the students' text. Students should be encouraged to check their results (for a few examples) by using the compound interest formula.

Assessment & Evaluation of Student Achievement
The focus of assessment should be on the consolidation of skills needed in problem solving. Students should be able to solve problems with and without the use of technology. Unless the teacher specifically requests the use of the formula, either type of solution should be acceptable.

Resources
TI-83+ *Graphing Calculator Guidebook*, Chapter 14 – Applications, Section 14-4

Activity 3.5: Introduction to Geometric Series
Time: 75 minutes

Description
Students investigate a geometric series using an activity requiring financial decision making. The formula for the sum of a geometric series is developed.

Strand(s) & Learning Expectations
Strand(s): Financial Applications of Sequences and Series

Overall Expectations
FAV.01 - solve problems involving arithmetic and geometric sequences and series.

Specific Expectations
FA1.05 - determine the sum of the terms of an arithmetic or a geometric series, using appropriate formulas and techniques.

Prior Knowledge & Skills
General term of a geometric sequence

Planning Notes
Access to a graphing calculator is helpful, but not essential.

Teaching/Learning Strategies
Make an overhead that states the problem only. Have the students investigate the problem. The students could make a table of values for each situation and use a graphing calculator to find an equation that models the total amount he is paid. Before completing the activity, discuss the fact that the amounts Sasha receives each day, in the second scenario, are the terms of a geometric series.

Student Activity

Sasha's neighbours are planning on taking a vacation for two weeks (14 days). They have asked him to look after their cat and to water their plants. The neighbours have offered to pay him $5 per day or $0.01 the first day, $0.02 the second day, $0.04 the third day, $0.08 the fourth day, etc. Which method of payment should Sasha choose?

1. Sasha notices that the amounts he will earn each day if he chooses the second plan are the terms of the geometric sequence

   $0.01, 0.01(2)^2, 0.01(2)^3, \ldots, 0.01(2)^{13}$

   He uses the following method to calculate the amount that he will have at the end of two weeks.

   $S_{14} = 0.01 + 0.01(2) + 0.01(2)^2 + 0.01(2)^3 + \ldots + 0.01(2)^{13}$

   $2S_{14} = \quad 0.01(2) + 0.01(2)^2 + 0.01(2)^3 + \ldots + 0.01(2)^{13} + 0.01(2)^{14}$

   $2S_{14} - S_{14} = 0.01(2)^{14} - 0.01$

   $$S_{14} = \frac{0.01[(2)^{14} - 1]}{2 - 1}$$

   $= \$163.83$
2. Use this method to develop a formula to find the amount that Sasha will earn if he works any number of days.
3. If the neighbours are away for 7 days which method of payment should Sasha choose?
4. The sum of the terms of a geometric sequence is called a geometric series. Any geometric series can be written using the following form $S_n = a + ar + ar^2 + ar^3 \ldots + ar^{n-1}$

Use the method that Sasha used to develop the formula for the sum of the terms of a geometric series.

Teacher Facilitation

Ensure that the students have developed the correct formula for the sum of the geometric series. Assign additional questions using the formula. Make sure that some of the questions are related to financial applications.

Follow up Questions

1. Find the amount that Sasha would earn if his neighbours were away for a month. Discuss whether this method of payment is reasonable.
2. Write the geometric series using summation notation.

Assessment & Evaluation of Student Achievement

A short quiz could be used to assess knowledge and understanding of expectations related to geometric sequences and series. Communication could be assessed using question 1 from the activity.

Accommodations

Hand outs should be provided for students who have difficulty reading off of the overhead

Activity 3.6: Applications: Finding the Amount and Present Value of an Annuity

Time: 150 minutes

Description

Using the example of a student who is saving money earned from a paper route, students investigate the amount of an annuity to determine interest, number of payments and the total amount. In the second part of the activity students calculate the present value of an annuity given the scenario of a hockey player who would like to set up a scholarship fund.

Strand(s) & Learning Expectations

Strand(s): Financial Applications of Sequences and Series

Overall Expectations
FAV.02 - solve problems involving compound interest and annuities;
FAV.03 - solve problems involving financial decision making using spreadsheets or other appropriate technology.

Specific Expectations
FA2.02 - solve problems involving compound interest and present value;
FA2.03 - solve problems involving the amount and the present value of an ordinary annuity;
FA3.05 - communicate the solutions to problems and the findings of investigations with clarity and justification.

Prior Knowledge & Skills

- Working knowledge of geometric series
- Solving equations

Planning Notes

- Students will need access to a graphing calculator and/or spreadsheet software.
- Before beginning the activity discuss situations where students encounter periodic payments, both in light of amount accumulated and the amount needed to accumulate a specified sum (for example savings plans, scholarship funds, car loans)
- Students need to be shown how to set up the first line diagram. Terms of the series should be left in unsimplified form (i.e., $100 + 100\left(1+\frac{0.06}{12}\right)+100\left(1+\frac{0.06}{12}\right)^2+\ldots$).

Teaching/Learning Strategies

The night before the activity students could be directed to investigate in the media any situations where periodic payments are used. This could be used to initiate discussion before the student activity begins.

Student Activity 3.6.1: How Much Do We Have?
Time: 75 minutes
Part A
Jeremy is in Grade 8. He has a paper route and wants to save for his college education. He determines that he has $100 per month to put into an account at 6%/a compounded monthly. How much will he have at the end of five years for his college education?

1. Set up a line diagram showing the amount paid each month and the amount accumulated for each payment at the end of the time period.
2. Show that the amounts form a geometric series.
3. Calculate the sum of this geometric series using $S_n = \frac{a(r^n - 1)}{r - 1}$

Part B
Based on an average cost per year to attend a university or college of your choice, how much should be invested each month to obtain the goal of that amount? Assume 6%/a compounded monthly for 5 years.
Teacher Facilitation
Part A: Make sure the students' line diagrams show several payments and the value of these payments at the end of the annuity. This will be important for the next activity when the amount invested at the end of each interval will not be known. The generality of determining amounts using the S_n formula is important.

Part B: If the students have not had the time to investigate actual amounts, suggest they use $10 000 per year. The teacher might suggest looking at other scenarios. For example, the student may be able to obtain a better interest rate.

Follow-up Questions

1. Ask if the amount accumulated for Jeremy's college education is realistic. Investigate the costs of going to university or college for one year at various educational institutions.
2. Have the students fill in a table such as the following.

Amount	Payment	Number of Payments	Conversion Period	Interest Rate pa
?	$300	20	Semi-annually	8%
?	$500	24	Quarterly	12%
?	$100	36	Monthly	12%
?	$4000	10	Annually	8%
$10 000	?	40	Semi-annually	8%
$3000	?	24	Quarterly	12%
$5000	?	18	Monthly	12%
$12 000	?	15	Annually	6%

Student Activity 3.6.2: How Much Do We Need?

Time: 75 minutes

A retired hockey star wants to set up a scholarship fund to assist an underprivileged child who would like to go to a post-secondary institution. He wants to ensure that the student will have $6000 per year for five years. How much should he give to the institution, now, to ensure that this can happen, if the institution is able to invest the money at 10%/a compounded annually.

1. Set up a line diagram showing the present value of each of the $6000 payments.
2. Explain why the amounts form a geometric series.
3. Calculate the sum of the geometric series using the formula $S_n = \frac{a(r^n - 1)}{r - 1}$

Teacher Facilitation

Remind the students that they will need to find the present value of an amount when calculating the present value of each $6000 payment.

Follow-up Questions

Set up a table similar to the one in Activity 3.6.1 to find the present value of an annuity given the same kind of conditions illustrated in the table. Students should also complete questions that involve knowing the present value of the annuity and find the periodic payment.

Assessment & Evaluation of Student Achievement

Student work could be collected and assessed using an inquiry rubric. Students' solutions should be organized and written using proper mathematical form. This is especially important when setting up the line diagram. Teachers could informally (or formally) assess progress and provide feedback to students to assist them to improve their communication skills.

Activity 3.7: What Happens When?: Changing the Time, Rate and Amount

Time: 150 minutes

Description

Students will solve problems involving periodic payments. They will investigate the results of changing different parameters.

Strand(s) & Learning Expectations

Strand(s): Financial Applications of Sequences and Series

Overall Expectations

FAV.03 - solve problems involving financial decision making using spreadsheets or other appropriate technology.

Specific Expectations

FA3.01 - analyse the effects of changing the conditions in long-term savings plans;
FA3.05 - communicate the solutions to problems and the findings of investigations with clarity and justification.

Prior Knowledge & Skills

- Formula for the sum of a geometric series
- Solve a quadratic equation

Planning Notes

Students will require access to a graphing calculator or spreadsheet software.

Teaching/Learning Strategies

Due to the number of parameters involved in these problems, it would be appropriate to complete a number of examples with the students. It may be helpful to use the overhead view screen projector with the graphing calculator, if it is available. The intent of this activity is to provide the students with an efficient method of investigating the result of changing some of the conditions when investing or borrowing. Students should be aware of the mathematics involved from the previous activity, but should make use of available technology to perform the calculations.

Student Activity

Example 1

Grant and Kera are both 75 years old. Kera is very money conscious. She was 20 years old when she began investing $1000 a year into an RRSP paying an average of 6%/a compounded annually. Grant, on the other hand, did not start to invest until age 50. He made an annual deposit of $3000 beginning at age 50. The average interest rate he received on his investment was 8%/a compounded annually. What amount does each have today? What should Grant have invested each year in order to have the same amount as Kera at age 75? If Grant could only afford to invest $3000 per month, what average rate of interest would result in his saving the same amount as Kera?

Solution

To find the amount that Grant has, use the TI-83+ graphing calculator. Select [APPS], [1],[1] to choose the financial applications.
Enter the following
N=25, I=8%, PV=0, PMT=-3000 (this indicates a cash outflow of $3000 per year), FV=0, P/Y=1, C/Y=1, Select PMT:END. Cursor to FV, then push [ALPHA], [solve].
Now try the rest of the problem using the calculator.

Example 2
Stacey buys a $1000 RRSP today. After one year she adds $2500. By the end of the second year, the money has grown to $3851 as it has earned interest over time. What rate of interest does the money earn?

1. Apply the compound interest formula to both the $1000 and $2500 investments. Assume that interest is compounded annually at the rate, i, to grow to $3851 to get
   $1000(1 + i)^2 + 2500(1 + i) = 3851$
2. If this equation is written $1000x^2 + 2500x - 3851 = 0$, what does x represent.
3. Use the quadratic formula and your calculator to solve for x and i.
4. There are two solutions to the quadratic equation. Which one would you choose? Why?

Teacher Facilitation
There are many types of variables that can be adjusted in problems related to finance. Teachers should spend time discussing other scenarios and assign similar problems from other resources. In example 2, the interest rate for both investments is compounded annually, consider what will happen if the interest rate were compounded more frequently for one of the two.

Follow up Questions

1. Maria received $50 on her 16th birthday, and $70 on her 17th birthday, both of which she immediately invested in the bank with interest compounded annually. On her 18th birthday, she had $134.97 in her account. Draw a time line and calculate the annual interest rate.
2. Raul's grandparents invested $1000 in a GIC for him on his 15th birthday and $2000 on his sixteenth birthday. During each of the two years the money earned 8.5% compounded annually.
   a) Draw a time line to show the situation.
   b) How much was in the fund on his 17th birthday?

Assessment & Evaluation of Student Achievement
Provide the students with two or three problems. Student ability to read, interpret, and solve the problems could be assessed. The students might be asked to hand in their solutions. Since a range of performances is likely, a rubric could be used to assess Application to Problem Solving and Communication.

Resources
TI-83+ Graphing Calculator Guidebook – Chapter 14.

Activity 3.8: Mortgages: How They Work
Time: 150 minutes

Description
In this activity students will investigate various scenarios related to mortgages. They will compare methods of interest calculation, generate amortization tables for mortgages and analyse the effects of changing conditions of a mortgage.

Strand(s) & Learning Expectations
Strand(s): Financial Applications of Sequences and Series

Overall Expectations
FAV.03 - solve problems involving financial decision making, using spreadsheets or other appropriate technology.

Specific Expectations
FA3.02 - describe the manner in which interest is calculated on a mortgage (i.e., compounded semi-annually but calculated monthly) and compare this with the method of interest compounded monthly and calculated monthly;
FA3.03 - generate amortization tables for mortgages, using spreadsheets or other appropriate software;
FA3.04 - analyse the effects of changing the conditions of a mortgage (e.g., the effect on the length of time needed to pay off the mortgage of changing the payment frequency or the interest rate);
FA3.05 - communicate the solutions to problems and the findings of investigations with clarity and justification.

Prior Knowledge & Skills
Students should have acquired an understanding of annuities.

Planning Notes
Access to the TI-83+ calculators is required. If different technology is used, the instructions in this activity will have to be modified.

Teaching/Learning Strategies
Students could be asked to visit a local financial institution to determine the method of interest calculation on loans and mortgages. An alternative would be to invite a guest speaker from a local financial institution to discuss these issues with the class. Before beginning this activity, teachers should introduce students to some of the language associated with mortgages. It is important for students to be comfortable using the financial applications of a graphing calculator or spreadsheet software. Teachers may choose to have the students use a computer spreadsheet to generate the amortization schedules.

Student Activity
Mortgages are a special type of annuity. They are also a fact of life for most Canadian families. When applying for a mortgage through a financial institution, it is important to know how much your family can afford to pay on a monthly basis, the term of the amortization, and the interest rate charged by the financial institution. The TI-83+ calculator has a software package that will help determine the value of some of these parameters.
Ron and Lynda are purchasing a new home. They will need to borrow $90 000 for the home. The amortization period, which best fits their financial situation is 25 years. Discuss why this might be so. The current interest rate is 8.7%/a. Interest is compounded semi-annually and calculated monthly. The couple wants to know how much it will cost them each month to pay off the loan. Assume that they will make one payment per month.
To solve this problem, select [APPS], [1], [1] (this selects the TVM solver). Enter, N=25x12, I=8.7, PV=90 000, PMT=0, FV=0, P/Y=12, C/Y=2, select PMT:END. Cursor to PMT, press [ALPHA], [solve]. This finds the payment of $727.52. (It will appear as a negative because this is a cash outflow.)

1. Create a schedule of payments for the first 5 years of the mortgage. Include a graph showing the declining balance at the end of each year.
2. After 5 years, the interest rate is reduced to 6.7%/a compounded semi-annually and calculated monthly. Find the amount of the monthly payments. (Remember that the new present value will be the amount owing at the end of the first five-year period.)
3. Create a new schedule of payments for 5 years.

Teacher Facilitation
Teachers may wish to prepare a handout, which covers the menu and the steps required to compute the amortization schedule on the TI-83+ calculator.

Assessment & Evaluation of Student Achievement
Learning skills and the ability to read and interpret problems might be informally assessed. Students could be asked to write a report on information obtained from visiting financial institutions or from listening to a guest speaker. This report could be assessed for Communication skills. The amortization tables could be handed in and assessed for Application and Knowledge/Understanding.

Activity 3.9: Financial Decision Making: A Case Study
Time: 300 minutes

Description
Students are presented with a realistic situation involving financial decision making. They analyse the situation and suggest financial decisions to be made. Students are required to write a report justifying their decisions using the mathematics learned in this unit.

Strand(s) & Learning Expectations
Strand(s): Financial Applications of Sequences and Series

Overall Expectations
FAV.01 - solve problems involving arithmetic and geometric sequences and series;
FAV.02 - solve problems involving compound interest and annuities;
FAV.03 - solve problems involving financial decision making, using spreadsheets or other appropriate technology.

Specific Expectations
It is expected that as many specific expectations as possible in the Financial Applications of Sequences and Series strand will be addressed in this summative assessment activity.

Planning Notes
Teachers may choose to review some of the major concepts discussed in the previous activities (e.g., computation of interest, annuities).

Teaching/Learning Strategies
Conduct a group discussion to enable students to generate an expense report for a typical month in the life of a family of four (two adults, two children in elementary school). Include in the report all expenses related to the management of a home. Emphasize in the student activity that a full and detailed report on the Johnston financial situation must be made, including calculations, all expenses and rationale. Communication skills should be emphasized.

Student Activity

Part 1
The Johnstons are married with two children, Josh, age 13 and Judy, age10. They have a 25-year mortgage of $75 000 at 8%/a compounded semi-annually and calculated monthly; a car loan of $18 000 taken out for five years at 10%/a compounded monthly. They also have the normal expenses of running a house (e.g., utilities, taxes, groceries, etc.). The Johnstons have a single income of $45000 per year and are contemplating having both parents return to work now that the children are both in school. Analyse the Johnstons' financial situation and decide if this is necessary. Justify your answers.
After 5 years, the mortgage must be renewed with an interest rate of 8.5%/a compounded semi-annually and calculated monthly. How will this effect their monthly expenses?

Part 2
Write a short report about long-term financial and educational planning. What have you learned? How has this information changed the way you view a mortgage? How could this influence your financial choices? You may wish to comment on the feelings that other people might have, or what they might know or not know about this matter. You may want to describe your own personal long-term plans in this area.

Assessment & Evaluation of Student Achievement

All categories of the Achievement Chart should be assessed in this summative assessment activity. Since a range of performances is likely, a rubric might be used. Teachers might choose to use a pencil-and-paper test to assess Knowledge/Understanding of expectations in the unit that are not specifically addressed in this activity.

Suggested Rubric to be used with Activity 3.9

Categories	Level 1 (50-59%)	Level 2 (60-69%)	Level 3 (70-79%)	Level 4 (80-100%)
Thinking/Inquiry and **Problem Solving** - analyses the effects of changing variables in financial applications	- takes into account only a limited number of factors in analysing the problem - is unable to devise a reasonable financial plan	- takes into account some of the factors in analysing the problem - offers a solution which is somewhat justified by the facts given	- takes into account most of the factors in analysing the problem - makes a reasonable suggestion based on the facts given	- takes into account all, or almost all, of the factors in analysing the problem - identifies more than one option based on the facts given
Communication - communicates with clarity and justification	- communicates with limited clarity and limited justification	- communicates with some clarity and some justification	- communicates with general clarity and justification	- communicates with a high degree of clarity and justification

Note: A student whose achievement is below level 1 (50%) has not met the expectations for this assignment of activity

Accommodations

Allow extra time for those students who need it.

Resources

Banks and trust companies
The personal financial planner at a local financial institution