Public and Catholic District School Board Writing Partnerships

Mathematics

Course Profile

Mathematics for College Technology

Grade 12
College Preparation
MCT4C

• *for teachers by teachers*

This sample course of study was prepared for teachers to use in meeting local classroom needs, as appropriate. This is not a mandated approach to the teaching of the course. It may be used in its entirety, in part, or adapted.

Spring 2002

Course Profiles are professional development materials designed to help teachers implement the new Grade 12 secondary school curriculum. These materials were created by writing partnerships of school boards and subject associations. The development of these resources was funded by the Ontario Ministry of Education. This document reflects the views of the developers and not necessarily those of the Ministry. Permission is given to reproduce these materials for any purpose except profit. Teachers are also encouraged to amend, revise, edit, cut, paste, and otherwise adapt this material for educational purposes.

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Course Overview
Mathematics for College Technology, MCT4C, Grade 12, College Preparation

Policy Document: *The Ontario Curriculum, Grades 11 and 12, Mathematics, 2000.*
Prerequisite: Functions, Grade 11, University/College Preparation, or
Functions and Relations, Grade 11, University Preparation

Course Description

This course equips students with the mathematical knowledge and skills needed for entry into college technology programs. Students will investigate and apply properties of polynomial, exponential, and logarithmic functions; solve problems involving inverse proportionality; and explore the properties of reciprocal functions. They will also analyse models of a variety of functions, solve problems involving piecewise-defined functions, solve linear-quadratic systems, and consolidate key manipulation and communication skills.

Students entering mathematics-focused programs at the college level benefit from MCT4C. This course enables students to consolidate and expand many pre-calculus concepts explored in previous mathematics courses. Contextual applications and technological tools are integrated throughout to support the development of new skills and the exploration of a variety of mathematical models.

How This Course Supports the Ontario Catholic School Graduate Expectations

Through the investigation and exploration of various mathematical models, students are challenged to discover a deeper understanding and appreciation of the mathematical nature of the world in which we live. These underlying mathematical principles provide the student another academic or intellectual lens for addressing possible problems and investigating potential solutions to situations around them. Students should have the opportunity to communicate these discoveries to others within a classroom environment that emphasizes collaboration rather than competition.

Course Notes

Making a Connection to Students' Career Paths

Providing a real-life context for the student in this course is important. Reference to students' Annual Education Plans (AEPs) assists the teacher in making possible connections with postsecondary programs, while aiding in the transition to students' new programs of study. Making this connection between career paths and curriculum also helps foster student interest.

Using Technology as a Tool for Learning

Students may use technology in discovering relationships and behaviours. Technology use should enhance and extend the understanding and communication of fundamental concepts presented in this course. Appropriate technology includes, but is not limited to scientific calculators, graphing calculators, and spreadsheet/graphing software.

A Focus on Mathematical Models

Mathematical models provide a realistic context for many of the concepts explored in this course. By the creation and examination of these models, students achieve an underlying understanding of the problem while working towards potential solutions. A focus on mathematical modelling provides the student with an opportunity to frame his/her skill development with potential applications to real-life problems.

Course Development

The organization of this course follows a logical progression of concepts. In Unit 1, students study the behaviour of polynomial functions of various degrees through the investigation of their graphs on the Cartesian plane. Unit 2 further explores polynomial graphs and equips the student with the algebraic skills required to understand and to manipulate polynomial functions. Unit 3 focuses on mathematical modelling and problem solving with both inverse and reciprocal functions. Unit 4 examines exponential and logarithmic functions and relates these functions to a variety of contexts. In Unit 5, students continue to investigate special functions classified as piecewise defined functions. Unit 6 focuses on linear and quadratic systems and extends students' knowledge of trigonometry. Unit 7 is a summative unit that allows students to demonstrate and apply their acquired skills and concepts of function behaviour.

Units: Titles and Time

Unit 1	Key Features of Polynomial Functions	17 hours
Unit 2	Exploring Polynomial Functions: Connecting Algebra and Geometry	18 hours
* Unit 3	Exploring Reciprocal and Inverse Functions	15 hours
* Unit 4	Exponential and Logarithmic Functions	20 hours
Unit 5	Piecing It Together	16 hours
Unit 6	Linear Functions, Quadratic Functions, and Trigonometry	16 hours
Unit 7	Mathematical Modelling – Summative Assessment	8 hours

* These units are fully developed in this Course Profile.

Any additional time can be allocated for remediation and consolidation of skills at the discretion of the teacher, depending on the needs of students.

Unit Overviews

Unit 1: Key Features of Polynomial Functions

Time: 17 hours

Unit Description

Students are introduced to the main concepts of graphing polynomial functions in order to explore them later in the course. Students examine the type and number of intercepts, the effects of changing numerical coefficients, the existence of symmetry, and the degree in relation to the shape of the function. Using skills from previous years, students explore curve sketching from a factored form.

Unit Overview Chart

Cluster	Learning Expectations	Assessment Categories	Focus
1	PFV.01, PFV.02, PF1.01, PF1.03, PF2.02 CGE2b, 5b	Knowledge/Understanding Thinking/Inquiry/Problem Solving Communication	Identifying properties of polynomial functions using technology
2	PFV.01, PF1.02, PF1.04 CGE2d	Knowledge/Understanding Application Communication	Using table of values to assist in the construction of graphs Using finite differences to describe the nature of function change

Cluster	Learning Expectations	Assessment Categories	Focus
3	PFV.01, ACV.04, PF1.05, AC4.01 CGE2e, 5b	Knowledge/Understanding Thinking/Inquiry/Problem Solving Application	Determining the equation of various graphs
4	PFV.01, PFV.02, PF1.01, PF2.08 CGE3c	Knowledge/Understanding Application Communication	Solving non-factorable polynomial inequalities using technology

Unit 2: Exploring Polynomial Functions: Connecting Algebra and Geometry

Time: 18 hours

Unit Description

Students explore polynomial equations and inequalities. Real and complex roots of both factorable and non-factorable polynomials are determined through graphical investigations and algebraic manipulation. Students use the remainder theorem, the factor theorem, and absolute-value notation.

Unit Overview Chart

Cluster	Learning Expectations	Assessment Categories	Focus
1	PFV.01, PFV.02, ACV.04, PF1.04, PF2.01, PF2.02, AC4.01 CGE2b, 2c	Knowledge/Understanding Thinking/Inquiry/Problem Solving Communication	Investigating factors and graphs of polynomials
2	PFV.02, ACV.04, PF2.02, PF2.03, PF2.05, AC4.01, AC4.04 CGE2d, 3c	Knowledge/Understanding Thinking/Inquiry/Problem Solving Application	Exploring real and complex roots
3	PFV.02, PF2.04 CGE2e, 4f	Thinking/Inquiry/Problem Solving Communication	Solving non-factorable polynomial equations using graphing technology
4	PFV.02, PF2.02, PF2.07, PF2.08 CGE2b, 2c, 2e	Knowledge/Understanding Thinking/Inquiry/Problem Solving Application	Solving inequalities algebraically and using graphing technology
5	PFV.02, PF2.06 CGE2b, 2c	Knowledge/Understanding Communication Application	Applying absolute value

Unit 3: Exploring Reciprocal and Inverse Functions

Time: 15 hours

Unit Description

Students explore the behaviour of reciprocal and inverse functions. Students apply their acquired knowledge to create mathematical models derived from realistic inverse-proportional relationships. Using extrapolation, students predict future behaviour and pose questions related to the generated models of these functions. Key features of linear and quadratic reciprocal functions are investigated through curve sketching. Technological aids, such as a graphing calculator, assist students in understanding key concepts of reciprocal and inverse functions.

Unit Overview Chart

Cluster	Learning Expectations	Assessment Categories	Focus
1	ACV.01, ACV.04, PFV.03, AC1.01, AC1.02, AC4.02, PF3.01 CGE2b, 3c, 3f	Knowledge/Understanding Application Thinking/Inquiry/Problem Solving	Exploring inverse proportionality
2	ACV.01, PFV.03, AC1.03, AC1.04, PF3.01, PF3.02 CGE2a, 2d, 2e	Communication Application Thinking/Inquiry/Problem Solving	Problem solving inverse proportionality with technology
3	PFV.04, PF4.01 CGE2c, 5g	Knowledge/Understanding Communication Thinking/Inquiry/Problem Solving	Sketching reciprocal functions
4	PFV.04, PF4.02, PF4.03 CGE2e, 5a	Communication Thinking/Inquiry/Problem Solving	Investigating properties of reciprocal functions with technology
5	ACV.01, PFV.03, AC1.01, AC1.02, AC1.03, PF3.01, PF3.02 CGE3c, 5a, 5g, 7b	Knowledge/Understanding Thinking/Inquiry/Problem Solving Application Communication	Summative assessment

Unit 4: Exponential and Logarithmic Functions

Time: 20 hours

Unit Description

Students investigate properties of exponential and logarithmic functions. The relationship between exponential and logarithmic functions is explored both graphically and algebraically. Students use the laws of logarithms to simplify and evaluate logarithmic expressions, and to solve problems. A variety of exponential and logarithmic applications and models is examined.

Unit Overview Chart

Cluster	Learning Expectations	Assessment Categories	Focus
1	ELV.01, EL1.01, EL1.02, EL1.03 CGE2a, 2d, 5a	Knowledge/Understanding Thinking/Inquiry/Problem Solving Communication	Investigating properties of exponential functions
2	ELV.01, ACV.01, EL1.04, EL1.05, AC1.01, AC1.02, AC1.03, AC1.04 CGE3c, 5g	Thinking/Inquiry/Problem Solving Application Communication	Modelling exponential growth and decay
3	ELV.01, ACV.04, EL1.05, AC4.04 CGE2b, 2c, 7b	Knowledge/Understanding Thinking/Inquiry/Problem Solving Application Communication	Applying exponential growth and decay
4	ELV.01, ELV.02, ACV.04, EL1.01, EL2.01, EL2.02, AC4.04 CGE2c, 2d	Knowledge/Understanding Communication	Determining connections between exponential functions and logarithmic functions

Cluster	Learning Expectations	Assessment Categories	Focus
5	ELV.02, ACV.01, EL2.04, AC1.01, AC1.02, AC1.03, AC1.04 CGE3c, 5g	Thinking/Inquiry/Problem Solving Communication	Modelling logarithmic functions
6	ELV.02, ACV.04, EL2.03, EL2.04, AC4.04 CGE2b, 2c	Knowledge/Understanding Thinking/Inquiry/Problem Solving Application Communication	Exploring laws of logarithms and applications of logarithmic scales
7	ELV.01, ELV.02, ACV.04, EL1.04, EL1.05, EL2.02, EL2.03, EL2.04, AC4.04 CGE2b, 2c, 5g, 7b	Knowledge/Understanding Thinking/Inquiry/Problem Solving Application Communication	Summative assessment

Unit 5: Piecing It Together

Time: 16 hours

Unit Description

Students identify and interpret different piecewise functions using paper and pencil, a graphing calculator, and graphing software. Piecewise functions are applied in a variety of contexts. Students analyse the key properties that set these functions apart from exponential, trigonometric, and quadratic functions.

Unit Overview Chart

Cluster	Learning Expectations	Assessment Categories	Focus
1	ACV.01, ACV.02, ACV.04, AC1.01, AC2.01, AC4.02 CGE2a, 2b	Knowledge/Understanding	Recognizing piecewise functions
2	ACV.02, AC2.02, AC2.03, AC2.04, AC2.05 CGE2c, 5g	Application Thinking/Inquiry/Problem Solving	Graphing piecewise functions
3	ACV.01, ACV.02, AC1.03, AC1.04, AC2.04 CGE3c, 5b	Thinking/Inquiry/Problem Solving Communication	Analysing and interpreting individual models of piecewise functions

Unit 6: Linear Functions, Quadratic Functions, and Trigonometry

Time: 16 hours

Unit Description

Students investigate properties of linear functions and quadratic functions. Students connect elements of a contextual problem to elements of the function representing the problem. Linear-quadratic systems are solved graphically and algebraically, and students interpret their solutions contextually. Students extend and apply their knowledge of trigonometry to solve problems involving right triangles and oblique triangles.

Unit Overview Chart

Cluster	Learning Expectations	Assessment Categories	Focus
1	ACV.03, ACV.04, AC3.01, AC4.04 CGE2b, 2c	Knowledge/Understanding Application	Determining the relationship between properties of linear functions and contextual examples
2	ACV.03, ACV.04, AC3.01, AC4.04 CGE2c, 2d	Knowledge/Understanding Application	Determining the relationship between properties of quadratic functions and contextual examples
3	ACV.03, ACV.04, AC3.02, AC3.03, AC4.04 CGE2e, 3c, 5g	Thinking/Inquiry/Problem Solving Application Communication	Solving linear-quadratic systems graphically and within a variety of contexts
4	ACV.04, AC4.03 CGE2b, 3c	Knowledge/Understanding Application Communication	Using trigonometry to solve problems involving right triangles and oblique triangles

Unit 7: Mathematical Modelling

Time: 8 hours

Ontario Catholic School Graduate Expectations: CGE 2c, 2e, 3c, 5b, 5g.

Unit Description

Students use their acquired knowledge to mathematically model real-world situations using the various functions explored in this course. Previous knowledge of the key properties that differentiate polynomial, inverse proportional, reciprocal, exponential, logarithmic, and piecewise functions assists students in deciding the function type that best approximates the situation under a given set of circumstances. The use of technology, such as a graphing calculator, assists students in their exploration of potential solutions.

Unit Overview Chart

Cluster	Learning Expectations	Assessment Categories	Focus
1	All Strands	Knowledge/Understanding Application Thinking/Inquiry/Problem Solving Communication	Using key properties of polynomial functions, inverse proportionality, reciprocal functions, exponential functions, logarithmic functions, and piecewise functions to mathematically model real-world situations

Teaching/Learning Strategies

Opportunities to apply a variety of both teaching and learning strategies are provided throughout the course. In applying the strategies, the teacher:

- provides regular, constructive feedback;
- integrates technological tools and software where appropriate;
- demonstrates the use of any technological tools or software used in the classroom;
- uses current and local information in contextual questions to promote relevance;
- teaches skills within a context as often as possible;
- uses a balance of whole-class, small-group, and individual instruction through student-centred and teacher-directed activities;
- uses a variety of instructional methods to address a variety of learning styles;
- uses positive reinforcement to foster a positive learning environment;
- provides opportunities for students to present mathematical results in a variety of different presentation formats;
- provides extension opportunities;
- provides review and remediation where appropriate.

In achieving the expectations of this course, students:

- investigate and explore concepts using technology;
- increase their proficiency with technology;
- demonstrate their knowledge and understanding using a variety of methods and mathematical/technological tools;
- communicate their understanding using a variety of mediums;
- summarize and support decisions using a variety of strategies;
- apply and develop individual and group learning skills;
- work individually and cooperatively;
- further develop their problem-solving strategies;
- develop responsibility for their own learning and decision-making.

Assessment & Evaluation of Student Achievement

Assessment is a systematic process of collecting information or evidence about student learning.

Assessment is defined in *Ontario Secondary Schools, Grades 9-12, Program and Diploma Requirements, 1999,* as "the process of gathering information from a variety of sources (including assignments, demonstrations, projects, performances, and tests) that accurately reflects how well students are achieving the curriculum expectations" (p. 31). Assessment is used for diagnostic, formative, and summative purposes.

Evaluation requires that the teacher not simply average marks. Evaluation is defined by *Ontario Secondary Schools, Grades 9-12, Program and Diploma Requirements, 1999,* as "the process of judging the quality of a student's work on the basis of established achievement criteria, and assigning a value to represent that quality. As part of assessment, teachers provide students with descriptive feedback that guides their efforts towards improvement" (p. 31). In forming an evaluative judgement, the teacher considers the student's performance in the four categories of the Achievement Chart.

The purpose of assessment and evaluation is to improve student learning. Assessment and evaluation strategies and tools must address the variety of learning styles and needs. A balanced assessment and evaluation program is based on the provincial curriculum expectations and the Achievement Chart levels.

The following can serve as a guide for assessing student achievement.

Knowledge/Understanding

Achievement in this category reflects the student's ability to demonstrate understanding of mathematical concepts and to perform algorithms. The teacher assesses:

- quizzes;
- short-answer and skill-based calculations on unit tests and exams;
- student-teacher conferencing;
- accuracy of mathematical answers in reports and presentations.

Thinking/Inquiry/Problem-Solving

Achievement in this category reflects the student's ability to demonstrate reasoning and to effectively apply the steps of an inquiry/problem-solving process. Rubrics may be used due to the open-ended nature of many of the problems. The teacher assesses:

- broad-based, open-ended problems on assessment tasks;
- rich assessment tasks and assignments;
- problem-solving strategies used in group work, through observation;
- student-teacher conferencing;
- tasks requiring mathematical reasoning.

Application

Achievement in this category reflects the student's ability to apply concepts and procedures to familiar and unfamiliar settings. The teacher assesses:

- appropriate application of technological tools;
- rich problems in unit tests and tasks;
- application of mathematical knowledge and understanding in reports and presentations;
- investigations in examinations.

Communication

Achievement in this category reflects the student's ability to communicate his/her reasoning using mathematical language, symbols, and conventions. Rubrics are effective, efficient tools for evaluating presentations and displays. The teacher assesses:

- verbal presentation of homework solutions;
- appropriate use of mathematical language and terminology on tests and assignments;
- visual aids during presentations;
- clarity of written expression in solutions;
- student interaction during group work, through observation;
- clarity of mathematical reasoning in reports and presentations;
- mathematical conventions on all written work.

Evaluation Notes

Seventy per cent of the grade will be based on assessments and evaluations conducted throughout the course. Thirty per cent of the grade will be based on a final evaluation in the form of an examination, performance, essay, and/or other methods of evaluation.

Learning Skills

Learning skills are not to be included in the determination of a student's percentage grade. Learning skills are assessed and reported separately from the student's percentage grade. Students should receive ongoing feedback concerning their demonstration of learning skills; therefore, learning skills should be tracked throughout the term. See Appendix A for examples of learning skill indicators.

Assessment Strategies

Though assessment strategies are listed for each activity, it is not intended that they all be used for summative purposes. Teachers use some of the strategies for formative purposes in order to build capacity and confidence in students in preparation for the unit summative assessment and evaluation. References to assessment of Learning Skills assume the understanding that these do not contribute to the final mark but may provide data for the Learning Skills section of the report card.

Accommodations

Teachers should consult individual student IEPs for specific direction on accommodation for individuals. Teachers work in consultation with resource teachers, where available, and parents/guardians to determine appropriate accommodations as students work to achieve the expectations in their IEPs.

Learning Disabilities

- Provide students with an overview of activities to anticipate issues that may arise.
- Assist with lesson-specific terminology.
- Modify handouts in terms of terminology, content, and font size. Allow plenty of space for written responses.
- Allow assignments to be completed in alternate formats or in longer timelines.
- Allow students to work in alternative settings.
- Provide a list of terms (possibly simplified) before an activity begins.
- Provide manipulatives, grid paper, formula sheets, and other aids.

ESL/ELD

- Review questions, assignments, or assessment instruments for language level.
- Pair written instructions with verbal instructions.
- Provide visual or auditory cues.
- Provide opportunities for students to practise oral presentations skills in low-risk settings.
- Use visuals to illustrate definitions.
- Simplify instructions and highlight key words and phrases.
- Have students work in pairs, with peer tutors, with classmates who share the same linguistic background, or in co-operative, supportive groups.
- Use peer conferencing to reinforce instructions or information.
- Reinforce main ideas by using the think/pair/share strategy.
- Brainstorm in groups using the student's first language if their usage of English is limited.
- Participate in ongoing student-teacher conferencing.
- Provide sets of reference notes; outlines of critical information; and models of charts, timelines, and diagrams.

Resources

Units in this Course Profile make reference to the use of specific texts, magazines, films, videos, and websites. The teachers need to consult their board policies regarding use of any copyrighted materials. Before reproducing materials for student use from printed publications, teachers need to ensure that their board has a Cancopy licence and that this licence covers the resources they wish to use. Before screening videos/films with their students, teachers need to ensure that their board/school has obtained the appropriate public performance videocassette licence from an authorized distributor, e.g., Audio Cine Films Inc. The teachers are reminded that much of the material on the Internet is protected by copyright. The copyright is usually owned by the person or organization that created the work. Reproduction of any work or substantial part of any work from the Internet is not allowed without the permission of the owner.

Software (Ministry-Licensed)

Geometer's Sketchpad (dynamic geometry)

Maple (word processor/programming)

Math Trek (concept and skill development)

Virtual Tiles (algebraic concept and skill development)

Zap-a-Graph (graphing)

Websites

The URLs for the websites were verified by the writers prior to publication. Given the frequency with which these designations change, teachers should always verify the websites prior to assigning them for student use.

Education Network of Ontario – www.enoreo.on.ca/

Hewlett-Packard – www.hp.com/calculators/

Internet Public Library – www.ipl.org

Math Forum – http://forum.swarthmore.edu

National Council of Teachers of Mathematics – www.nctm.org

Ontario Association of Mathematics Educators – www.oame.on.ca

Texas Instruments – www.ti.com/calc/docs

Books

Brueningsen, C., et al. *Real-World Math with the CBL System – 25 Activities Using the CBL and TI-82*. Texas Instruments, 1994.

Brueningsen, C., et al. *Real-World Math with the CBL System – Activities for the TI-83 and TI-83 Plus*. Texas Instruments, 1994.

Bush, W.S. and A.S. Greer, eds. *Mathematics Assessment – A Practical Handbook for Grades 9-12*. Retson, VA: The National Council of Teachers of Mathematics, 1999.

Garland, T. and C. Kahn. *Math and Music – Harmonious Connections*. Dale Seymour Publications, 1995.

Gregory, K., C. Cameron, and A. Davies. *Knowing What Counts: Setting and Using Criteria*. Meriville, BD: Connections Publishing, 1999.

High School Assessment: Balanced Assessment for the Mathematics Curriculum, Package 1. Dale Seymour Publications, 2000.

High School Assessment: Balanced Assessment for the Mathematics Curriculum, Package 2. Dale Seymour Publications, 2000.

National Council of Teachers of Mathematics. *Assessment Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics, 1997.

O'Connor, K. *The Mindful School: How to Grade for Learning*. Palatine, IL: Skylight Training and Publishing Inc., 1998.

O.S.S.T.F. *Quality Assessment*. Toronto: Educational Services Committee, 1999.

Rogers, S. and S. Graham. *The High Performance Toolbox*. Evergreen, CO: Peak Learning Systems, 1997.

Stiggins, R. *Classroom Assessment for Student Success*. Washington, DC: National Education Association of the United States, 1998

Stiggins, R. *Student-Centered Classroom Assessment, 2nd ed*. Columbus OH: MacMillan, 1997.

Taggart, G., ed. *Rubrics – A Handbook for Construction and Use*. Lancaster, PA: Techonomic Publishing, 1998.

The Ministry of Education has also published several resource documents, brochures, and policy/program memoranda in support of its OSS policies. They are available online at the Ministry of Education website (http://www.edu.gov.on.ca/eng/document/document.html)

Publications Concerning Faith Development:

- Catholic Curriculum Cooperative (Central Ontario Region). *Blueprints.*
- Ontario Catholic School Trustees' Association. *Catholicity Across the Curriculum*.
- Institute for Catholic Education. *Educating the Soul.*
- Institute for Catholic Education. *Ontario Catholic Secondary School Graduate Expectations*.
- Ontario Conference of Catholic Bishops. *This Moment of Promise*.

Career Goals/Cooperative Education Programs:

- Ontario Youth Apprenticeship Program
- Youth Employment Skills Program

Community Partnerships:

- Refer to local board policies, e.g., Relations with Business – Corporate Donations, Sponsorships, and Agreements.

OSS Considerations

The following resources support many of the Ontario Secondary School policies, as well as the Ontario Catholic School Graduate Expectations.

Ministry of Education Policy and Reference Documents:

Choices Into Action: Guidance and Career Education Program Policy, 2000.

Cooperative Education: Policies and Procedures for Ontario Secondary Schools, 2000.

Individual Education Plans: Standards for Development, Program Planning, and Implementation, 2000.

The Ontario Curriculum, Grades 9-10, Mathematics, 1999.

The Ontario Curriculum, Grades 11-12, Mathematics, 2000.

Ontario Schools Code of Conduct.

Ontario Secondary Schools, Grades 9-12, Program and Diploma Requirements, 1999.

Program Planning and Assessment, Grades 9-12, 2000.

Violence-Free Schools Policy.

Appendix A

Indicators of Learning Skills

Organization

The student:
- uses a planning process;
- brings the required materials to class;
- shows organization in his/her notebook;
- uses appropriate resources.

Work Habits

The student:
- completes class work and homework;
- works with attention to detail;
- shows thought and revision in written work;
- reviews and studies appropriately;
- follows instructions of assigned work;
- uses class time effectively and submits work on time.

Teamwork

The student:
- listens actively;
- shows respect for all group members;
- completes the appropriate portion of the group's work;
- co-operates to complete the task;
- uses conflict-management skills;
- adopts a variety of roles in group-work settings;
- shares ideas;
- works constructively towards group goals.

Initiative

The student:
- actively and constructively participates in class discussions;
- takes responsibility for his/her own learning;
- demonstrates classroom leadership;
- acts to solve problems;
- tries new techniques or approaches to learning;
- reflects on his/her own progress and adapts strategies;
- shows interest in new learning.

Works Independently

The student:
- demonstrates commitment to the task;
- uses a variety of problem-solving strategies;
- accepts responsibility for his/her own behaviour;
- plans and executes tasks with minimal teacher assistance.

Coded Expectations, Mathematics for College Technology, Grade 12, College Preparation, MCT4C

Polynomial Functions and Inverse Proportionality

Overall Expectations

PFV.01 · determine, through investigation, the characteristics of the graphs of polynomial functions of various degrees;
PFV.02 · demonstrate facility in the algebraic manipulation of polynomials;
PFV.03 · demonstrate an understanding of inverse proportionality;
PFV.04 · determine, through investigation, the key properties of reciprocal functions.

Specific Expectations

Investigating the Graphs of Polynomial Functions

PF1.01 – determine, through investigation, using graphing calculators or graphing software, various properties of the graphs of polynomial functions (e.g., determine the effect of the degree of a polynomial function on the shape of its graph; the effect of varying the coefficients in the polynomial function; the type and the number of x-intercepts; the behaviour near the x-intercepts; the end behaviours; the existence of symmetry);
PF1.02 – describe the nature of change in polynomial functions of degree greater than two, using finite differences in tables of values;
PF1.03 – compare the nature of change observed in polynomial functions of higher degree with that observed in linear and quadratic functions;
PF1.04 – sketch the graph of a polynomial function whose equation is given in factored form;
PF1.05 – determine an equation to represent a given graph of a polynomial function, using methods appropriate to the situation (e.g., using the zeros of the function; using a trial-and-error process on a graphing calculator or graphing software; using finite differences).

Manipulating Algebraic Expressions

PF2.01 – demonstrate an understanding of the remainder theorem and the factor theorem;
PF2.02 – factor polynomial expressions of degree greater than two, using the factor theorem;
PF2.03 – determine, by factoring, the real or complex roots of polynomial equations of degree greater than two;
PF2.04 – determine the real roots of non-factorable polynomial equations by interpreting the graphs of the corresponding functions, using graphing calculators or graphing software;
PF2.05 – write the equation of a family of polynomial functions, given the real or complex zeros [e.g., a polynomial function having non-repeated zeros 5, –3, and –2 will be defined by the equation $f(x) = (x - 5)(x + 3)(x + 2)$, for $k \in \mathbf{R}$;
PF2.06 – describe intervals and distances, using absolute-value notation;
PF2.07 – solve factorable polynomial inequalities;
PF2.08 – solve non-factorable polynomial inequalities by graphing the corresponding functions, using graphing calculators or graphing software and identifying intervals above and below the x-axis.

Understanding Inverse Proportionality

PF3.01 – construct tables of values, graphs, and formulas to represent functions of inverse proportionality derived from descriptions of realistic situations (e.g., the time taken to complete a job varies inversely as the number of workers; the intensity of light radiating equally in all directions from a source varies inversely as the square of the distance between the source and the observer);
PF3.02 – solve problems involving relationships of inverse proportionality.

Determining the Key Properties of Reciprocal Functions

PF4.01 – sketch the graph of the reciprocal of a given linear or quadratic function by considering the implications of the key features of the original function as predicted from its equation (e.g., such features as the domain, the range, the intervals where the function is positive or negative, the intervals where the function is increasing or decreasing, the zeros of the function);

PF4.02 – describe the behaviour of a graph near a vertical asymptote;

PF4.03 – identify the horizontal asymptote of the graph of a reciprocal function by examining the patterns in the values of the given function.

Exponential and Logarithmic Functions

Overall Expectations

ELV.01 · demonstrate an understanding of the nature of exponential growth and decay;

ELV.02 · define and apply logarithmic functions.

Specific Expectations

Understanding the Nature of Exponential Growth and Decay

EL1.01 – identify, through investigations, using graphing calculators or graphing software, the key properties of exponential functions of the form a^x ($a > 0$, $a \neq 1$) and their graphs (e.g., the domain is the set of the real numbers; the range is the set of the positive real numbers; the function either increases or decreases throughout its domain; the graph has the x-axis as an asymptote and has y-intercept = 1);

EL1.02 – describe the graphical implications of changes in the parameters a, b, and c in the equation $y = ca^x + b$;

EL1.03 – compare the rates of change of the graphs of exponential and non-exponential functions (e.g., those with equations $y = 2x$, $y = x^2$, $y = x^{\frac{1}{2}}$, and $y = 2^x$);

EL1.04 – describe the significance of exponential growth or decay within the context of applications represented by various mathematical models (e.g., tables of values, graphs, equations);

EL1.05 – pose and solve problems related to models of exponential functions drawn from a variety of applications, and communicate the solutions with clarity and justification.

Defining and Applying Logarithmic Functions

EL2.01 – define the logarithmic function $\log_a x$ ($a > 1$) as the inverse of the exponential function a^x, and compare the properties of the two functions;

EL2.02 – express logarithmic equations in exponential form, and vice versa;

EL2.03 – simplify and evaluate expressions containing logarithms, using the laws of logarithms;

EL2.04 – solve simple problems involving logarithmic scales (e.g., the Richter scale, the pH scale, the decibel scale).

Applications and Consolidation

Overall Expectations

ACV.01 · analyse models of linear, quadratic, polynomial, exponential, or trigonometric functions drawn from a variety of applications;

ACV.02 · analyse and interpret models of piecewise-defined functions drawn from a variety of applications;

ACV.03 · solve linear-quadratic systems and interpret their solutions within the contexts of applications;

ACV.04 · demonstrate facility in carrying out and applying key manipulation skills.

Specific Expectations

Analysing Models of Functions

AC1.01 – determine the key features of a mathematical model (e.g., an equation, a table of values, a graph) of a function drawn from an application;

AC1.02 – compare the key features of a mathematical model with the features of the application it represents;

AC1.03 – predict future behaviour within an application by extrapolating from a given model of a function;

AC1.04 – pose questions related to an application and use a given function model to answer them.

Analysing and Interpreting Models of Piecewise-Defined Functions

AC2.01 – demonstrate an understanding that some naturally occurring functions cannot be represented by a single formula (e.g., the temperature at a particular location as a function of time);

AC2.02 – graph a piecewise-defined function, by hand and by using graphing calculators or graphing software;

AC2.03 – analyse and interpret a given mathematical model of a piecewise-defined function, and relate the key features of the model to the characteristics of the application it represents;

AC2.04 – make predictions and answer questions about an application represented by a graph or formula of a piecewise-defined function;

AC2.05 – determine the effects on the graph and formula of a piecewise-defined function of changing the conditions in the situation that the function represents.

Solving Linear-Quadratic Systems

AC3.01 – determine the key properties of a linear function or a quadratic function, given the equation of the function, and interpret the properties within the context of an application;

AC3.02 – solve linear-quadratic systems arising from the intersections of the graphs of linear and quadratic functions;

AC3.03 – interpret the solution(s) to a linear- quadratic system within the context of an application.

Consolidating Key Skills

AC4.01 – perform numerical computations effectively, using mental mathematics and estimation;

AC4.02 – solve problems involving ratio, rate, and percent drawn from a variety of applications;

AC4.03 – solve problems involving trigonometric ratios in right triangles and the sine and cosine laws in oblique triangles;

AC4.04 – demonstrate facility in using manipulation skills related to solving linear, quadratic, and polynomial equations, simplifying rational expressions, and operating with exponents.

Ontario Catholic School Graduate Expectations

The graduate is expected to be:

A Discerning Believer Formed in the Catholic Faith Community who

CGE1a -illustrates a basic understanding of the **saving story** of our Christian faith;
CGE1b -participates in the **sacramental life** of the church and demonstrates an understanding of the centrality of the Eucharist to our Catholic story;
CGE1c -actively reflects on **God's Word** as communicated through the Hebrew and Christian scriptures;
CGE1d -develops attitudes and values founded on Catholic **social teaching** and acts to promote social responsibility, human solidarity and the common good;
CGE1e -speaks the **language of life**... "recognizing that life is an unearned gift and that a person entrusted with life does not own it but that one is called to protect and cherish it." (Witnesses to Faith)
CGE1f -seeks intimacy with God and celebrates **communion** with God, others and creation through prayer and worship;
CGE1g -understands that one's purpose or **call in life** comes from God and strives to discern and live out this call throughout life's journey;
CGE1h -respects the **faith traditions**, world religions and the life-journeys **of all people of good will**;
CGE1i -integrates faith with life;
CGE1j -recognizes that "sin, human weakness, conflict and forgiveness are part of the human journey" and that the cross, the ultimate sign of forgiveness is at the heart of **redemption**. (Witnesses to Faith)

An Effective Communicator who

CGE2a -listens actively and critically to understand and learn in light of gospel values;
CGE2b -reads, understands and uses written materials effectively;
CGE2c -presents information and ideas clearly and honestly and with sensitivity to others;
CGE2d -writes and speaks fluently one or both of Canada's official languages;
CGE2e -uses and integrates the Catholic faith tradition, in the critical analysis of the arts, media, technology and information systems to enhance the quality of life.

A Reflective and Creative Thinker who

CGE3a -recognizes there is more grace in our world than sin and that hope is essential in facing all challenges;
CGE3b -creates, adapts, evaluates new ideas in light of the common good;
CGE3c -thinks reflectively and creatively to evaluate situations and solve problems;
CGE3d -makes decisions in light of gospel values with an informed moral conscience;
CGE3e -adopts a holistic approach to life by integrating learning from various subject areas and experience;
CGE3f -examines, evaluates and applies knowledge of interdependent systems (physical, political, ethical, socio-economic and ecological) for the development of a just and compassionate society.

A Self-Directed, Responsible, Life Long Learner who

CGE4a -demonstrates a confident and positive sense of self and respect for the dignity and welfare of others;
CGE4b -demonstrates flexibility and adaptability;
CGE4c -takes initiative and demonstrates Christian leadership;
CGE4d -responds to, manages and constructively influences change in a discerning manner;
CGE4e -sets appropriate goals and priorities in school, work and personal life;
CGE4f -applies effective communication, decision-making, problem-solving, time and resource management skills;
CGE4g -examines and reflects on one's personal values, abilities and aspirations influencing life's choices and opportunities;
CGE4h -participates in leisure and fitness activities for a balanced and healthy lifestyle.

A Collaborative Contributor who

CGE5a -works effectively as an interdependent team member;
CGE5b -thinks critically about the meaning and purpose of work;
CGE5c -develops one's God-given potential and makes a meaningful contribution to society;
CGE5d -finds meaning, dignity, fulfillment and vocation in work which contributes to the common good;
CGE5e -respects the rights, responsibilities and contributions of self and others;
CGE5f -exercises Christian leadership in the achievement of individual and group goals;
CGE5g -achieves excellence, originality, and integrity in one's own work and supports these qualities in the work of others;
CGE5h -applies skills for employability, self-employment and entrepreneurship relative to Christian vocation.

A Caring Family Member who

CGE6a -relates to family members in a loving, compassionate and respectful manner;
CGE6b -recognizes human intimacy and sexuality as God given gifts, to be used as the creator intended;
CGE6c -values and honours the important role of the family in society;
CGE6d -values and nurtures opportunities for family prayer;
CGE6e -ministers to the family, school, parish, and wider community through service.

A Responsible Citizen who

CGE7a -acts morally and legally as a person formed in Catholic traditions;
CGE7b -accepts accountability for one's own actions;
CGE7c -seeks and grants forgiveness;
CGE7d -promotes the sacredness of life;
CGE7e -witnesses Catholic social teaching by promoting equality, democracy, and solidarity for a just, peaceful and compassionate society;
CGE7f -respects and affirms the diversity and interdependence of the world's peoples and cultures;
CGE7g -respects and understands the history, cultural heritage and pluralism of today's contemporary society;
CGE7h -exercises the rights and responsibilities of Canadian citizenship;
CGE7i -respects the environment and uses resources wisely;
CGE7j -contributes to the common good.

Unit 3: Exploring Reciprocal and Inverse Functions

Time: 15 hours

Unit Description

Students explore the behaviour of reciprocal and inverse functions. Students apply their acquired knowledge to create mathematical models derived from realistic inverse proportional relationships. Using extrapolation, students predict future behaviour and pose questions related to the generated models of these functions. Key features of linear and quadratic reciprocal functions are investigated through curve sketching. Technological aids, such as a graphing calculator, assist students in understanding key concepts of reciprocal and inverse functions.

Unit Synopsis Chart

Activity	Time	Learning Expectations	Assessment Categories	Tasks
1. Part-Time Jobs	3.75 hours	ACV.01, ACV.04, PFV.03, AC1.01, AC1.02, AC4.02, PF3.01 CGE2b, 3c, 3f	Knowledge/Understanding Application Thinking/Inquiry/Problem Solving	Exploring inverse proportion using students' part-time jobs
2. The Weightless Student	3.75 hours	ACV.01, PFV.03, AC1.03, AC1.04, PF3.01, PF3.02 CGE2a, 2d, 2e	Communication Application Thinking/Inquiry/Problem Solving	Problem solving using an Earth/moon gravity model
3. The Ups and Downs of Curve Sketching	2.5 hours	PFV.04, PF4.01 CGE2c, 5g	Knowledge/Understanding Communication Thinking/Inquiry/Problem Solving	Sketching reciprocal functions
4. Key Reciprocal Properties	2.5 hours	PFV.04, PF4.02, PF4.03 CGE2e, 5a	Communication Thinking/Inquiry/Problem Solving	Investigating key properties of reciprocal functions with technology
5. Summative – From Start to Finish	2.5 hours	ACV.01, PFV.03, AC1.01, AC1.02, AC1.03, PF3.01, PF3.02 CGE3c, 5a, 5g, 7b	Knowledge/Understanding Thinking/Inquiry/Problem Solving Application Communication	Investigating the inverse relationship between running speed and time to complete a given distance

Activity 3.1: Part-Time Jobs

Time: 3.75 hours

Description

Students are introduced to inverse proportion through a series of activities that focus on part-time jobs. Students investigate the key features of tables and graphs that are generated from inversely proportional relationships.

Strand(s) & Learning Expectations

Ontario Catholic School Graduate Expectations

CGE3c - a reflective and creative thinker who thinks reflectively and creatively to evaluate situations and solve problems;
CGE3f - a reflective and creative thinker who examines, evaluates, and applies knowledge of interdependent systems (physical, political, ethical, socio-economic, and ecological) for the development of a just and compassionate society;
CGE5b - a collaborative contributor who thinks critically about the meaning and purpose of work.

Strand(s): Polynomial Functions and Inverse Proportionality, Applications and Consolidation

Overall Expectations

ACV.01 - analyse models of linear, quadratic, polynomial, exponential, or trigonometric functions drawn from a variety of activities;
ACV.04 - demonstrate facility in carrying out and applying key manipulation skills;
PFV.03 - demonstrate an understanding of inverse proportionality.

Specific Expectations

AC1.01 - determine the key features of a mathematical model of a function drawn from an application;
AC1.02 - compare the key features of a mathematical model with the features of the application it represents;
AC4.02 - solve problems involving ratio, rate, and percent drawn from a variety of applications;
PF3.01 - construct tables of values, graphs, and formulas to represent functions of inverse proportionality derived from descriptions of realistic situations.

Prior Knowledge & Skills

- Construction and analysis of data in tables and graphs
- Conversion of fractions to decimals and vice-versa
- Recognition of linear and non-linear relationships graphically and from first differences
- Scatter plot skills using a graphing calculator and spreadsheet

Planning Notes

- Review direct variation between two variables within real-world contexts.
- Review the definition of a function.
- Graphing the relationships within each activity can be done on a graphing calculator.
- These activities may be completed in groups or by pairing students.
- Photocopy student worksheets as required.

Teaching/Learning Strategies

Teacher Facilitation

- Because this activity focuses on part-time jobs, the teacher should initially survey the class to discover who has a part-time job.
- Students share their job types and provide brief descriptions of job-related duties.
- Since many students have part-time jobs in the service industry, the teacher uses their duties as a suitable starting point to introduce inverse proportion.
- In many service jobs, such as a fast-food restaurant worker, students are responsible for maintaining a clean work environment. Provide students with an hypothetical scenario where they and their co-workers are responsible for cleaning the workplace after closing.
- Provide an estimated time required to clean a typical student's workplace without any assistance from co-workers. Compare this time with the estimated time required to clean the workplace with help from one co-worker, two co-workers, and so on. Generate a table with this data.

Number of Workers	1	2	3	4	5	6
Clean-up Time (hours)	18	9	6	4.5	3.6	3

- Note that this relationship assumes that all workers work at the same pace.
- Ensure students recognize that when the number of workers is doubled the time is divided by two; when the number of workers is tripled the time is divided by three.
- Note that the product of the number of workers and clean-up time is a constant.
- Sketch this relationship by hand or using a graphing calculator. Note changes in the shape of the graph for very small and large values of the number of workers. Introduce the idea of asymptotes – the horizontal and vertical limits of a curve.
- Note that this graph will not have a vertical asymptote, since the number of workers starts at $x = 1$. The teacher could generalize beyond this physical situation and extend the curve for small values of x, illustrating the graph's vertical asymptote.
- Students describe the sketched relationship, e.g., linear, non linear.
- Students generalize the relationship between the time required to clean up and the number of workers.
- Generate an equation to represent this relationship.
- Students conclude that the time it takes to clean the restaurant is inversely proportional to the number of workers.

Student Activity 3.1.1

Frank and Harold are evening crew chiefs at the same fast-food restaurant. The manager makes the following charts to assess the efficiency of each crew.

Frank's Crew

Number of Workers	2	3	4	5	6
Clean-up Time (hours)		5		3	

Harold's Crew

Number of Workers	2		4		6
Clean-up Time (hours)		4.5		3.4	2.5

1. Complete each chart.
2. Graph each relation on the same axis.
3. Can we tell from the graphs which crew is fastest? Explain.
4. Which crew is the fastest?
5. On the same axis, sketch the relationship of a crew that is slower than both Frank's and Harold's.
6. On the same axis, sketch the relationship of a crew that is faster than both Frank's and Harold's.

Teacher Facilitation

- Introduce students to other inversely proportional relationships, such as inverse squared ($1/x^2$) or inverse cubed ($1/x^3$).
- Generate a table of values and sketch an inverse proportional relationship, such as $1/x^2$.
- Discuss properties of the curve, such as vertical and horizontal asymptotes, and link them to the function's domain and range by asking: Are there any values of x or y that the curve cannot have?
- Introduce Student Activity 3.1.2 by discussing the problem with reference to the chart. What is the relationship between the average distance the trees are apart and the number of trees per section?
- Direct students towards an inverse proportional relationship.
- Have students try different inverse proportional possibilities.
- As an option, the teacher may choose to have students, individually or with a partner, draw on grid or graph paper trees 1 metre apart, 2 metres apart, 3 metres, etc., to determine the expected relationship.
- It may be necessary to lead students to the required relationship: the number of trees varies inversely as the square of the distance between the trees.

Student Activity 3.1.2

Lynn and Sue work at a Christmas tree farm planting trees. Before planting, the girls survey the number of trees on the farm. While surveying, they discover that the older trees are generally further apart than the newer stands of trees. Breaking the farm into 9 equal sections, Lynn and Sue gather the following data:

Region	1	2	3	4	5	6	7	8	9
Average Distance Apart (metres)	2	2.5	3	3.5	4	4.5	5	5.5	6
Number of Trees in Region	2250	1440					360		

1. Trying various inverse proportional relationships between average distance apart and number of trees, complete the table.
2. Graph the relationship between average distance apart and number of trees.
3. Describe the relationship between the distance the trees are apart and the number of trees.
4. Why might the trees in the older part of the farm be further apart?
5. Is there an optimum distance apart for planting trees?
6. Why, financially, is it important to plant the trees as close together as possible?
7. What are the environmental benefits of Christmas tree farms?

Assessment & Evaluation of Student Achievement

- Assess students' Knowledge/Understanding of inversely proportional relationships using a quiz.
- Thinking/Inquiry/Problem Solving and Application skills of inverse proportion can be assessed formatively within the written reports using criteria outlined in Appendix 3.2.

Accommodations

Students with visual impairments can use computer and graphing software.

Appendices

Appendix 3.2 – Written Report Rubric

Activity 3.2: The Weightless Student

Time: 3.75 hours

Description

In this activity, students mathematically model an Earth/moon gravity system. Using the inverse proportional law of gravitation, students use both algebraic and graphic methods to investigate, hypothesize, and discover the unique point of weightlessness between the Earth and moon.

Strand(s) & Learning Expectations

Ontario Catholic School Graduate Expectations

CGE2a - an effective communicator who listens actively and critically to understand and learn in light of gospel values;
CGE2d - an effective communicator who writes and speaks fluently one or both of Canada's official languages;
CGE2e - an effective communicator who uses and integrates the Catholic faith tradition, in the critical analysis of the arts, media, technology, and information systems, to enhance the quality of life.

Strand(s): Applications and Consolidation, Polynomial Functions and Inverse Proportionality

Overall Expectations

AVC.01 - analyse models of linear, quadratic, polynomial, exponential, or trigonometric functions drawn from a variety of applications;
PFV.03 - demonstrate an understanding of inverse proportionality.

Specific Expectations

AC1.03 - predict future behaviour within an application by extrapolation from a given model of a function;
AC1.04 - pose questions related to an application and use a given function model to answer them;
PF3.01 - construct tables of values, graphs, and formulas to represent functions of inverse proportionality derived from descriptions of realistic situations;
PF3.02 - solve problems involving relationships of inverse proportionality.

Prior Knowledge & Skills

- Algebraic and graphic understanding of direct proportionality and inverse proportionality relationships
- Use of scientific notation, a graphing calculator, and spreadsheet software
- Calculation of values using the quadratic formula
- Use of board or school computer and Internet etiquette

Planning Notes

- Discuss metric units surrounding this topic.
- Reserve computer time to provide access to the Internet and to spreadsheet software.
- An historical review of the evolution of astronomical data can enrich the lesson. Review significant figures that have been introduced in science courses.
- Photocopy student worksheets as required.
- Students should know their approximate mass. Teachers should use a generic mass if weight is a sensitive issue to one or more students in the class.

Teaching/Learning Strategies

Teacher Facilitation

- The class discusses why an apple falls towards the Earth, or why we as individuals are held to the surface of the Earth.
- Introduce the attractive force of gravitation between the apple and the Earth, or us and the Earth. Students can provide other examples.
- Compare the gravitational force holding us on the surface of the Earth to that of astronauts in space, such as in the space shuttle. Ask students to provide the variables that affect this force.
- Introduce a direct relationship between gravitational force of attraction and what we generally classify as weight.
- Prompt students to answer what would happen to our force of attraction (or weight) to the Earth as we go further into space.
- Lead discussion towards an inverse proportional relationship between the force (our weight) holding us on the Earth and the distance from the Earth's centre.
- Generalize inverse proportionality relationship to all mass in the universe, e.g., student on the surface of the moon.
- Present students with the inverse proportionality: $F_{1,2} \propto \frac{1}{d^2}$

  where $F_{1,2}$ is the force of attraction, in Newtons, between objects 1 and 2,
  d is the distance, in metres, between the two objects.
- Prompt students for other factors that influence the size of this force.
- Present students with the Law of Universal Gravitation: $F_{1,2} = \frac{GM_1M_2}{d^2}$,

  where $F_{1,2}$ is the force of attraction, in Newtons, between objects 1 and 2,
  G is the constant of proportionality,
  M_1 is the mass, in kg, of object 1,
  M_2 is the mass, in kg, of object 2, and
  d is the distance, in metres, between the two objects' centres.
- Students go on a hypothetical, mathematical, astronomical trip to the moon, exploring the effects of gravitational attraction between themselves and the Earth/moon system. Ask: Who wants to experience true weightlessness?
- Ask students where on this trip between the Earth and the moon a student space traveler would be truly weightless. Guide students towards the understanding that this point is where the force of attraction between the Earth and the student equals or balances the force of attraction between the moon and the student. A large diagram would greatly assist student understanding of the physical situation.
- Ask students to hypothesize where this point would be between the Earth and the moon, e.g., $\frac{1}{2}$ way, $\frac{3}{4}$ of the way to the moon, and explain why.
- Students require various astronomical data to calculate/generate this point. Using the Internet as a resource, students complete the table in Student Activity 3.2.2.

Student Activity 3.2.1

You require data on the earth/moon system before you begin your investigation of weightlessness. Using the Internet as a resource for some of this, find the following information:

1. Student mass (kg) = 100
2. a) Earth mass (kg) = 5.972 x 1024 b) Earth radius (m) = 6 378 000
3. a) Moon mass (kg) = 7.35 x 1022 b) Moon radius (m) = 1 738 000

4. Distance from centre of Earth to centre of moon (m) = 384 404 000
5. Universal gravitational constant (Nm^2/kg^2) = 6.67×10^{-11}
6. Law of Gravitation: $F_{1,2} = \dfrac{GM_1M_2}{d^2}$

Teacher Facilitation

- Review data to ensure all student quantities are approximately equal, as they may vary depending upon source (www.seds.org/nineplanets/nineplanets/).
- After collecting the data, students use their understanding of inverse proportionality to recognize that this unique location is much closer to the moon than the Earth.
- Students should re-hypothesis where this point of weightlessness will be in relation to the Earth and moon based upon new data.
- Discuss variable units associated with Student Activity 3.2.1.
- Suggest that to find the solution (the point of weightlessness), we can employ two methods – graphic and algebraic. Students may use one or both methods.
- Considering the graphic method, ask students what quantities to plot on a graph to find this unique point.
- Explain the headings of the table in Student Activity 3.2.2. Teachers should perform a sample calculation (e.g., provide answers for 100 000 000 metres from the Earth's surface).

<u>Student Activity 3.2.2</u>

1. Create a visual model (scale not important) indicating distances and respective masses of Earth, moon, and student from collected data for reference. The following steps will allow you to approximate this unique point in space graphically.
2. Using the Universal Law of Gravitation and the data collected, complete the following using a spreadsheet.

Distance From Earth's Centre (m)	**Distance From Moon's Centre (m)**	**Force Towards Earth, F_E (N)**	**Force Towards Moon, F_M (N)**	**Net Force (N)**	*** Direction of Net Force**
Earth's Surface 6 378 000					
50 000 000					
100 000 000	284 404 000	3.983	0.006	3.977	Towards Earth
150 000 000					
200 000 000					
250 000 000					
300 000 000					
350 000 000					
Moon's Surface 382 666 000					

* direction of force is either towards the Earth or moon

3. Construct a graph of F_E vs. distance from the Earth's centre.
4. On the same graph, plot F_M vs. distance from the Earth's centre.
5. Investigate by zooming in on the point where F_E intersects F_M. Approximate, from your graph, this unique point.

6. What is the physical significance of this point?
7. Is this point close to your original hypothesis?

The following steps will allow you to solve for this unique point in space algebraically.

8. Equate the formulas you used in your spreadsheet for F_E to F_M.
9. Be sure you are using the same *d* on each side of the new formula. Recall, the distance to Earth plus distance to moon equal about 384 404 000 m. Solve for *d* in this new equation (use the quadratic formula).
10. Would the location of this point change if your mass was greater, or less? If yes, how would it change – towards the Earth or towards the moon? If no, explain why not.
11. Hypothesize how this inverse proportionality relationship between force and distance may assist astronomical/interplanetary travel.

Assessment & Evaluation of Student Achievement

- Use the spreadsheets to assess students' Application of inverse proportionality.
- Assess Application by assessing students' ability to find the point of weightlessness between the earth and moon.
- Assess student Thinking/Inquiry/Problem Solving based upon their hypothesis, model creation, and both graphical and algebraic conclusions to the question posed.
- Assess Communication by assessing individual students's oral contributions within teacher-facilitated discussions and in students' written reports.
- Assess Knowledge/Understanding of inverse proportionality with a quiz.
- Assess work habits as a result of their completion of Student Activity 3.2.2.

Resources

http://sohowww.nascom.nasa.gov/explore/lessons (astronomical data, NASA lesson plans)

www.seds.org/nineplanets/nineplanets/ (solar system data)

www.stardate.org/resources/ssguide/ (solar system guide)

www.pd.astro.it/ (Internet links on astronomy)

www.wilders.force9.co.uk/BeyondEarth/ (solar system guide)

Activity 3.3: The Ups and Downs of Curve Sketching

Time: 2.5 hours

Description

Students create graphs of reciprocal functions. Through this creation, students identify key elements of the graphs while making connections to their respective equations.

Strand(s) & Learning Expectations

Ontario Catholic School Graduate Expectations

CGE2c - an effective communicator who presents information and ideas clearly and honestly and with sensitivity to others;

CGE5g - a collaborative contributor who achieves excellence, originality, and integrity in one's own work and supports these qualities in the work of others.

Strand(s): Polynomial Functions and Inverse Proportionality

Overall Expectations

PFV.04 - determine, through investigation, the key properties of reciprocal functions.

Specific Expectations
PF4.01 - sketch the graph of the reciprocal of a given linear or quadratic function by considering the implications of the key features of the original function as predicted from its equation.

Prior Knowledge & Skills

- Creation of tables of values given a relationship between two variables
- An understanding of domain and range
- The definition of a function and finding the zeros of a function
- Knowledge of x- and y-intercepts
- Vertical and horizontal asymptotes on the Cartesian plane and their relation to a function's domain and range

Planning Notes

- Students can work alone or with a partner.
- Graphing the relationships can be done on a graphing calculator, though the activities are designed to foster paper-and-pencil curve-sketching skill.
- Photocopy student worksheets as required.

Teaching/Learning Strategies

Teacher Facilitation

- Students explore the key features of reciprocal functions.
- The teacher checks student progress and gives assistance where needed.
- Note that the x-values provided in the activity worksheets provide a starting point for possible values.
- The teacher may choose to work with students on the first column of each worksheet and assign the remainder to be done independently or with a partner.

Student Activity 3.3.1

Part A

1. Using a table of values, sketch the graphs of the following functions on the same axis.

$y=\frac{1}{x}$		$y=\frac{1}{x+3}$		$y=\frac{1}{x-3}$	
x	y	x	y	x	y
-1		-1		-1	
0		0		0	
1		1		1	
...		...		...	

2. What do you notice about these graphs?
3. State the domain and range for each graph.
4. Do the graphs have an x-intercept? Why or why not?
5. Do the graphs have a y-intercept? Why or why not?
6. Do the graphs have vertical asymptotes? If yes, state their equations.
7. Do the graphs have horizontal asymptotes? If yes, state their equations.

Part B

1. Using a table of values, sketch the graphs of the following functions on the same axis.

$y=\frac{1}{x}+1$		$y=\frac{1}{x}-3$		$y=\frac{1}{x+2}-6$		$y=\frac{1}{x-4}+3$	
x	y	x	y	x	y	x	y
-1		-1		-1		-1	
0		0		0		0	
1		1		1		1	
...		...		...		...	

2. What do you notice about these graphs?
3. What change in the equation makes the graph move up or down? Provide an example.
4. State the domain and range for each relationship.
5. Do the graphs have an x-intercept? Why or why not?
6. Do the graphs have a y-intercept? Why or why not?
7. Do the graphs have vertical asymptotes? If yes, state their equations.
8. Do the graphs have horizontal asymptotes? If yes, state their equations.
9. Choose one of the graphs and state the intervals where the graph is positive and where the graph is negative.

Part C

1. Use a table of values to create the graphs of the following functions on the same axis.

$y=\frac{1}{x^2}$		$y=\frac{1}{x^2}+1$		$y=\frac{1}{x^2+x-12}$	
x	y	x	y	x	y
-1		-1		-1	
0		0		0	
1		1		1	
...		...		...	

2. What do you notice about these graphs?
3. How are these graphs similar to the graphs in Part A? How are they different?
4. State the domain and range for each graph.
5. Do the graphs have an x-intercept? Why or why not?
6. Do the graphs have a y-intercept? Why or why not?
7. Do the graphs have vertical asymptotes? If yes, state their equations.
8. Do the graphs have horizontal asymptotes? If yes, state their equations.
9. Choose one of the graphs and state the intervals in which the graph is increasing and those in which the graph is decreasing.

Part D

1. Using the key features of the graphs, sketch the graphs of the following:
   $y=\frac{1}{x+3}-2,\quad y=\frac{1}{x^2-2x-3}+3,\quad y=\frac{1}{x^2-16}$
2. Predict what the graph of $y=\frac{1}{9-x^2}$ will look like. Provide a sketch.
3. Now graph $y=\frac{1}{9-x^2}$. Were your predictions correct?
4. What changes this function from the previous functions?

Assessment & Evaluation of Student Achievement

- Student Knowledge/Understanding and Communication of curve sketching in Parts A, B, and C can be evaluated using a rubric (see Appendix 3.1).
- Assess Knowledge/Understanding of curve sketching with a quiz and marking scheme.
- Thinking/Inquiry/Problem Solving can be assessed through student responses to questions within Student Activity 3.3.1 based on a rating scale or checklist.
- Thinking/Inquiry/Problem Solving and Application can be assessed by making Part D a formal hand-in assignment.

Accommodations

- For students who require additional time for graphing, teachers may wish to assign fewer questions, without reducing difficulty level.
- Key x values (for table of values), already in place, may be necessary for some students.

Appendices

Appendix 3.1 – Rubric for Evaluation of Student's Graphs and Responses

Activity 3.4: Key Reciprocal Properties

Time: 2.5 hours

Description

Students create graphs of reciprocal functions using graphing calculators or graphing software. Through the creation of reciprocal graphs, students identify the relationships between key elements of the graphs and their respective equations. Students also describe the nature of the curve as it approaches asymptotes.

Strand(s) & Learning Expectations

Ontario Catholic School Graduate Expectations

CGE2e - an effective communicator who uses and integrates the Catholic faith tradition, in the critical analysis of the arts, media, technology, and information systems to enhance the quality of life;
CGE5a - a collaborative contributor who works effectively as an interdependent team member.

Strand(s): Polynomial Functions and Inverse Proportionality

Overall Expectations

PFV.04 - determine, through investigation, the key properties of reciprocal functions.

Specific Expectations

PF4.02 - describe the behaviour of a graph near a vertical asymptote;
PF4.03 - identify the horizontal asymptote of the graph of a reciprocal function by examining the patterns in the values of the given function.

Prior Knowledge & Skills

- Use of graphing calculator, including table of values function, or graphing software skills
- Understanding of asymptotes

Planning Notes

- Students can work alone or with a partner.
- Graphing relationships should be done with a graphing calculator or graphing software.
- Prepare a brief review of asymptotes.
- Teachers may provide graphing calculator or software window settings for some graphs.
- Photocopy student worksheets as required.

Teaching/Learning Strategies

Teacher Facilitation

- Students explore the key features of reciprocal functions and investigate their asymptotes using technology with the guidance of worksheets.
- If using graphing calculators, ensure that students can properly use the table and trace functions to assist in their explanation of the behaviour of functions near asymptotes.
- If using graphing calculators, set the graphing mode to "dot" versus "connected," to see function asymptotic behaviour more clearly.

Student Activity 3.4.1

Part A

1. Using a graphing calculator or graphing software, sketch the graphs of $y=\frac{1}{x}, y=\frac{1}{x+6}, y=\frac{1}{x-2}$ on the same axis. Window settings: x_{min}: -8, x_{max}: 8, y_{min}: -5, y_{max}: 5
2. What do you notice about these graphs?
3. What are the equations of the asymptotes?
4. State an equation of a reciprocal function that has asymptotes of:
   a) $x = -2, y = 0$ b) $x = 5, y = 0$.

Part B

1. Using a graphing calculator or graphing software, sketch the graphs of $y=\frac{1}{x-4}-2,\ \ y=\frac{1}{x+3}+1,\ \ y=\frac{1}{x+2}-6,\ \ y=\frac{1}{x-4}+3$.
   Window settings: x_{min}: -10, x_{max}: 10, y_{min}: -10, y_{max}: 10
2. What do you notice about these graphs?
3. What are the equations of the asymptotes?
4. What in the equation causes the graph to move up or down?
5. For each equation, describe what is happening to the curve as it approaches the horizontal asymptote from above and from below the asymptote.
6. For each equation, describe what is happening to the curve as it approaches the vertical asymptote from the left side and from the right side.
7. State a possible reciprocal function that has asymptotes of:
   a) $x = 5, y = -2$ b) $x = -1, y = 4$ c) $x = -7$, y = -5.

Part C

1. Using a graphing calculator or graphing software, sketch the graphs of $y=\frac{1}{x^2+2}-1,\ \ y=\frac{1}{x^2-3x+2}+3,\ \ y=\frac{1}{x^2+12x-13}$ on the same axis.
   Window settings: x_{min}: -1, x_{max}: 3, y_{min}: -2, y_{max}: 4
2. What do you notice about these graphs?
3. What is similar to the graphs in Part A? What is different?
4. What are the equations of the asymptotes?
5. What would make these graphs move up or down? Provide an example.

6. For each equation above, describe what is happening to the curve as it approaches the horizontal asymptote from the top and from the bottom.
7. For each equation, describe what is happening to the curve as it approaches the vertical asymptote from the left side and from the right side.

Assessment & Evaluation of Student Achievement

- Assess student Knowledge/Understanding and Communication of reciprocal function properties with a rubric (see Appendix 3.1).
- Assess student Application of reciprocal functions by responses in Student Activity 3.4.1.
- Assess work habits based upon the completion of the activity.

Accommodations

For students who require additional time for graphing, teachers may assign fewer questions, without reducing difficulty level, to assist with task completion.

Appendices

Appendix 3.1 – Rubric for Evaluation of Student's Graphs and Responses

Activity 3.5: To the Finish Line

Time: 2.5 hours

Description

In this summative activity, students investigate the inverse proportional relationship between running speed and time required to complete a given distance. Students generate graphs and equations to model and compare the key properties each situation presents.

Strand(s) & Learning Expectations

Ontario Catholic School Graduate Expectations

CGE3c - a reflective and creative thinker who thinks reflectively and creatively to evaluate situations and solve problems;
CGE5a - a collaborative contributor who works effectively as an interdependent team member;
CGE5g - a collaborative contributor who achieves excellence, originality, and integrity in one's own work and supports these qualities in the work of others;
CGE7b - a responsible citizen who accepts accountability for one's own actions.

Strand(s): Applications and Consolidation, Polynomial Functions and Inverse Proportionality

Overall Expectations

AVC.01 - analyse models of linear, quadratic, polynomial, exponential, or trigonometric functions drawn from a variety of applications;
PFV.03 - demonstrate an understanding of inverse proportionality.

Specific Expectations

AC1.01 - determine the key features of a mathematical model;
AC1.02 - compare the key features of a mathematical model with the features of the application it represents;
AC1.03 - predict future behaviour within an application by extrapolation from a given model of a function;
PF3.01 - construct tables of values, graphs, and formulas to represent functions of inverse proportionality derived from descriptions of realistic situations;
PF3.02 - solve problems involving relationships of inverse proportionality.

Prior Knowledge & Skills

- Algebraic and graphic understanding of direct proportionality and inverse proportionality relationships
- Graphing calculator skills
- Knowledge of dependent and independent variables

Planning Notes

- Graphing the relationships can be done on a graphing calculator or graphing software.
- Consider gathering race data from the Internet, newspaper, or school track results to substitute for the activity data.
- Photocopy student worksheets as required.

Teaching/Learning Strategies

Teacher Facilitation

- Begin the class by introducing and discussing various track-and-field running events, such as the 100-metre event.
- Introduce average speed with appropriate units for each runner over an entire race.
- Provide students with this example from the men's 100-m final at the World Track and Field Championships in Seville, 1999:

Runner	M. Greene	B. Surin	D. Chambers	O. Thompson	T. Harden	T. Mongomery	J. Gardener	K. Streete
Average Speed (m/s)	10.2	10.16	10.03	10.00	9.98	9.96	9.93	9.77
Time (s)	9.81	9.84	9.97	10.00	10.02	10.04	10.07	10.24

- Discuss the relationship between average speed and time. Note that time depends on the speed; time is the dependent variable and average speed is the independent variable.
- Graph the relationship by hand or with a graphing calculator.
- Examine the graph and state the relationship: time is inversely proportional to speed.
- Students mathematically model this relationship in the form of an equation. **Note:** speed multiplied by time equals a constant.
- Prompt students to comment on the relevance of this constant.
- With reference to the graph, inquire if there any limits on the variables, e.g., can we extend the function further in each direction? Why or why not?
- Ask students for the physical interpretation of such extrapolation.
- Approximate a maximum limit on men's average speed over 200 metres and calculate the respective time.
- Students read through all activities before starting to ensure proper scaling of axes on graphs.

Student Activity 3.5.1

Part A

The following are the results of the women's 400-m final at the World Track and Field Championships in Seville, 1999:

Runner	C. Freeman	A. Rucker	L. Graham	F. Ogunkoya	K. Merry	N. Nazarova	G. Breuer	O. Kotlyarova
Average Speed (m/s)	8.05	8.04	8.01	8.00	7.92	7.90	7.90	7.89
Time (s)	49.67	49.74	49.92	50.03	50.52	50.61	50.67	50.72

1. Graph the relationship between average speed and time.
2. Mathematically model this relationship in the form of an equation.
3. What is the relevance of this constant?
4. From the graph, approximate a maximum limit on women's average speed over 400 metres, and calculate the respective time.
5. Hypothesize, in a sentence, the graph of the men's 400-m final on the same axis.
6. Graph the men's 400-m final on the same axis:

Runner	M. Johnson	S. Parrella	A. Cardenas	J. Young	A. Pettigrew	M. Richarson	G. Haughton	J. Baulch
Average Speed (m/s)	9.26	9.03	9.03	8.97	8.98	8.96	8.88	8.85
Time (s)	43.18	44.29	44.31	44.36	44.54	44.65	45.07	45.18

7. Do the actual locations of the runners match where you hypothesized they would be on the graph?
8. Mathematically model this relationship in the form of an equation.
9. If you placed ninth in the 400-m race at the World Track and Field Championships in Seville, where would you place yourself on this graph? Why?

Part B

The following are some of the results of the women's 200-m final

Runner	I. Millar	B. McDonald	M. Frazer	A. Phillip	D. Ferguson	F. Yusuf	L. Hewitt	J. Campbell
Average Speed (m/s)	9.19		9.98		8.93		8.88	8.83
Time (s)	21.77	22.22		22.26		22.42	22.53	

1. Model this relationship in the form of an equation.
2. Complete the table.
3. Graph the relationship between average speed and time.
4. Given the times of the men's 200-m final, plot this relationship on the same axis:

Runner	M. Greene	C. Silva	F. Obadele	O. Thompson	M. Urbas	K. Little	J. Golding	F. Fredericks
Time (s)	19.90	20.00	20.11	20.23	20.30	20.37	20.48	DNF

5. What can we conclude about the constant of proportionality when graphing average speed vs. time?

Assessment & Evaluation of Student Achievement

- Use student tables and calculations to assess their Application of inverse proportionality.
- Assess students' Knowledge/Understanding based on their graphs. Students may also demonstrate their knowledge of inverse proportionality by completing a table of values and a graph in a paper-and-pencil performance task.
- Assess students' Thinking/Inquiry/Problem Solving using their hypotheses, approximations, model creation, and conclusions to the questions.
- Assess Communication by assessing students' oral contributions within teacher-facilitated discussions and in students' written reports, using the criteria in rubric Appendix 3.1.
- Thinking/Inquiry/Problem Solving, Communication, and Application skills can be assessed within the written reports using criteria outlined in Appendix 3.2.

Accommodations

Teachers may assign fewer questions or allow extra time to assist with task completion.

Resources

www.runnersworld.com (links to race results from around the world)

Appendices

Appendix 3.1 – Rubric for Evaluation of Student's Graphs and Responses
Appendix 3.2 – Written Report Rubric

Appendix 3.1 – Rubric for Evaluation of Student's Graphs and Responses

Category	Level 1 (50-59%)	Level 2 (60-69%)	Level 3 (70-79%)	Level 4 (80-100%)
Knowledge/ Understanding Graphing	- completes graphs with limited accuracy	- completes graphs with some accuracy	- completes graphs with considerable accuracy	- completes graphs with a high degree of accuracy
Communication Explanation of graph PF4.01	- explains with limited clarity and limited justification of reasoning	- explains with some clarity and some justification of reasoning	- explains with considerable clarity and justification of reasoning	- explains with a high degree of clarity and full justification of reasoning

Note: A student whose achievement is below Level 1 (50%) has not met the expectations for this assignment or activity.

Appendix 3.2 – Written Report Rubric

(Adapted from the Grade 10 Mathematics Applied: Catholic Course Profile)

Category	Level 1 (50-59%)	Level 2 (60-69%)	Level 3 (70-79%)	Level 4 (80-100%)
Knowledge/ Understanding - Understanding of concepts - Accuracy of data in tables and graphs PF3.01	- demonstrates limited understanding of concepts - presents data in tables and graphs with limited accuracy	- demonstrates some understanding of concepts - presents data in tables and graphs with some accuracy	- demonstrates considerable understanding of concepts - presents data in tables and graphs with considerable accuracy	- demonstrates a thorough understanding of concepts - presents data in tables and graphs with a high degree of accuracy
Thinking/ Inquiry/Problem Solving Presentation of arguments PF3.01	- presents arguments with limited logic and organization	- presents arguments with some logic and organization	- presents arguments with considerable logic and organization	- presents arguments with logic and organization
Communication Use of appropriate mathematical terms AVC.01, AC1.02	- uses limited appropriate mathematical terminology	- uses some appropriate mathematical terminology	- uses considerable appropriate mathematical terminology	- uses appropriate mathematical terminology consistently
Application Application of concepts or procedures AC1.03, PF3.02	- applies concepts or procedures to problems in familiar settings in limited ways	- applies concepts to problems in familiar settings sometimes	- applies concepts and procedures to problems in familiar and unfamiliar settings regularly	- applies concepts and procedures to problems in familiar and unfamiliar settings consistently

Note: A student whose achievement is below Level 1 (50%) has not met the expectations for this assignment or activity.

Unit 4: Exponential and Logarithmic Functions

Time: 20 hours

Unit Description

Students investigate properties of exponential and logarithmic functions. The relationship between exponential and logarithmic functions is explored both graphically and algebraically. Students use the laws of logarithms to simplify and evaluate logarithmic expressions, and to solve problems. A wide variety of exponential and logarithmic applications and models are examined.

Unit Synopsis Chart

Activity	Time	Learning Expectations	Assessment Categories	Tasks
1. Exploring Exponential Functions	3 hours	ELV.01, EL1.01, EL1.02, EL1.03 CGE2a, 2d, 5a	Knowledge/Understanding Communication	Investigating properties of exponential functions
2. Modelling Population	3 hours	ELV.01, ACV.01, EL1.04, EL1.05, AC1.01, AC1.02, AC1.03, AC1.04 CGE3c, 5g	Thinking/Inquiry/Problem Solving Communication	Modelling exponential growth and decay
3. Ed's Exponential Existence	3 hours	ELV.01, ACV.04, EL1.05, AC4.04 CGE2b, 2c, 7b	Knowledge/Understanding Thinking/Inquiry/Problem Solving Application Communication	Applications of exponential growth and decay
4. Bridging the Gap with Logs	3 hours	ELV.01, ELV.02, ACV.04, EL1.01, EL2.01, EL2.02, AC4.04 CGE2c, 2d	Knowledge/Understanding Communication	Determining connections between exponential functions and logarithmic functions
5. A Log "Jam Session"	3.5 hours	ELV.02, ACV.01, EL2.04, AC1.01, AC1.02, AC1.03, AC1.04 CGE3c, 5g	Thinking/Inquiry/Problem Solving Communication	Modelling logarithmic functions
6. Logarithms and Applications	3 hours	ELV.02, ACV.04, EL2.03, EL2.04, AC4.04 CGE2b, 2c	Knowledge/Understanding Thinking/Inquiry/Problem Solving Application Communication	Laws of logarithms and applications of logarithmic scales

Activity	Time	Learning Expectations	Assessment Categories	Tasks
7. Summative Assessment: Springfield and Shelbyville's History	1.5 hours	ELV.01, ELV.02, EL1.04, EL1.05, EL2.02, EL2.03, EL2.04, AC4.04 CGE2b, 2c, 5g, 7b	Knowledge/Understanding Thinking/Inquiry/Problem Solving Application Communication	Summative assessment

Note: The order of exercises 4.1 and 4.2 can be switched if the teacher would like to introduce modelling before algebra.

Activity 4.1: Exploring Exponential Functions

Time: 3 hours

Description

Students use graphing technology to explore properties of exponential functions. The graphical effect of the parameters *a, b,* and *c* in the equation $y = ca^x + b$ is examined. The rates of change of exponential functions are compared to the rates of change of non-exponential functions.

Strand(s) & Learning Expectations

Ontario Catholic School Graduate Expectations

CGE2a - an effective communicator who listens actively and critically to understand and learn in light of gospel values;
CGE2d - an effective communicator who writes and speaks fluently one or both of Canada's official languages;
CGE5a - a collaborative contributor who works effectively as an interdependent team member.

Strand(s): Exponential and Logarithmic Functions, Applications and Consolidation

Overall Expectations

ELV.01 - demonstrate an understanding of the nature of exponential growth and decay.

Specific Expectations

EL1.01 – identify, through investigations, using graphing calculators or graphing software, the key properties of exponential functions of the form a^x ($a > 0$, $a \neq 1$) and their graphs (e.g., the domain is the set of the real numbers; the range is the set of the positive real numbers; the function either increases or decreases throughout its domain; the graph has the *x*-axis as an asymptote and has *y*-intercept = 1);
EL1.02 - describe the graphical implications of changes in the parameters *a, b,* and *c* in the equation $y = ca^x + b$;
EL1.03 - compare the rates of change of the graphs of exponential and non-exponential functions (e.g., those with equations $y = 2x$, $y = x^2$, $y = x^{\frac{1}{2}}$, and $y = 2^x$).

Prior Knowledge & Skills

- Knowledge of graphing calculators and graphing a function
- Rates of change of a graph and properties of graphs, including domain, range, increasing, decreasing, asymptotes, and intercepts
- Use of First Differences chart;
- Understanding of positive and rational exponents, and exponent laws.

Planning Notes

- Students require graph paper and graphing calculators. Provide students with window settings for all graphing calculator activities.
- An overhead and a graphing calculator projection unit are required for class demonstrations.
- Prepare worksheets and a review of the relevant concepts of transformation of functions.

Teaching/Learning Strategies

Teacher Facilitation

- Review various types of graphical relationships, i.e., linear, quadratic, exponential, etc. with the class by projecting the graphical relationships using a graphing calculator and an overhead projecting unit. Then, discuss the properties of each type.
- Students may either work in pairs or individually to complete graphing activities.
- Distribute graphing calculators and graph paper.
- Do a numerical example of exponential growth with the class to review this concept from Grade 11 e.g., If you were investing in a mutual fund, describe the rate of change of the accumulated amount of your investment over time if the rate of growth is exponential.
- Distribute Student Activity 4.1.1.

Student Activity 4.1.1

1. Using graphing technology, graph the following functions: $y = 2x$, $y = 10x$, $y = x^2$, $y = x^{\frac{1}{2}}$, $y = 2^x$, and $y = 7^x$. Use the information from the graph to complete the following chart.

Equation	What type of relationship is this?	Sketch the graph	Describe the shape of the graph
$y = 2x$			
$y = 10x$			
…			

2. Describe a situation that could be modelled by each of the following equations:
   a) $y = 10x$
   b) $y = x^2$
   c) $y = 2^x$

Teacher Facilitation

- Project the graphs in #1 on an overhead and take up the chart. In a teacher-directed discussion, discuss students' answers to #2.
- Review domain and range and then distribute Student Activity 4.1.2.

Student Activity 4.1.2

1. Using graphing technology, graph the following functions: $y = 2^x$, $y = 3^x$; $y = 6^x$; $y = 10^x$; $y = (\frac{1}{2})^x$, $y = (\frac{1}{3})^x$, $y = (\frac{1}{6})^x$, and $y = (\frac{1}{10})^x$. Use the information from each graph to complete the following chart.

Equation	Describe the shape of the graph	Domain	Range	Intervals where increasing (if any)	Intervals where decreasing (if any)	Equation of asymptotes	Intercepts
$y = 2^x$							
$y = 3^x$							
…							

2. Use the results in your chart to summarize your findings for exponential functions $y = a^x$, $a > 0$, $a \neq 1$.
   a) State the domain of $y = a^x$.
   b) State the range of $y = a^x$.
   c) State the equation of the asymptote of $y = a^x$.
   d) State the intercepts of $y = a^x$.
   e) Describe the graphical effects of different values for a for the equation $y = a^x$ ($a > 0$, $a \neq 1$). Be specific about the increasing or decreasing nature of the function.
   f) Describe the graph $y = a^x$ if $a = 1$. How does this graph differ from the graphs where $a \neq 1$?
3. a) Without graphing, explain how the graphs of $y = 5^x$ and $y = (\frac{1}{5})^x$ differ, and describe what they have in common.
   b) Verify your answer using graphing technology.
4. a) Without graphing, explain how the graphs of $y = 4^x$ and $y = 7^x$ differ, and describe what they have in common.
   b) Verify your answer using graphing technology.
5. a) Without graphing, explain how the graphs of $y = (\frac{1}{2})^x$ and $y = (\frac{1}{9})^x$ differ, and describe what they have in common.
   b) Verify your answer using graphing technology.
6. Summarize your conclusions about the graphical significance of changes in parameter a for the equation $y = a^x$, $a > 0$, $a \neq 1$.

Teacher Facilitation

- In a teacher-directed discussion, discuss students' conclusions as a class.
- Summarize the key properties of $y = a^x$ for $a > 0$, $a \neq 1$ from question #2. Demonstrate the properties visually using the overhead and graphing calculator projection unit.
- Summarize the graphical effect of parameter a for the equation $y = a^x$ for $a > 0$, $a \neq 1$, from question #6. Demonstrate the conclusions visually using the overhead and graphing calculator projection unit.
- Introduce and distribute Student Activity 4.1.3.

Student Activity 4.1.3

1. a) Using graphing technology, graph the following functions: $y = 2^x$, $y = (4)(2^x)$, $y = (2)(2^x)$, $y = (\frac{1}{2})(2^x)$, $y = (-4)(2^x)$, $y = (-3)(2^x)$, $y = (-\frac{1}{2})(2^x)$. Use the information from the graphs to complete the following chart.

Equation	**Describe graphical changes to the graph of $y = 2^x$**	**Domain**	**Range**	**Intervals where increasing (if any)**	**Intervals where decreasing (if any)**	**Equation of asymptotes**	**Intercepts**
$y = 2^x$							
$y = (4)(2^x)$							
…							

b) Using graphing technology, graph the following functions: $y = (\frac{1}{2})^x$, $y = (3)(\frac{1}{2})^x$, $y = (5)(\frac{1}{2})^x$, $y = (-1)(\frac{1}{2})^x$, $y = (-3)(\frac{1}{2})^x$, $y = (-5)(\frac{1}{2})^x$. Use the information from the graphs to complete the following chart.

Equation	**Describe graphical changes to the graph of $y = (\frac{1}{2})^x$**	**Domain**	**Range**	**Intervals where increasing (if any)**	**Intervals where decreasing (if any)**	**Equation of asymptotes**	**Intercepts**
$y = (\frac{1}{2})^x$							
$y = (3)(\frac{1}{2})^x$							
etc.							

2. Use the results in your chart to explain your findings for exponential functions $y = ca^x$, $a > 0$, $a \neq 1$. Describe how changes in the parameter c in $y = ca^x$ affects the equation $y = a^x$ for the following properties:
   a) shape of the graph;
   b) the domain;
   c) the range;
   d) intervals of increasing or decreasing;
   e) equations of asymptotes;
   f) intercepts.
3. Summarize your conclusions by describing the graphical effects of parameter c for $y = ca^x$, $a > 0$, $a \neq 1$.
4. a) Without graphing $y = (5)(3^x)$, state the shape of the graph, domain, range, intervals of increase/decrease, equation of asymptote, and intercepts.
   b) Verify your answer using graphing technology.
5. a) Without graphing $y = (-2)(4^x)$, state the shape of the graph, domain, range, intervals of increase/decrease, equation of asymptote, and intercepts.
   b) Verify your answer using graphing calculator technology.
6. a) Without graphing $y = (3)\ (\frac{1}{6})^x$, state the shape of the graph, domain, range, intervals of increase/decrease, equation of asymptote, and intercepts.
   b) Verify your answer using graphing technology.
7. a) Use a graphing calculator to graph the following functions on the same set of axes:
      i) $y = 2^x$, $y = 2^x + 1$, and $y = 2^x - 1$
      ii) $y = 3^x$, $y = 3^x + 8$, and $y = 3^x – 8$
      iii) $y = (\frac{1}{4})^x$, $y = (\frac{1}{4})^x - 2$, and $y = (\frac{1}{4})^x + 2$
   b) Summarize your findings by describing the graphical effects of parameter b for $y = a^x + b$, $a > 0$, $a \neq 1$.

Teacher Facilitation

- In a teacher-directed discussion, discuss students' conclusions as a class.
- Summarize the graphical implications of parameter c for the equation $y = ca^x$ for $a > 0$, $a \neq 1$ (question #3). Demonstrate the conclusions visually using the overhead and graphing calculator projection unit.
- Summarize the graphical implications of parameter b for the equation $y = a^x + b$, for $a > 0$, $a \neq 1$ (question #7b). Demonstrate the conclusions visually.

- Distribute Student Activity 4.1.4. This worksheet consolidates skills learned from Student Activities 4.1.1 to 4.1.3. Student Activity 4.1.4 can be handed in for assessment; teachers may wish to instruct students to work individually rather than in pairs.

Student Activity 4.1.4

1. a) Without using graphing technology, use the base graph $y = 4^x$ to sketch the graphs of the following functions: $y = (-1)(4^x)$, $y = (3)(4^x)$, $y = 4^x + 5$, $y = 4^x - 2$, $y = (\frac{1}{3})(4^x)$.
   b) Verify your results using graphing technology.
   c) What properties do all of the graphs have in common?
   d) What graphical properties are different between these graphs?
2. Explain why $a \neq 1$ for $y = a^x$. Use words and graphs in your explanation.
3. a) Use graphing technology to graph each of the following functions: $y = (2)(6^x) + 4$, $y = (-1)(\frac{1}{2})^x + 3$, $y = (-5)(3^x) - 8$.
   b) State the domain, range, and intercept of each of the functions.
4. a) Is the domain for all exponential functions the same? Use examples to support your explanation.
   b) Is the range for all exponential functions the same? Use examples to support your explanation.
5. Consider all of the graphical properties of the equation $y = (3)(4^x)$. Could this equation be used to model:
   a) the depreciation of a speed boat? Explain why or why not.
   b) the increase of a city's population? Explain why or why not.
   c) the depletion of the balance of a bank account that decreases exactly $100.00 each week? Explain why or why not.

Assessment & Evaluation of Student Achievement

- If students work in pairs, they can be assessed for Teamwork.
- Students write a summary of their findings from the activity.
- Focus on formative assessment and self-assessment, rather than on marks.

Accommodations

- Enlarge the student activity charts for any student with spatial difficulties.
- Use computer technology rather than a small screen calculator.

Activity 4.2: Modelling Population

Time: 3 hours

Description

Students expand their knowledge of exponential functions to model exponential growth and decay in various population contexts. Data representing both growth and decay are analysed graphically and algebraically. Students develop equations to model the data and then interpret their findings.

Strand(s) & Learning Expectations

Ontario Catholic School Graduate Expectations

CGE3c - a reflective and creative thinker who thinks reflectively and creatively to evaluate situations and solve problems;

CGE5g - a collaborative contributor who achieves excellence, originality, and integrity in one's own work and supports these qualities in the work of others.

Strand(s): Exponential and Logarithmic Functions, Applications and Consolidation

Overall Expectations
ELV.01 - demonstrate an understanding of the nature of exponential growth and decay;
ACV.01 - analyse models of linear, quadratic, polynomial, exponential, or trigonometric functions drawn from a variety of applications.

Specific Expectations
EL1.04 - describe the significance of exponential growth or decay within the context of applications represented by various mathematical models (e.g., tables of values, graphs, equations);
EL1.05 - pose and solve problems related to models of exponential functions drawn from a variety of applications, and communicate the solutions with clarity and justification;
AC1.01 - determine the key features of a mathematical model (e.g., an equation, a table of values, a graph) of a function drawn from an application;
AC1.02 - compare key features of a mathematical model with the features of the application it represents;
AC1.03 - predict future behaviour within an application by extrapolating from a given model of a function;
AC1.04 - pose questions related to an application and use a given function model to answer them.

Prior Knowledge & Skills

- Understanding of positive and rational exponents, and exponent laws
- Knowledge of graphs of exponential functions and solving exponential equations algebraically
- Geometric sequences and related formulas
- Use of a graphing calculator

Planning Notes

- Prepare worksheets.
- Students require graph paper and graphing calculators. Provide students with window settings for all graphing calculator activities.
- An overhead and a graphing calculator projection unit are required for class demonstrations.
- Students require access to the Internet to gather data for Student Activity 4.2.3. Research websites that contain population data to recommend for data collection.

Teaching/Learning Strategies

Teacher Facilitation

- Student Activity 4.2.1 models linear population growth.
- For Student Activities 4.2.1, 4.2.2, and 4.2.3, students can work individually or in pairs.
- Review concepts related to geometric series to assist with Student Activity 4.2.1, question #2.
- Set a context for the exploration of exponential growth and decline by discussing reasons why population research is of interest to government agencies and to private industries. Discuss ways that population growth or decline would affect students' neighbourhoods, e.g., roads, traffic, industry, housing, school size, etc.
- Distribute graphing calculators, graph paper, and Student Activity 4.2.1.

Student Activity 4.2.1 - Population Growth

The following chart records the population of a city from 1997 to 2001.

Year	Population (thousands)	First Differences	Ratio of Growth
1997	29 987.2		
1998	30 248.2		$r = 30\ 248.2 \div 29\ 987.2$ =
1999	30 499.2		
2000	30 769.7		
2001	31 081.0		

1. a) Complete the first differences column in the chart.
   b) Is the graph of Population versus Year linear or non-linear? Explain your reasoning.
   c) Use the graphing calculator to graph Population versus Year to determine the accuracy of your answer to (b).
   d) What type of equation, i.e., linear, quadratic, or exponential, represents this graph?
2. a) Complete the last column of the chart to calculate the ratio for the population increases each year. What is the significance of this ratio?
   b) Calculate the average ratio over the 5 years. What does this ratio represent?
3. Write an equation, in the form $y = ca^x$, that represents the population, P, after n years.
4. Describe how your original equation would change if the initial population were 15 000 000.
5. Describe how your original equation would change if the population doubled every year.
6. a) Use the equation to predict the population in the year 2005.
   b) Use your graph to verify your solution.
7. a) Use your graph to predict how long it would take the population to increase to 33 million.
   b) Use your equation to verify your solution.
8. Is it reasonable to use your graph or equation to accurately predict the population in the year 2050? Why or why not? Suggest reasons for the limitations of both the graphical and algebraic models.

Teacher Facilitation

- In a teacher-directed discussion, discuss students' conclusions as a class. Take up the questions, providing visual demonstrations using the overhead and the graphing calculator projection unit. Alternatively, students could briefly present their solutions to the class.
- Direct students to the conclusion that for exponential growth, the equation $y = ca^x$ has $a > 1$.
- Lead into Student Activity 4.2.2 by discussing/brainstorming possible reasons for population growth and population decline. Have students predict the differences in both the algebraic models and graphical models between exponential growth and exponential decay. Distribute Student Activity 4.2.2.

Student Activity 4.2.2 – Population Decline

1. Suppose that the population of a city, recorded in the chart below, began to decrease after the year 2001. Complete the chart.

Year	Population (thousands)	Ratio of Decline
2001	31 081.0	
2002	30 366.1	
2003	29 637.3	
2004	28 985.3	
2005	28 260.7	

2. Determine the average ratio over the 5-year period.
3. a) Use the graphing calculator to graph Population versus Year.
   b) Explain how the shape of this graph differs from the graph in Student Activity 4.2.1 that represented population growth.

4. a) Write an equation that represents population, P, after n years.
   b) Use the graphing calculator to confirm the accuracy of your equation by graphing it and comparing it with the graph in #3a.
5. a) Use the equation to determine the population after 10 years.
   b) Use your graph to verify your solution.
6. a) Use the graph to determine when the population will have decreased to 27 000 000.
   b) Use your equation to verify your solution.
7. Describe how your original equation would change if the population in the year 2001 was 50 000 000.
8. Describe how your original equation would change if the population declined 5% each year. Will the population ever reach 0? Explain.
9. Discuss possible limitations of both the graphical model and the algebraic model for population decline.

Teacher Facilitation

- In a teacher-directed discussion, discuss students' conclusions as a class. Take up the questions, providing visual demonstrations. Alternatively, students could present their solutions to the class.
- In a class discussion, compare the algebraic models of exponential growth (Student Activity 4.2.1) and exponential decay (Student Activity 4.2.2). Direct students to the conclusion that for exponential decay, the equation $y = ca^x$ has $0 < a < 1$; for exponential growth, the equation $y = ca^x$ has $a > 1$.
- In a class discussion, compare the characteristics of the graphical models of exponential growth and exponential decay. Similarities and the differences should be explored.
- Distribute Student Activity 4.2.3.

Student Activity 4.2.3 – Population Research

1. Use the websites provided by your teacher to locate population data for any country, province, or city over a 50-year span. Record your data in a chart.
2. Graph the relationship between Population and Year.
3. Determine an equation, in the form $y = ca^x$, that represents population, P, of your chosen region after n years. Show your exploration of algebraic models and justify your choice of model.
4. a) Use your algebraic model to predict the population of your chosen region in the year 2010.
   b) Use your graph to determine the accuracy of your prediction.
5. a) Describe any restrictions on your algebraic model.
   b) Describe any restrictions on your graphical model.
6. Despite limitations on your algebraic and graphical models, they are still useful sources of information. City planners are interested in population trends. Based on your chosen location, write a letter to a city planner outlining the population trends for this location. Include in your letter several recommendations about what should be done to prepare for the upcoming population trends in that region. Provide data to support your conclusions and recommendations.

Assessment & Evaluation of Student Achievement

- Students write a summary of their findings in this activity. The summary can be assessed formatively by the teacher or shared with a peer.
- Focus on formative assessment and peer or self-assessment, rather than on marks. Assess Thinking/Inquiry/Problem Solving and Communication using a rubric (see Appendix 4.1).

Accommodations

- Students can work in pairs if they are having difficulty with the investigation. If students require further guidance, it may be beneficial for the teacher to complete Student Activity 4.2.1 as a class and have the students complete Student Activities 4.2.2 and 4.2.3 independently or in pairs. Alternatively, the teacher can provide students with specific data.
- Use computer technology rather than a small screen calculator.

Resources

Useful websites for data collection for Part C include:

- www.statcan.ca
- http://www.region.peel.on.ca/planning/stats/popproj.htm

Activity 4.3: Ed's Exponential Existence

Time: 3 hours

Description

Students expand on concepts investigated in the previous activity to include a variety of growth and decay applications. Problem-solving skills are developed as students work through applications. Students further develop an understanding of restrictions on both algebraic models and graphical models.

Strand(s) & Learning Expectations

Ontario Catholic School Graduate Expectations

CGE2b - an effective communicator who reads, understands, and uses written material effectively;
CGE2c - an effective communicator who presents information and ideas clearly and honestly and with sensitivity to others;
CGE7b - a responsible citizen who accepts accountability for one's own actions.

Strand(s): Exponential and Logarithmic Functions, Applications and Consolidation

Overall Expectations

ELV.01 - demonstrate an understanding of the nature of exponential growth and decay;
ACV.04 - demonstrate facility in carrying out and applying key manipulation skills.

Specific Expectations

EL1.05 - pose and solve problems related to models of exponential functions drawn from a variety of applications, and communicate the solutions with clarity and justification;
AC4.04 - demonstrate facility in using manipulation skills related to solving linear, quadratic, and polynomial equations, simplifying rational expressions, and operating with exponents.

Prior Knowledge & Skills

- Understanding of positive and rational exponents, and exponent laws
- Knowledge of graphs of exponential functions, and solving exponential equations algebraically and graphically
- Geometric sequences and related formulas
- Use of graphing calculator

Planning Notes

- Prepare worksheets.
- Students require graph paper and graphing calculators. Provide students with window settings for all graphing calculator activities.
- If students are presenting their solutions to the class, chart paper and markers are required.
- An overhead projector and projection unit for the graphing calculator is needed to complete Student Activity 4.3.1 as a class. Prepare overhead transparencies.

Teaching/Learning Strategies

Teacher Facilitation

- Students may need to be reminded how to determine a growth/decay ratio.
- Complete Student Activity 4.3.1 together as a class in a teacher-directed activity. Use an overhead projection unit for the graphing calculator to facilitate demonstration and class discussion. Alternatively, Student Activity 4.3.1 could be completed by the students in pairs, and then taken up as a class prior to beginning the next activity, or teachers can structure a Jigsaw in which students teach others in small groups.
- Distribute graphing calculators, graph paper, and Student Activity 4.3.1.

Student Activity 4.3.1

When Ed was born, his town of Edenville had a population of 35 000. The average yearly growth rate since then has been 1.5%.

1. Assuming this growth rate continues, construct a table of values relating the population, *P*, of Edenville and time, *t*.
2. a) Use your graphing calculator to graph the Population versus Year. Use window settings $x_{\min} = 0,\ x_{\max} = 100,\ x_{scl} = 10,\ y_{\min} = 0,\ y_{\max} = 100\,000,\ y_{scl} = 10\,000$.
   b) Explain how the characteristics of the graph indicate exponential growth.
3. Determine an equation, in the form $y = ca^x$, for the population of Edenville.
4. Explain how your equation would change if the population were declining at a rate of 1.5%.
5. a) Use your equation to determine the population of Edenville on Ed's 15th birthday.
   b) Use your graphical model to verify your solution.
6. Consider the limitations of both the graphical model and the algebraic model in this context. Summarize these restrictions.

Teacher Facilitation

- For Student Activity 4.3.2, students are placed in groups of two to four.
- The problem sets can be set up as a circuit; students can work through the stations in any order. An alternative is to have each group work through one or two problem sets and present their solutions to the class.
- Distribute Student Activity 4.3.2.

Student Activity 4.3.2

1. Ed got very sick one day and decided to go to the doctor. Dr. B. Better told him that he had a bacterial infection and put Ed on penicillin. The doctor hypothesised that he presently had 10 000 bacterium in his body. The net effect of penicillin killing the bacteria and the bacteria growing results in an overall decrease of bacteria 5% every hour.
   a) Construct a table of values relating the number of bacteria and time.
   b) Use your graphing calculator to graph the relationship between the number of bacteria and time.
   c) Determine an equation, in the form $y = ca^x$, to represent the number of bacteria remaining, *N*, after *t* hours. Enter this equation into a graphing calculator to determine how well it fits your data.

d) Explain how your original equation would change if the bacteria decreased by 5% every 3 hours.
e) Use your equation to determine the number of bacteria remaining after 16 hours. Verify your answer using your graphical model.
f) Can you use your equation to determine the number of bacteria present after 7 days? Explain why or why not.
g) Use your graph to determine when the number of bacteria in Ed will be reduced to 0. Explain why this is or is not realistic.

2. Ed decided to invest $1 000 for college. Presently, the banks are offering 4%/a compounded yearly.
   a) Construct a table of values relating the amount of money, A, and the number of years n.
   b) Use your graphing calculator to graph the relationship between the accumulated amount of Ed's money and the number of years the money is invested.
   c) Determine an equation, in the form $y = ca^x$, to represent the accumulated amount of money, A, over n years of investment.
   d) Explain how your original equation would change if the interest were compounded quarterly.
   e) Use your original equation to determine how much money Ed is predicted to have after 3 years. Verify your solution using your graphical model.
   f) State any limitations of your algebraic model and on your graphical model.
3. At basketball practice, Ed noticed that when he drops the basketball it only bounces back up to 60% of its original height.
   a) Construct a table of values relating the height of the ball, h, and the number of bounces, n. Use a starting height of 2 m.
   b) Use your graphing calculator to graph the relationship between the height of the ball and the number of bounces.
   c) Determine an equation, in the form $y = ca^x$, to represent the height, h, of the ball after n bounces.
   d) Use your equation to determine the height of the ball after 4 bounces. Verify your solution using your graphical model.
   e) Is it realistic to use your equation or graph to determine the height after 80 bounces? Explain why or why not.
4. Ed's parents bought a car for $25 000. He was told that in any given year, this particular car depreciates to 70% of its value of the previous year.
   a) Construct a table of values relating the value of the car, V, and the number of years, n.
   b) Use your graphing calculator to graph the relationship between the value of the car and the number of years after its purchase.
   c) Determine an equation, in the form $y = ca^x$, to represent the value of the car n years after it is purchased.
   d) Explain how your original equation would change if the car depreciated to 70% of its value every second year, rather than every year.
   e) Explain how your original equation would change if the car depreciated to 80% of its value in the previous year.
   f) Use your equation to determine the value of the car after 10 years. Verify your answer using your graphical model.
   g) Explain any restrictions or limitations of either your algebraic model or graphical model.
5. Using his microscope, Ed counted 30 bacterium in his petri dish at the beginning of biology class. After carefully watching the bacteria, he observed that they double every hour.
   a) Construct a table of values relating the number of bacteria, n, and time, t.
   b) Use your graphing calculator to graph the relationship between the number of bacteria and time.
   c) Determine an equation, in the form $y = ca^x$, for the number of bacteria after t hours.
   d) Explain how your original equation would change if the bacteria doubled every 3 hours.

e) Use your equation to determine how many bacteria there will be after 24 hours. Verify your answer using your graphical model.
f) Would your equation be able to accurately predict the number of bacterium in the petri dish after six months? Explain why or why not.

6. The amount of time it takes for a radioactive element to decay to one half of its original amount is known as half-life. In 1970, Ed's science teacher purchased 500 g of cobalt-60 to show students. The half-life of cobalt-60 is 5 years.
   a) Construct a table of values relating the amount of cobalt-60, A, and number of years, n.
   b) Use your graphing calculator to graph the relationship between the amount of cobalt-60 and the number of years.
   c) Determine an equation, in the form $y = ca^x$, for the amount of cobalt-60 after n years.
   d) Explain how your equation would change if the half-life of cobalt-60 were 10 years.
   e) Use your equation to determine how much cobalt was present in the year 2000. Verify your answer using your graphical model.
   f) Explain any restrictions or limitations of either your algebraic model or graphical model.

Assessment & Evaluation of Student Achievement

- Students can be assessed on Teamwork throughout the activity.
- If students present their solutions in groups, these can be assessed formatively on Application of knowledge as well as Communication skills.

Activity 4.4: Bridging the Gap with Logs

Time: 3 hours

Description

Students investigate the connection between exponential functions and logarithmic functions. Students extend their knowledge of inverses and apply it to exponential functions to discover the logarithmic function.

Strand(s) & Learning Expectations

Ontario Catholic School Graduate Expectations

CGE2c - an effective communicator who presents information and ideas clearly and honestly and with sensitivity to others;
CGE2d - an effective communicator who writes and speaks fluently one or both of Canada's official languages.

Strand(s): Exponential and Logarithmic Functions, Applications and Consolidation

Overall Expectations

ELV.01 - demonstrate an understanding of the nature of exponential growth and decay;
ELV.02 - define and apply logarithmic functions;
ACV.04 - demonstrate facility in carrying out and applying key manipulation skills.

Specific Expectations

EL1.01 - identify, through investigations, using graphing calculators or graphing software, the key properties of exponential functions of the form $a^x (a > 0, a \neq 1)$ and their graphs (e.g., the domain is the set of real numbers; the range is the set of the positive real numbers; the function either increases or decreases throughout its domain; the graph has the x-axis as an asymptote and has y-intercept = 1);
EL2.01 - define the logarithmic function $\log_a x$ ($a > 1$) as the inverse of the exponential function a^x, and compare the properties of the two functions;

EL2.02 - express logarithmic equations in exponential form, and vice versa;
AC4.04 - demonstrate facility in using manipulation skills related to solving linear, quadratic, and polynomial equations, simplifying rational expressions, and operating with exponents.

Prior Knowledge & Skills

- Understanding of properties of exponential functions and exponential graphs
- The concept of the inverse as an interchange of *x* co-ordinates with *y* co-ordinates of the original function
- Use of the zoom and trace features of a graphing calculator to determine points on a graph

Planning Notes

- Students require graph paper, MIRAs, and graphing calculators. Provide window settings for graphing calculator activities, e.g., for Student Activity 4.4.1;
  use $x_{\min} = -3$, $x_{\max} = 5$, $x_{sclx} = 1$, $y_{\min} = -1$, $y_{\max} = 10$, $y_{scl} = 1$.
- Prepare worksheets.
- Do a class activity using the temperature probe to practise gathering and graphing authentic data. For example, use a temperature probe to measure the temperature of a cup of coffee as it cools. Then graph the data to determine the pattern of cooling, and compare this with Newton's law of cooling. Refer to Resources for a variety of calculator-based laboratory (CBL) activities appropriate for this purpose.

Teaching/Learning Strategies

Teacher Facilitation

- Review with the class the characteristics of exponential functions and real-life models that are exponential in nature (population growth, compound interest, radioactive decay, etc.).
- Review the key properties of the graphs of exponential functions.
- Distribute graphing calculators, graph paper and Student Activity 4.4.1.
- Students work through the questions independently or in pairs.

Student Activity 4.4.1

1. Create a table of values for each of the following functions
   a) $y = 2^x$ b) $y = 3^x$ c) $y = 5^x$
2. Graph each of the functions on a separate grid.
3. Use your graphs to answer the following questions:
   a) What point is common to all graphs?
   b) Do these graphs represent functions? Explain why or why not.
   c) State the Domain and Range of each function.
4. Graph the line $y = x$ on each grid.
5. Place the MIRA along the line $y = x$. Graph the reflection on the same grid as the original function. Describe the relationship between the graph of the original exponential function and its reflected graph.
6. Identify several co-ordinates of the reflected graph and record them in a table of values.
7. Use your reflected graphs to answer the following questions:
   a) What point is common to all of the reflected graphs?
   b) Do these graphs represent functions? Explain why or why not.
   c) State the Domain and Range of each function.
8. For each of the original graphs and its reflected graph, compare the co-ordinates from the table of values created in #1 with the table of values created in #6. Is there a relationship? Explain your findings.

9. Determine the inverse of the function $y = 7^x$ by interchanging the x and y co-ordinates. Graph the original equation and its inverse on the same set of axes. Use the MIRA to confirm the accuracy of your inverse.

Teacher Facilitation

- Discuss students' findings in a teacher-directed discussion.
- The teacher facilitates consolidation of students' findings regarding the following properties of the graph that is the inverse of the exponential function: Domain, Range, the x-intercept (1,0) as a common point of each of the graphs for the inverse function, the shape of the graph, and the behaviour of the function around the x-axis.
- Discuss the term *logarithm* and its meaning.
- In Student Activity 4.4.2, students are introduced to the definition of the logarithm, "log", as the inverse of the exponential function. Graphing calculators are used to visually illustrate exponential and logarithmic functions as inverse functions of each other. Students also compare the properties of exponential and logarithmic functions.
- Distribute graphing calculators and Student Activity 4.4.2.

<u>Student Activity 4.4.2</u>

1. Use graphing calculators to graph the function $y = 10^x$. Sketch the graph in your notes.
2. Use the zoom feature from the graphing calculator or the trace function to fill in the missing values in the following table.

x	0	1	2	3	4
y					

Confirm the co-ordinates in the completed table of values using the equation.

3. a) Determine the inverse of the function $y = 10^x$ by interchanging the x and y co-ordinates in the table in question #2. Complete the table of values for the inverse function:

x	1		100		10 000
y		1		3	

b) Examine the table of values and describe the shape of the inverse function.
c) Graph $y = \log_{10}x$ on the graphing calculator. Compare the co-ordinates of this graph to the table in (a). What relationship exists between the functions $y = 10^x$ and $y = \log_{10}x$?
d) Verify that the inverse of $y = 10^x$ is $y = \log_{10}x$ by graphing both functions on the same set of axes with the graphing calculator.

4. Given the equation $y = 8^x$:
a) What is the equation of the inverse function?
b) Verify your inverse function visually by graphing both functions on the same set of axes.

5. a) Use graphing technology to graph $y = 4^x$, $y = 9^x$, $y = 11^x$, and $y = 20^x$ on the same set of axes. Use your graphs to complete the following chart:

Exponential Equation	**Describe the shape of the graph**	**Domain**	**Range**	**Intervals where increasing (if any)**	**Intervals where decreasing (if any)**	**Equation of asymptote**	**Intercepts**
$y = 3^x$							
$y = 7^x$							
$y = 11^x$							
$y = 20^x$							

b) For each function in (a), determine the equation of the inverse function (the logarithmic equation).

c) Use the line $y = x$ to graph the inverse of the equations in (a) on the same set of axes. Use your graphs to complete the following chart:

Logarithmic Equation	**Describe the shape of the graph**	**Domain**	**Range**	**Intervals where increasing (if any)**	**Intervals where decreasing (if any)**	**Equation of asymptote**	**Intercepts**

d) Compare the properties of the graph of the exponential functions with the properties of the graphs of the inverse, logarithmic functions. How are they alike? How are they different?

Teacher Facilitation

- In a teacher-directed discussion, discuss students' findings. Students must be clear that the logarithmic function $y = \log_a x$, a > 1, $a \neq 1$, is the inverse of the exponential function $y = a^x$. During the discussion, provide visual demonstrations using the overhead.
- Discuss students' results for question #5. Students must be clear about the graphical properties of $y = \log_a x$ and how these graphical properties compare with the exponential functions $y = a^x$. During the discussion, provide visual demonstrations.
- Provide context for the usefulness of determining the inverse function through a discussion of compound interest where the time of an investment must be determined instead of accumulated amount.
- Discuss the following concepts relating to the relationship between exponential equations and logarithmic equations:
  1. To find the inverse of $y = a^x$, switch the x and y co-ordinates: $x = a^y$.
  2. The function $x = a^y$ is called a logarithmic function, and $x = a^y \Leftrightarrow y = \log_a x$. Note that $\log_a x$ is the exponent to which the base a must be raised to give x.
  3. Examples
     a) The exponential equation $x = 5^y$ can be written in logarithmic form $y = \log_5 x$.
     b) The logarithmic equation $y = \log_7 x$ can be written in exponential form $x = 7^y$.
     c) The exponential equation $2^4 = 16$ can be written in logarithmic form $4 = \log_2 16$.
     d) The logarithmic equation $2 = \log_3 9$ can be written in exponential form $9 = 3^2$.
- Discuss why logarithms are useful (they were created to help us solve exponential equations; they were the only way to isolate the exponent).
- In Student Activity 4.4.3, students solve exponential equations both graphically (with graphing technology) and logarithmically.
- Distribute Student Activity 4.4.3.

Student Activity 4.4.3

1. a) Use graphing technology to graph $y = 6^x$ and its inverse on the same set of axes. Remember that the graph of the inverse function can be determined by interchanging the x and y values of the original function, or by reflecting the original function in the line $y = x$.
   b) The graph of the inverse can be used to solve for the exponent of the original function. Look at the original graph and its inverse, and explain how to use the graph of the inverse to solve $36 = 6^x$.
   c) Using the equation of the inverse function, approximate a solution for the original function $y = 6^x$ for the following values of y:
      i) $y = 6$ ii) $y = 1$ iii) $y = 88$ iv) $y = 15$
   d) Explain how to solve exponential equations graphically.

2. Find an approximate solution by solving graphically:
   a) $2^x = 6$ b) $5^x = 11$ c) $8^x = 23$
3. Use the relationship $x = a^y \Leftrightarrow y = \log_a x$ to solve each of the following:
   a) $\log_5 25 = x$ b) $\log_4 64 = x$ c) $\log_2 128 = x$
   d) $\log_{10} 100 = x$ e) $\log_3 x = 0$ f) $\log_6 x = 2$
   g) $\log_7 x = 4$ h) $\log_8 x = 3$ i) $\log_5 x = 5$
4. The equation $y = 15^x$ represents the growth of a population of dust mites, where x represents the number of days. Determine, to the nearest day, how long it will take for the population to reach:
   a) 150 b) 600 c) 3000
   What assumption did you make? (*started with one dust mite.*)
5. The equation $y = 4^x$ represents the population increase of the number of algae, where x represents the number of weeks. Determine, to the nearest week, how long it will take for the population to reach:
   a) 80 b) 400 d) 2000
   What assumption did you make?

Teacher Facilitation

- Additional questions like #3 should be assigned if further practice is needed for students to work with the relationship $x = a^y \Leftrightarrow y = \log_a x$.
- Extension
  - Students could explore the restrictions of logarithmic functions.
  - Students could explore the graphs of logarithmic functions of different bases including $a = 1$, $a < 0$, and $0 < a < 1$.

Assessment & Evaluation of Student Achievement

- Learning Skills, such as Works Independently, or Teamwork can be assessed in Student Activity 4.4.1 and Student Activity 4.4.2.
- Questions in Student Activity 4.4.3 that require explanations can be assessed for Communication.
- Student Activity 4.4.3 can be assessed for Knowledge/Understanding using a marking scheme.

Resources

Brueningsen, C., et al. *Real-World Math with the CBL System – 25 Activities Using the CBL and TI-82.* Texas Instruments, 1994.

Brueningsen, C., et al. *Real-World Math with the CBL System – Activities for the TI-83 and TI-83 Plus*. Texas Instruments, 1994.

Activity 4.5: A Log "Jam Session"

Time: 3.5 hours

Description

Students extend their knowledge of logarithms and logarithmic graphs, and apply logarithms in context to real-life situations involving sound. Students experiment with different sound levels from a portable radio or CD/tape player to determine the intensity of sound at various levels of volume.

Strand(s) & Learning Expectations

Ontario Catholic School Graduate Expectations

CGE3c - a reflective and creative thinker who thinks reflectively and creatively to evaluate situations and solve problems;

CGE5g - a collaborative contributor who achieves excellence, originality, and integrity in one's own work and supports these qualities in the work of others.

Strand(s): Exponential and Logarithmic Functions, Applications and Consolidation

Overall Expectations
ELV.02 - define and apply logarithmic functions;
ACV.01 - analyse models of linear, quadratic, exponential, or trigonometric functions drawn from a variety of applications.

Specific Expectations
EL2.04 - solve simple problems involving logarithmic scales;
AC1.01 - determine the key features of a mathematical model (e.g., an equation, a table of values, a graph) of a function drawn from an application;
AC1.02 - compare the key features of a mathematical model with the features of the application it represents;
AC1.03 - predict future behaviour within an application by extrapolating from a given model of a function;
AC1.04 - pose questions related to an application and use a given function model to answer them.

Prior Knowledge & Skills

- Understanding of the key properties of logarithmic functions and their graphs;
- How to identify the model (graphical or algebraic) used in an investigation;
- How to interpolate, extrapolate, and apply their results from an investigation.

Planning Notes

- Prepare worksheets.
- Discuss with students the definitions of decibel, Richter Scale, and pH.
- Make sure that students know how to use and read a sound-level meter.
- Students need access to the Internet to complete Student Activity 4.5.2. Teachers may investigate websites to recommend to students. Refer to Resources for suggested websites.
- To collect data, students require a radio or a CD/tape player (one per group) and a sound-level meter (one per group). If the Mathematics Department does not have a sound-level meter, sound-level meters may be available through the Science or the Technology Department.
- **Note:** An alternate method of collecting data is to use a calculator-based laboratory (CBL) and a microphone.
- Students require graph paper.
- Students should work in groups of two to four for Student Activity 4.5.3.

Teaching/Learning Strategies

Teacher Facilitation

- Review with the class applications of logarithms and real life models that are logarithmic in nature (compound interest, pH levels, Richter scale, decibel scale). Briefly explain the decibel scale and discuss its usefulness in a variety of contexts.
- Discuss the function of logarithms, i.e., they allow us to make very large or very small numbers more manageable to work with, and discuss the function of logarithmic scales, i.e., they measure quantities that can have a very large range.
- Discuss the concept of the intensity of sound. Relate different levels of sounds to music concerts, machinery, a car horn, barely audible whispers, and to sounds not detectable by the human ear.

- Prepare students for the activity by discussing hearing: The human ear is capable of hearing a wide range of sounds. The intensity of sounds, and related electronic measurements are often expressed in decibels (abbreviated as dB). The dB is not an absolute measurement; it is based upon the relative intensity between two sounds. Furthermore, it is a logarithmic concept, so that when comparing very large ratios, it can be expressed with small numbers.
- Students need practice and instruction on how to use a sound-level meter properly.
- Student Activity 4.5.1 provides exercises designed to introduce decibels and how decibels relate to different levels of loudness.
- Divide students into small groups of two to four. Provide each group with a sound-level meter to gather data and distribute Student Activity 4.5.1.
- Complete #1(a) together as a class; students complete the remaining questions in their groups.

Student Activity 4.5.1

The formula for computing the decibel relationship between two sounds of intensity A and B is given by the formula $X = 10 \log\left(\frac{A}{B}\right)$, where A is the intensity at sound level X and B is a standard reference intensity near the lower level of human hearing.

Questions

1. Table 1 shows a comparison of intensity ratios and their sound level equivalents measured in decibels. Note that if the intensity of a sound is increased by a multiple of 10, the sound level increases by 10 dB, but if the intensity is multiplied by 100, the sound level only increases by 20 dB.

   Table 1 – Ratio of Intensity of Sound Compared to dB level

$\frac{A}{B}$	2	3	5	10	20	32	100	1000
	3	5	7	10	13	15	20	30

   Using the table, how many times more intense is a sound of 20 dB than a sound of
   a) 15 dB, b) 10 dB, c) 5 dB

2. a) Choose two additional noisy events in your daily life at school to measure. Add them to the first column. Use sound-level meters to measure the noise level (both low and high level readings) of each of the following:

Event measured	**Low Reading (dB)**	**High Reading (dB)**	**Average Reading (dB)**
Conversational Speech in the classroom			
Cafeteria during a lunch period			
Announcements			

   b) Determine an average reading for each event measured.
   c) Which aspects of your daily life at school are the noisiest? Propose measures that could be used to reduce noise levels in your environment. Provide justification why these measures would be both effective and necessary.

Teacher Facilitation

- Discuss the findings of the class for Student Activity 4.5.1. Discuss students' surroundings and the effects of prolonged exposure of high intensity to sound (gathered in #2). Brainstorm ways of reducing or eliminating loud sounds in the classroom, community, work, and home environments.
- Student Activity 4.5.2 is designed for students to consider the noise in their own environment and provides an opportunity for research into the effects of noise pollution and measures to control noise pollution. Students require time to complete their Internet research.

- Distribute graph paper and Student Activity 4.5.2.
- Two columns in Table 2 are already complete. Illustrate to the class how the number of maximum hours for these two sound levels (90 dB and 95 dB) are calculated. Students complete the rest of the worksheet.

Student Activity 4.5.2

Part A

Listening to very loud sounds over a sustained period of time can permanently damage a person's hearing. Table 2 below shows various dB levels and the approximate corresponding maximum number of hours of exposure recommended in order to avoid hearing loss. Note that for every 5 dB increase in sound, the number of hours of maximum exposure at that sound level is reduced by one-half (in order to avoid hearing loss). Use this information to complete the following table:

Table 2 – Protecting your hearing

Sound Level (dB)	90	92	95	97	100	102	105	110
Maximum number of hours, *h*, of exposure per day in order to avoid hearing loss	8		4					

Part B

1. Automobiles and Traffic are high contributors to noise in the environment. The following table summarizes the noise pollution of cars and trucks at various speeds.

Speed (km/h)	**Noise at 20 m (dB)**		
	Auto	**Medium Truck**	**Heavy Truck**
10	15	20	31
48	60	70	81
64	68	80	84
80	71	83	86
96	74	86	88
112	78	87	89

   a) Graph the following relationships:
      i) speed versus noise at 20 m of an auto   ii) speed versus noise at 20 m of a medium truck
      iii) speed versus noise at 20 m of a heavy truck.
   b) Use the table and your graph to describe the relationship between speed and noise level.

Teacher Facilitation

- Discuss students' findings. As an alternative to a teacher-led discussion, students could briefly present their solutions to the class.
- For Student Activity 4.5.3, students need to attach the sound-level meter to their headphones using masking tape. Students take the sound level at equal increments on the volume control of the radio/CD player/tape player. If the volume setting does not have numbers, students can use correction fluid or some marking device to make eight to ten equal increments on the volume control. At each increment, students measure the sound level produced and record it in a table.
- Students continue to work in groups. Each group needs a radio or CD/tape player, a sound level meter, and graph paper. Distribute Student Activity 4.5.3.

Student Activity 4.5.3

Follow the instructions to complete the investigation

1. Test the sound-level meter to make sure that it is working properly and that you know how to measure the readings correctly.
2. Using the numeric settings on your volume control, determine the increments on the CD/tape player you are going to measure with the sound-level meter. If there are no settings, make eight to ten equally spaced marks on the control dial, beginning with no volume.
3. Attach the headphones to the microphone of the sound-level meter using masking tape.
4. Beginning in the off position (volume setting 0), measure the sound level at each increment on the volume control. Record the sound level in Table 3.
5. Repeat the procedure for other radio/CD/tape player sounds as additional trials.

Table 3 – Sound Level Readings

Volume Setting	**Sound Level**		
	Trial 1	**Trial 2**	**Trial 3**
0			
1			
2			
3			
etc.			

6. a) Make sure the volume is turned to a low level. Remove the sound-level meter from the headphones and place them on your head. Adjust the volume on the radio/CD/tape player until it reaches the volume at which you prefer to listen to music. Record this volume setting in Table 4.
   b) In Table 4, record the estimated number of hours in a typical day that you would listen to music at this level.

   Table 4 – Personal Data

Preferred volume setting for listening	
Average Time spent listening to music	

7. a) Use Table 3 to make a graph of sound level (y-axis) versus volume setting (x-axis).
   b) Is there any obvious relationship that exists between the sound level and volume setting?
8. Use the graph in #7 to determine the sound level corresponding to your preferred listening volume setting
9. a) Use Table 2 to determine the maximum time you should be listening to your music at the volume you prefer.
   b) Are your music habits potentially dangerous for your hearing? Explain.
   c) If you lower the volume by two settings from your preferred volume setting, determine the maximum length of time you can now listen to your music without causing any permanent hearing damage.
10. a) For each volume setting, suggest the maximum number of hours of listening and justify your results.
    b) Summarize your findings by drawing a volume line (like a number line). On it, place the decibel rating and the suggested number of hours.

Assessment & Evaluation of Student Achievement

- Thinking/Inquiry/Problem Solving can be assessed formatively by having students hand in (as groups) the investigation of Student Activity 4.5.3.
- Student Activity 4.5.1 can be assessed for Application.
- Knowledge/Understanding can be assessed using a quiz.

Extension

1. A student is considering the purchase of a new car stereo and thinks he needs 150 W amplifier producing 100 W of power. The student has asked you to determine if this is a good choice. To aid the student, you need to investigate:
   a) the possible intensity of the sound from this system at a distance of r = 1 m (r represents radius or distance from the sound source);
   b) the corresponding sound level. Use the equation $I = \frac{P}{4\pi r^2}$, where P is the power in watts and I is the intensity measured in W/m^2. $I_0 = 10^{-12}$ W/m^2. Show all the calculations and recommend if the purchase of this stereo is a wise choice. Explain.
2. Using the Internet, research:
   a) five major noise polluters in our community;
   b) five effects that noise has on human health and hearing;
   c) five occupations where hearing loss can be a major hazard. Research the safety measures these occupations use to attempt to protect the hearing of employees.

Accommodations

The activity needs to be adapted for students with hearing impairments.

Resources

http://www.nonoise.org/resource/trans/highway/spnoise.htm
http://www.noisesolutions.com/
http://www.lhh.org/noise/index.htm
http://www.nonoise.org
http://www.noisesolutions.com/ .

Activity 4.6: Logarithms and Applications

Time: 2 hours

Description

Students use previous knowledge of logarithms to develop and examine the laws of logarithms. Logarithms are then used to solve problems involving logarithmic scales. Authentic data relating to pH scales, decibel scales, and Richter scales is gathered and used in problem solving.

Strand(s) & Learning Expectations

Ontario Catholic School Graduate Expectations

CGE2b - an effective communicator who reads, understands, and uses written material effectively;
CGE2c - an effective communicator who presents information and ideas clearly and honestly and with sensitivity to others.

Strand(s): Exponential and Logarithmic Functions, Applications and Consolidation

Overall Expectations

ELV.02 - define and apply logarithmic functions;
ACV.04 - demonstrate facility in carrying out and applying key manipulation skills.

Specific Expectations

EL2.03 - simplify and evaluate expressions containing logarithms, using the laws of logarithms;
EL2.04 - solve simple problems involving logarithmic scales.

Prior Knowledge & Skills

- Understanding of the key properties of logarithmic functions and their graphs
- The decibel scale, the Richter Scale, and the pH scale
- How to calculate logarithms on a scientific calculator
- The relationship $x = a^y \Leftrightarrow y = \log_a x$
- Exponent laws

Planning Notes

- Prepare worksheets.
- Students require scientific calculators.
- Students require the Internet to research data for Student Activity 4.6.2. Students may search for their own data using search engines. However, if the teacher is recommending websites, they should be screened.
- An alternate to researching pH levels for substances may be to use the pH meter from the Science Department. In this case, simply dip the meter into the solution and the digital output shows the pH level.

Teaching/Learning Strategies

Teacher Facilitation

- In Student Activity 4.6.1, students develop an understanding of the product law of logarithms, the quotient law of logarithms, and the power law of logarithms. Students can work in pairs.
- Distribute Student Activity 4.6.1.

Student Activity 4.6.1 - Investigating the laws of logarithms

1. a) Use your knowledge of logarithms to complete the chart.

$\log_a B$	$\log_a C$	$\log_a B + \log_a C$	$\log_a (B \times C)$
$\log_2 16 =$	$\log_2 4 =$		$\log_2 64 =$
$\log_3 81 =$	$\log_3 9 =$		$\log_3 729 =$
$\log_4 64 =$	$\log_4 16 =$		$\log_4 1024 =$

   b) What pattern do you notice in each row?
   c) Write instructions, in words, for evaluating $\log_a(B \times C)$.
   d) Use the information in the chart to write the product law of logarithms.
2. Evaluate using the product law of logarithms:
   a) $\log_6 2 + \log_6 108$ b) $\log_{10} 5 + \log_{10} 20$ c) $\log_4 2 + \log_4 32$
3. a) Use your knowledge of logarithms to complete the chart.

$\log_a B$	$\log_a C$	$\log_a B - \log_a C$	$\log_a \frac{B}{C}$
$\log_5 125 =$	$\log_5 5 =$		$\log_5 \frac{125}{5} =$
$\log_2 256 =$	$\log_2 64 =$		$\log_2 \frac{256}{64} =$
$\log_3 243 =$	$\log_3 27 =$		$\log_3 \frac{243}{27} =$

   b) What pattern do you notice in each row?
   c) Write instructions, in words, for evaluating $\log_a \frac{B}{C}$.
   d) Use the information in the chart to write the quotient law of logarithms.

4. Evaluate using the quotient law of logarithms:
   a) $\log_8 1024 - \log_8 2$ b) $\log_3 108 - \log_3 4$ c) $\log_2 160 - \log_2 10$
5. Complete the following chart

$\log_a B$	$\log_a B^D$	$D(\log_a B)$
$\log_8 8$ =	$\log_8 8^3$ =	$3(\log_8 8)$ =
$\log_3 3$ =	$\log_3 3^7$ =	$7(\log_3 3)$ =
$\log_2 4$ =	$\log_2 4^5$ =	$5(\log_2 4)$ =

   b) What pattern do you notice in each row?
   c) Write instructions, in words, for evaluating $\log_a B^D$.
   d) Use the information in the chart to write the power law of logarithms.
6. Evaluate:
   a) $\log_5 25^{16}$ b) $\log_9 81^{43}$ c) $\log_4 64^9$

Teacher Facilitation

- Discuss students' findings. Summarize the laws of logarithms on the board. Ensure that the laws of logarithms are explained and clarified as needed.
- Students may need additional time to work with practice questions that use the laws of logarithms. Provide students with practice questions resembling questions #2, #4, and #6.
- Explain or review the purpose of pH, decibel, and Richter logarithmic scales. Discuss the concept of hydrogen concentration $[H^+]$ and its application with pH scales.
- Complete examples of simple problems involving pH, decibel, and Richter logarithmic scales together with the class. The following formulas are useful: $pH = -\log[H^+]$; $M = \log \frac{I}{S}$, where M is the magnitude of an earthquake, I is the intensity of the earthquake and S is the intensity of a "standard" earthquake; $X = 10dB\ log\left(\frac{A}{B}\right)$ for decibels, used in an earlier activity.
- Sample examples:
  a) Lemon Juice has pH 2.3. Calculate the $[H^+]$.
  b) The $[H^+]$ level of tomato juice is 3.2×10^{-4}mol/L. Calculate the pH.
  c) Port Hope experienced an earthquake of magnitude 1.7 on March 22, 2001. What would be the measure of an earthquake that is double the intensity?
  d) How many times louder than a mosquito buzzing at 40 dB is a hair dryer at 70 dB?
  Give students additional examples of similar questions to complete in pairs.
- For Student Activity 4.6.2, students use the Internet to gather authentic data relevant to logarithmic scales. Students may work in pairs. Distribute Student Activity 4.6.2.

Student Activity 4.6.2

1. The pH scale measures the acidity of a substance. Complete the chart by first researching the pH level of the given substances. Choose two additional substances to add to the last two rows. Then, calculate the associated $[H^+]$ for each substance.

Substance	Researched pH level	Calculated [H+].
Milk		
Diet Cola		
Regular Cola		
Water		
Orange juice		
Grapefruit juice		
Vinegar		
Cleaning Fluid		

2. The Richter Scale is used to measure the relative magnitude of earthquakes.
   a) Locate information (location, date, and magnitude) for six earthquakes that are less than 8.9 on the Richter Scale and complete the first two columns in the chart:

Date and location	Magnitude on the Richter Scale	Calculate the magnitude of an earthquake twice as intense	How many times more intense is this earthquake than an earthquake measuring 8.9 on the Richter Scale?
1.			
2.			
3.			
4.			
5.			
6.			

   b) Use logarithms to complete the third column of the chart.
   c) Use logarithms to complete the fourth column of the chart.
3. The Decibel Scale is used to measure the intensity level of sound.
   a) Locate the decibel level for five sounds greater than 30 dB and complete the first two columns of the chart:

Sound	dB level	How many times more intense is this sound than a soft whisper of 30 dB?	How many times more intense is this sound than a wristwatch ticking at 20 dB?
1.			
2.			
3.			
4.			
5.			

   b) Use logarithms to complete the third column of the chart.
   c) Use logarithms to complete the fourth column of the chart.

Assessment & Evaluation of Student Achievement

- A quiz can be used to assess Knowledge/Understanding of Student Activity 4.6.1.
- For Student Activity 4.6.2, assess Teamwork if students work in pairs
- For Student Activity 4.6.2, use questions #1 and #2 as learning tasks. Collect question #3 to assess for Application, and have students write a summary of their findings for question #3 to be assessed for Communication.

Resources

http://www.nal.usda.gov:8001/Safety/SISAppen.pdf (for pH information)
http://www.gp.uwo.ca/docs/eqlist.html (for earthquake information)
http://www.pgc.nrcan.gc.ca/seismo/table.htm (for earthquake information)
http://www.equakealert.com/bc_earthquakes/intense.htm (for earthquake information)
http://www.pgc.nrcan.gc.ca/seismo/eqinfo/eq-westcan.htm (for earthquake information)

Activity 4.7: Summative Assessment: Springfield and Shelbyville's History

Time: 1.5 hours

Description

Students demonstrate their knowledge of exponential functions and logarithms in a variety of applications.

Strand(s) & Learning Expectations

Ontario Catholic School Graduate Expectations

CGE2b - an effective communicator who reads, understands, and uses written material effectively;
CGE2c - an effective communicator who presents information and ideas clearly and honestly and with sensitivity to others;
CGE5g - a collaborative contributor who achieves excellence, originality, and integrity in one's own work and supports these qualities in the work of others;
CGE7b - a responsible citizen who accepts accountability for one's own actions.

Strand(s): Exponential and Logarithmic Functions, Applications and Consolidation

Overall Expectations

ELV.01 - demonstrate an understanding of the nature of exponential growth and decay;
ELV.02 - define and apply logarithmic functions;
ACV.04 - demonstrate facility in carrying out and applying key manipulation skills.

Specific Expectations

EL1.04 - describe the significance of exponential growth and decay within the context of applications represented by various mathematical models;
EL1.05 - pose and solve problems related to models of exponential functions drawn from a variety of applications, and communicate the solutions with clarity and justification;
EL2.02 - express logarithmic equations in exponential form, and vice versa;
EL2.03 - simplify and evaluate expressions containing logarithms, using the laws of logarithms;
EL2.04 - solve simple problems involving logarithmic scales;
AC4.04 - demonstrate facility in using manipulation skills related to solving linear, quadratic, and polynomial equations, simplifying rational expressions, and operating with exponents.

Prior Knowledge & Skills

- Knowledge of the concepts introduced and examined throughout Activities 4.1 to 4.6
- Applications related to exponential growth/decay and logarithmic scales

Planning Notes

- Prepare worksheets.
- Students require graph paper and graphing calculators. Provide students with window settings for all graphing calculator activities.
- For Part B, each group needs a temperature probe, a calculator-based laboratory (CBL) unit with unit-to-unit link cable, and a cup of hot water.

Teaching/Learning Strategies

Teacher Facilitation

- Students can be placed in groups of two or three or they may complete the activity individually.
- If students are completing the activity individually, the teacher may allow them time to brainstorm ideas in groups of three or four at the beginning of each class.
- Distribute graphing calculators, graph paper, and Student Activity 4.7.1.

Student Activity 4.7.1

Part A

1. The neighbouring towns of Springfield and Shelbyville were both founded in the year 1900. Springfield started with 100 settlers and had an average yearly growth rate of 5%. Shelbyville started with only 40 settlers but had a growth rate of 8%. Assume a constant rate of growth for each town.
   a) Determine a formula relating the population of Springfield, P, and the number of years, t.
   b) Determine a formula relating the population of Shelbyville, P, and the number of years, t.
   c) Determine the total population of each town in the year 1950.
   d) Use a graphical model to determine the number of years it will take each town to reach a population of 75 000. Verify your solution using your algebraic model.
   e) Use a graphical model to determine in what year Springfield and Shelbyville will have the same population. Verify your solution using your algebraic model.
2. Distance in kilometres above sea level is given by the formula $d = \frac{500(\log_{10} P - 2)}{27}$, where P is the atmospheric pressure measured in kiloPascals, kPa.
   a) At the top of the highest mountain in Shelbyville, the atmospheric pressure was recorded as being 220 kPa. Calculate the height of the mountain above sea level.
   b) The town of Springfield has a mountain with a peak 4.5 km above sea level. Calculate the atmospheric pressure at the top of the mountain.
3. In the year 1980, both towns had an earthquake. Springfield's earthquake measured 7.5 on the Richter Scale while the earthquake in Shelbyville measured a 6.4. Determine the difference in magnitude of the two earthquakes.
4. The Earthquake uncovered an archeological find in Shelbyville and a fossil was uncovered. The formula for the amount of carbon-14 remaining in a fossil is $M(t) = M_0\left(2^{-\frac{t}{5760}}\right)$, where $M(t)$ is the amount of carbon-14 in the fossil at time t, and M_0 is the original amount of carbon-14. Use a graphical model to calculate the age of the fossil if 20% of the original amount of carbon is remaining. Verify your solution using the formula.
5. In 1960, the city of Springfield set up a disaster relief fund based on donations. The amount of money, A, that Springfield must invest compounded annually at 8%/a in order to have B dollars in 20 years is represented by the equation $20(\log 1.08) + \log A = \log B$.
   a) How much money should Springfield have invested in 1960 in order to have its investment grow to $1 000 000 in 1980 when the earthquake hit?
   b) How would this information be represented in an exponential equation?
6. In 1970, Shelbyville's budget was at a surplus. City council decided to invest the $400 000 surplus at 4.25%/a compounded annually.
   a) Create a table of values and graph the relationship between the amount of money, A, and the number of years, n, using a graphing calculator.
   b) Describe how the graph would change if the interest rate were 7%.
   c) Describe how the graph would change if $400 000 were invested initially.
   d) Determine an equation relating the amount of money and the number of years.
   e) Use the equation to determine the accumulated amount of money available when the earthquake hit.
   f) Use a graphical model to determine how long would it take for the money in the fund to reach $1 000 000. Verify your solution using your equation in part (d).

Part B (to be completed in groups of two or three)

1. Use a temperature probe to gather data about the temperature of your cup of hot water as it cools. Measure the temperature each minute for 10 minutes.
2. Create a graphical model of your results. Describe the features of your graph (include domain, range, intervals where increasing, intervals where decreasing, equation of asymptote, and intercepts). What type of graph does it represent?
3. Create an algebraic equation to model your data. Are there any restrictions on your equation? Explain fully.
4. Write a summary of your findings about the rate of cooling of the cup of hot water.

Assessment & Evaluation of Student Achievement

- Use a rubric for the assessment and evaluation of student achievement (see Appendix 4.2).

Appendix 4.1

Rubric for Assessment of Student Activity 4.2.3

* Use this rubric to provide formative feedback for student Activity 4.2.3

Category	Level 1 (50-59%)	Level 2 (60-69%)	Level 3 (70-79%)	Level 4 (80-100%)
Thinking/Inquiry/ Problem Solving Creation of an algebraic model to represent the data	- develops an algebraic model with limited effectiveness	- develops an algebraic model with some effectiveness	- develops an algebraic model with considerable effectiveness	- develops an algebraic model with a high degree of effectiveness
Communication Clarity of conclusions and recommendations made to the city planner	- makes conclusions and recommendations with limited clarity	- makes conclusions and recommendations with some clarity	- makes conclusions and recommendations with considerable clarity	- makes conclusions and recommendations with a high degree of clarity

Note: A student whose achievement is below Level 1 (50%) has not met the expectations for this assignment or activity.

Appendix 4.2

Rubric for Evaluation of Activity 4.7.1

Category/ Criteria	Level 1 (50-59%)	Level 2 (60-69%)	Level 3 (70-79%)	Level 4 (80-100%)
Knowledge/ Understanding Understanding Concepts	- demonstrates a limited understanding of concepts relating to exponents and logarithms	- demonstrates some understanding of concepts relating to exponents and logarithms	- demonstrates considerable understanding of concepts relating to exponents and logarithms	- demonstrates a thorough understanding of concepts relating to exponents and logarithms
Application Application of concepts or procedures	- applies concepts or procedures with limited effectiveness	- applies concepts or procedures with some effectiveness	- applies concepts or procedures effectively	- applies concepts or procedures with considerable effectiveness and efficiency
Communication Clarity of explanations	- demonstrates limited clarity in explanations	- demonstrates some clarity in explanations	- demonstrates considerable clarity in explanations	- demonstrates clear explanation consistently
Thinking/ Inquiry/Problem Solving Creation of a model (either graphical or algebraic)	- creates exponential or logarithmic models that address few aspects of the problem	- creates exponential or logarithmic models that address some aspects of the problem	- creates appropriate exponential or logarithmic models to address the problem	- creates appropriate exponential or logarithmic models that integrate all or almost all aspects of the context

Note: A student whose achievement is below Level 1 (50%) has not met the expectations for this assignment or activity.