Course Profile

Principles of Mathematics

Grade 9
Academic

! *for teachers by teachers*

Course Profiles are professional development materials designed to help teachers implement the new Grade 9 secondary school curriculum. These materials were created by writing partnerships of school boards and subject associations. The development of these resources was funded by the Ontario Ministry of Education and Training. This document reflects the views of the developers and not necessarily those of the Ministry. Permission is given to reproduce these materials for any purpose except profit. Teachers are encouraged to amend, revise, edit, cut, paste, and otherwise adapt this material for educational purposes.

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Academic Course Profile Overview
Principles of Mathematics, Grade 9

Identifying Information

School: **Course Developer:**

Department:

District:

Course Title: Principles of Mathematics

Grade: 9 **Development Date:**

Course Type: Academic **Course Revisor(s):**

Ministry Course Code: MPM1D

Credit Value: 1 Credit **Revision Date:**

Description/Rationale

This course enables students to develop generalizations of mathematical ideas and methods through the exploration of applications, the effective use of technology, and abstract reasoning. Students will investigate relationships to develop equations of straight lines in analytic geometry, explore relationships between volume and surface area of objects in measurement, and apply extended numeric and algebraic skills in problem solving. Students will engage in abstract extensions of core learning that will deepen their mathematical knowledge and enrich their understanding.

This grade nine mathematics course is designed to develop students' proficiency as learners and thinkers in mathematics, and their ability to apply mathematics in their daily lives. Students will interpret, analyze, and create mathematical models to solve realistic problems, within and beyond the mathematics classroom.

Unit Titles and Time

Unit 1:	Constructing Graphical Models through Investigation	35 hours
Unit 2:	Algebraic Models and Rates of Change	30 hours
Unit 3:	Dynamic Geometry and Measurement	30 hours
Unit 4:	Summative Assessment Activities	15 hours

Unit Descriptions

Unit 1: Constructing Graphical Models through Investigation
Time: 35 hours

Description

This Unit will introduce abstract concepts through activities which will engage grade 9 students, and embed the teaching of skills within contexts.

Students will gather, analyze, manipulate, and display data from primary and secondary sources to model and communicate results about both linear and non-linear situations. Many contextual problems will be studied to ensure that students gain depth of understanding through meeting the same specific expectations in different contexts. Students will conduct investigations to verify or refute their own conjecture, using lines or curves of best fit, tables and pattern descriptions. They will communicate their findings and describe trends. A rich contextual foundation for subsequent algebraic studies will be built in this unit. Several different types of technologies will be introduced for gathering, analyzing and displaying data.

Overall Expectations: all those from the Relationships Strand
Specific Expectations: all those from the Relationships Strand and as identified in the activities from each of the other Strands

Unit 2: Algebraic Models and Rates of Change
Time: 30 hours

Description

This unit is designed to:

- weave the expectations of the Analytic Geometry strand together with expectations from each of the other strands in the policy document.
- introduce the abstraction of *x*'s, *y*'s, and vocabulary such as 'slope' and 'intercepts'.
- allow the teacher and students to tie these abstractions back to contexts from the first unit.
- highlight the power of these abstract symbols and concepts as a means to summarize similar models (e.g., $y = 2x$ summarizes H = 2A, total points are double the number of baskets, etc.) and to communicate with technology.
- help students consolidate and extend their algebraic and numeration skills.

Students will use the ideas and contexts of the first unit to develop algebraic models of linear relations. Students will explore and determine the characteristics of lines and their corresponding equations through the use of spreadsheets, graphing technology, and paper and pencil. To solve problems, students will recognize and model realistic situations that involve constant rates of change. The need for algebraic techniques, numeric skills and the laws of exponents will emerge from problems in context.

Overall Expectations: all those from the Analytic Geometry Strand
Specific Expectations: all those from the Analytic Geometry Strand and some from each of the other Strands

Unit 3: Dynamic Geometry and Measurement
Time: 30 hours

Description

This unit is designed to:

- weave the expectations of the Measurement and Geometry strand together with expectations from each of the other strands in the policy document.
- help students to extend their skills in exploring geometric relationships, forming and testing reasonable conjectures, using dynamic geometry software and other means to manipulate and transform, communicating their findings and applying geometric relationships to solve problems.
- help students to consolidate and extend their algebraic and numeration skills through work with formulas and multi step problems.

Students will use concrete materials, diagrams, drawings and dynamic geometric software to investigate the properties of three dimensional objects, optimal measurements and geometric relationships of two dimensional figures. Students will confirm and extend their intuitive understanding of geometric properties through inquiry. They will pose questions and confirm or deny statements using observations made with the help of technology, judge the reasonableness of answers and solve multi-step problems.

Overall Expectations: all those from the Measurement and Geometry Strand
Specific Expectations: all those from the Measurement and Geometry Strand and some from other Strands

Unit 4: Summative Assessment Activities
Time: 15 hours

Description

This unit will be used to model a final assessment in grade 9 mathematics. Individual and group performance skills will be assessed using traditional and performance based tasks, over a period of several days. Thirty percent of the final evaluation for the course will be based on this summative assessment unit and it is recommended that at least 2/3 be based on performance tasks, and at most 1/3 be based on pencil and paper tests. It is suggested that the form and substance of this summative assessment unit be shared with students and their parents near the beginning of the course, so that their energies can be directed towards acquisition of the required skills and knowledge.

In this summative assessment unit, students will demonstrate their achievement of the expectations of the course. They will do this by solving problems which require them to:

- form and test conjectures.
- model situations.
- shift a point of view, defend an alternate viewpoint on the solution of a problem.
- gather, organize, and display data for a purpose.
- identify necessary and/or sufficient conditions in a problem.
- decide, with awareness, what is important and what can be ignored in a problem.
- communicate reasoning and results.

- demonstrate their skills using technology for a purpose.
- carry out pencil and paper routines.

Course Notes

Key messages:

- Mathematical modeling is a primary focus.

  The high school mathematics curriculum is designed to ensure that students understand the power of mathematics in modeling authentic problems and situations, and acquire the various skills needed to create, interpret, and analyze such models.

- The problem comes first.

  Traditionally in mathematics, common practice would be to first teach skills in isolation or limited contexts, and then use the skills to solve problems, contextual or not. In this sample course profile, the problem to come first, although it is still the desired mathematical skills and knowledge that drive the choice of problems and activities.

- Teach skills as they are needed.

  It is intended that the need for the skills (including basic manipulative skill) of the course emerge from rich, contextual settings rather than in isolation. Teachers may well have to take time to develop specific skills, once the need has been identified. An inquiry approach to teaching and learning demands that teachers use flexibility to capture "teachable moments." An analogy might help:

  > The goal of this course is similar to creating a beautiful piece of furniture. It might be necessary, at times, to take time to practise the use of a lathe or sanding on scraps of wood, separate from the finished product. However, all work is directed towards making our fine piece of furniture.

- New classroom routines are needed.

  A range of teaching strategies is expected. To implement the inquiry-related expectations of the curriculum, teachers will likely find it necessary to broaden their repertoire of strategies to facilitate investigations, explorations, and communication of findings. Because there will not be a lot of class time available to engage in large amounts of review of skills, it is recommended that diagnostic testing be used to determine the extent of focus required on numerical skills. Practice and other remediation activities can be assigned as homework. As implementation of the grade 1-8 mathematics program proceeds, teachers of grade 9 mathematics may find that the need for diagnosis and remediation of student background knowledge and skills will become less time-consuming and leave more time for students to work on enhancements and extensions. Research and preparation for the next day's investigation might also be included as homework assignments, along with consolidation and extension of classroom activities

Other points to consider:

- Much of the material in the profiles is common to both the Grade 9 Applied and Grade 9 Academic courses. The content, procedures and processes are important for all students to learn in both programs. The two programs allow for different approaches and emphases as related to time allocation, instructional approaches, and needed supports and extensions.

- To ensure that all students performing at or above the provincial standard have the necessary knowledge and skills to succeed in either the Grade 10 Academic or Grade 10 Applied Mathematics course, summative assessment activities should be similar or identical for the two grade 9 courses except where additional expectations are involved in the Grade 9 Academic course.
- Different developments for some activities within a unit are provided for the applied and academic streams, but students might benefit from the teacher's 'borrowing' from the other stream.
- Unit 1 is very similar in the Academic and Applied profiles since it is important for all students to develop conceptual understandings of linear and non-linear relationships from a concrete point of view. Unit 4 is also very similar in the Academic and Applied versions since students who are working at level 3 or 4 in either course should be well prepared for either the Applied or Academic grade 10 course.
- It is envisioned that Units 2 and 3 will show more differences between the Academic and Applied courses. Unit 2 introduces the abstract concepts of lines. The Applied version will return more frequently to concrete examples and applications in different contexts while the academic version will extend farther and more frequently into abstract applications. In Unit 3, geometric properties will be explored more concretely in the Applied version and more abstractly in the Academic. Problems will be posed in a more open-ended fashion in the Academic course and contain more scaffolding (guiding, supporting) questions in the Applied course.
- When doing investigations, teachers are advised to carry out the experiments themselves, beforehand. Considerations should include timing, questions to pose, means of drawing closure, prerequisite skills, appropriate connections and links to other disciplines, and the choosing of investigations having regional or current interest. Availability of computer resources may determine the sequencing for Unit 3, since computer labs may not be available for all of the grade 9 classes at the same time.
- Technology can be useful in learning, doing, and assessing achievement in mathematics is desirable and essential. When using technology for an activity, teachers are advised to practise its use beforehand. This profile will identify which graphic calculator and dynamic geometry software skills are needed for each activity. Occasionally, teachers may wish to demonstrate the use of technology as a tool for gathering, organizing and displaying data, and at other times students must learn to use the technology themselves.
- Not all Specific Expectations are of equal value. Those which are critical to the development of mathematical literacy are emphasized in the learning activities, and are often revisited. These are expectations which are taught, assessed, evaluated and, where necessary, revisited using alternate instructional strategies in a cyclic process that stops only when students have achieved the Expectations to their optimal Levels.
- Use of the Achievement Levels Chart of Mathematics is the basis of assessment of all aspects of the course.
- The implementation of Grade 9 Mathematics is a process, not an event. It is necessary that those at all levels of the system make continuous and measurable progress towards full implementation.
- The expectations of the Number Sense and Algebra strand will be woven into the first three units rather than separated out.

Teaching/Learning Strategies

Only through the use of a wide variety of teaching, learning, and assessment strategies and tools can the wide range of expectations in this course be addressed.

Teachers will:

- include a balance of whole class, small group and individual instruction.
- include a balance of student-centred and teacher-directed activities.
- provide students with materials, technological tools and software for use in experiments, demonstrations, and investigations.
- address a variety of learning styles in each unit.
- plan so that sufficient time is spent engaging students in the solution of rich contextual problems.
- be accountable to addressing the overall and specific expectations in their planning, and accountable to tracking student progress in the overall expectations, including the most important specific expectations.
- assume a variety of roles in the classroom, including guide, facilitator, and director of learning.
- act as guide and facilitator in the classroom.
- provide many opportunities for students to demonstrate their ability to meet course expectations.
- ensure that the culmination of an activity helps the students to build a solid understanding of the mathematical concepts arising from that activity and sets the stage for future learning.
- prompt at the beginning of an activity, provide suggestions in the middle, and support and challenge at the end, as needed by individual students, and by the class as a whole.
- provide verbal instruction to accompany written procedures to avoid the frustration and uncertainty that so often undermine the learning opportunities afforded by a complex task.
- use learning/performance tasks that are designed to link several expectations and give the students occasion to demonstrate their optimal levels of achievement through the communication of results, the ability to pose extending questions following an inquiry, and to provide the solution to unfamiliar problems.
- provide remediation or extension opportunities.
- provide opportunities for students to practise or extend their skills and knowledge, outside of the classroom.
- provide regular, informal assessment which provides the feedback that students need in order to improve their achievement.
- modify instructional and assessment strategies for special needs students.

Students will:

- develop increasing responsibility for their own learning.
- carry out investigations and engage in the inquiry process.

- explore, hypothesize or formulate, manipulate or transform, infer or conclude, and communicate during an inquiry.
- engage in explorations involving the use of technology (e.g., graphing software, dynamic geometric software, data bases, the Internet, statistical programs, spreadsheets and multimedia resources) and the collection of data.
- follow examples and Socratic developments of concepts and take notes provided by the teacher.
- apply individual and group learning skills.
- pose and answer questions in a context.
- describe the patterns that emerge verbally, algebraically and visually (using tables, graphs and posters).
- demonstrate an understanding of concepts, and ability to select and perform algorithms accurately in. order to solve problems.
- practise prerequisite skills.

Assessment/Evaluation

Assessment is a systematic process of collecting information or evidence about student learning; evaluation is the judgement we make about the assessments of student learning based on established criteria. This profile will focus on providing specific examples of assessment strategies and tools, and general statements about how these assessments might be used in evaluation. Evaluation requires that the teacher not simply average marks. In forming an evaluative judgement, the student's highest, most consistent level of achievement should be used.

The focus of this course is on inquiry, problem solving, communication, acquisition of high levels of knowledge and skills and application of mathematics. Knowledge and understanding continue to be important. Assessment looks at students meeting course expectations at a variety of levels, with an emphasis on growth over time. Assessment should be used to gather information for diagnostic, formative and summative purposes. It is important to note that assessment and evaluation will be criterion referenced, comparing student performance to the Ministry standard, not to other students. Level 3 is defined as the provincial standard. A student achieving at this level is well-prepared for work in the Grade 10 Academic or Applied course. Level 4 performance requires a consistent demonstration of well-communicated higher level thinking and not simply technically correct solutions.

Assessment strategies and tools must address the variety of teaching and learning styles as well as the variety of expectations. High quality assessment can measure individual and group performance, and individual performance within a group. A balanced assessment program will include **methods**:

- to assess Understanding of Conceptual and Procedural Knowledge/Understanding: tests, quizzes, and observation of performance tasks.
- to assess Thinking/Inquiry/Problem Solving, and Application in unfamiliar settings: performance assessment, observation, and conferencing.
- to assess Communication: journals, portfolios, performance assessments, observations and presentations

- to assess Application in familiar settings: tests, quizzes, performance assessments
- to assess Learning Skills and to set goals: journals, portfolios, observations and conferencing

Assessment **tools** to be used throughout the course include:

- the four level Achievement Chart.
- rubrics (both teacher-created and student-generated).
- checklists.
- rating scales.
- anecdotal comments.
- analytic marking schemes.

A selection of these tools has been designed to accompany specific assessment activities. Teachers are encouraged to use them, then develop similar tools for other assessment activities. Some suggestions for increasing scoring consistency include:

- involve other teachers in the department in the creation of rubrics for assessment.
- involve students in the setting of criteria, and the use of self and peer assessments.
- gather exemplars of student work at the four levels, so that teachers and students can get a better image of what achievement at these levels looks like.

Assessment of the expectations using the 4 levels of the Achievement Chart will be ongoing throughout the unit. There will be a summative performance activity and a summative pencil and paper test for each unit. Assessment tools will be designed to allow students to demonstrate performance at the full range of their learning (levels 1 to 4). Some smaller pencil and paper assessment tools may not allow for demonstration of Level 4 performance.

Accommodations

The following accommodations for students with special needs should be made throughout the course:

Accommodations for ESL/ESD students:

- The unit provides many activities to learn that are not language driven, such as data gathering and recording. When students make submissions, they should have opportunities to use point form rather than prose, and graphical or pictorial representations. Alternatively, a member of their group could help them record their observations and conclusions. A glossary of terms may assist with explanations and definitions.

Accommodations for students with learning disabilities:

- Students may have directions read to them by a member of their group or the teacher. Some students may benefit from being given more detailed directions identifying key components of problems. Students with communications difficulties may be given the option to present their solutions orally to the teacher, practising with a group member first. Information can be taped or the student provided scribed notes. Accommodation for extra time should be made. Time has been allotted for

consolidation of numeric skills from earlier grades. Teachers could modify the amount of work assigned. Calculators can be used throughout the course as needed.

Resources

Practical

Flewelling, Gary and Lemenchick, Chuck, Mathematics Assessment Grade 9. Gage

NCTM Activities For Active Learning and Teaching

EQAO Grade 9 Sample Assessment Document. 1998

OMCA, OAME Linking Assessment and Instruction in Mathematics: Connecting to the Ontario Provincial Standards. 1995

ProQuest, http://www.umi.com/proquest, This website provides access to more than 3000 journals, magazines, dissertations, newspapers, and other publications, for a fee. This is a good source of secondary data. There are several similar services available

Professional Reading

NCTM, Mathematics Assessment: Myths, Models, Good Questions and Practical Suggestions

Stenmark, Jean Kerr, Assessment Alternatives in Mathematics. Assessment Committee of the California Mathematics Council

Bennett, Barrie et. al., Cooperative Learning: Where Hearts Meet Minds. 1991 (available from Educational Connections, P.O. Box 249, Station P, 704 Spadina Avenue, Toronto, Ontario)

Clarke, Judy et. al., Together We Learn: Co-operative Small Group Learning, Prentice Hall, 1990

Course Evaluation

Course improvement should be viewed as an ongoing and collaborative process among mathematics teachers. As new resources, new technology, and new insights on the programs develop, teachers will adapt their programs to better serve the needs of their students.

To meet these goals, teachers should evaluate the effectiveness of their courses using a variety of information sources. While students' performance on summative evaluations such as class tests, the final assessment, and the grade 9 EQAO mathematics assessments are obvious indicators of a course's success, many other sources of information are available to teachers as well. These include students' reflections on their learning in their mathematics journals, parental feedback, and student performance in subsequent mathematics courses as well as other subject disciplines which build on grade nine mathematics.

Anecdotal evidence can be gathered from observing the following indicators:

- the care taken by the students in their work
- students' efforts to complete their work and seek help as needed
- students' pursuit of extension activities
- students' growth in independence and persistence when completing tasks

Coded Expectations: Principles of Mathematics, Grade 9, Academic

Number Sense and Algebra

Overall Expectations

NAV.01D
– solve multi-step problems requiring numerical answers, using a variety of strategies and tools;

NAV.02B
– demonstrate understanding of the three basic exponent rules and apply them to simplify expressions;

NAV.03B
– manipulate first-degree polynomial expressions to solve first-degree equations;

NAB.04B
– solve problems, using the strategy of algebraic modelling.

Specific Expectations

Solving Numerical Problems

NA1.01
– demonstrate facility with critical numerical skills, including mental mathematics, estimation, operations with integers (as necessary for working with equations and analytic geometry), and operations with rational numbers (as necessary in analytic geometry, measurement, and equation solving);

NA1.02
– distinguish between exact and approximate representations of the same quantity and choose appropriately between them in given situations (e.g., use the symbol ð instead of 3.14 in determining the effect on the volume of a sphere of doubling its diameter; determine the perimeter of a square having an area of 2);

NA1.03
– solve multi-step problems involving applications of percent, ratio, and rate as they arise throughout the course;

NA1.04
– use a scientific calculator effectively for applications that arise throughout the course;

NA1.05
– judge the reasonableness of answers to problems by considering likely results within the situation described in the problem;

NA1.06
– judge the reasonableness of answers produced by a calculator, a computer, or pencil and paper, using mental mathematics and estimation.

Operating With Exponents

NA2.01

– evaluate numerical expressions involving natural-number exponents with rational-number bases;

NA2.02

– substitute into and evaluate algebraic expressions involving exponents, to support other topics of the course (e.g., measurement, analytic geometry);

NA2.03

– determine the meaning of negative exponents and of zero as an exponent from activities involving graphing, using technology, and from activities involving patterning;

NA2.04

– represent very large and very small numbers, using scientific notation;

NA2.05

– enter and interpret exponential notation on a scientific calculator, as necessary in calculations involving very large and very small numbers;

NA2.06

– determine, from the examination of patterns, the exponent rules for multiplying and dividing monomials and the exponent rule for the power of a power, and apply these rules in expressions involving one and two variables.

Manipulating Polynomial Expressions and Solving Equations

NA3.01

– add and subtract polynomials;

NA3.02

– multiply a polynomial by a monomial, and factor a polynomial by removing a common factor;

NA3.03

– expand and simplify polynomial expressions involving one variable;

NA3.04

– solve first-degree equations, including equations with fractional coefficients, using an algebraic method;

NA3.05

– calculate sides in right triangles, using the Pythagorean theorem, as required in topics throughout the course (e.g., measurement);

NA3.06

– rearrange formulas involving variables in the first degree, with and without substitution, as they arise in topics throughout the course (e.g., analytic geometry, measurement).

Using Algebraic Modelling to Solve Problems

NA4.01

– use algebraic modelling as one of several problem-solving strategies in various topics of the course (e.g., relations, measurement, direct and partial variation, the Pythagorean theorem, percent);

NA4.02

– compare algebraic modelling with other strategies used for solving the same problem;

NA4.03

– communicate solutions to problems in appropriate mathematical forms (e.g., written explanations, formulas, charts, tables, graphs) and justify the reasoning used in solving the problems.

Relationships

Overall Expectations

REV.01B
– determine relationships between two variables by collecting and analyzing data;

REV.02D
– compare the graphs and formulas of linear and non-linear relations;

REV.03B
– describe the connections between various representations of relations.

Specific Expectations

Determining Relationships

RE1.01
– pose problems, identify variables, and formulate hypotheses associated with relationships (*Sample problem*: If you look through a paper tube at a wall, you can see a region of a certain height on the wall. If you move farther from the wall, the height of that region changes. What is the relationship between the height of the visible region and your distance from the wall? Describe the relationship that you think will occur);

RE1.02
– demonstrate an understanding of some principles of sampling and surveying (e.g., randomization, representivity, the use of multiple trials) and apply the principles in designing and carrying out experiments to investigate the relationships between variables (*Sample problem*: What factors might affect the outcome of this experiment? How could you design the experiment to account for them?);

RE1.03
– collect data, using appropriate equipment and/or technology (e.g., measuring tools, graphing calculators, scientific probes, the Internet) (*Sample problem*: Acquire or construct a paper tube and work with a partner to measure the heights of visible regions at various distances form a wall);

RE1.04
– organize and analyze data, using appropriate techniques (e.g., making tables and graphs, calculating measures of central tendency) and technology (e.g., graphing calculators, statistical software, spreadsheets) (*Sample problem*: Enter the data into a spreadsheet. Decide what analysis would be appropriate to examine the relationship between the variables – a graph, measures of central tendency, ratios);

RE1.05
– describe trends and relationships observed in data, make inferences from data, compare the inferences with hypotheses about the data, and explain the differences between the inferences and the hypotheses (*Sample problem*: Describe any trend observed in the data. Does a relationship seem to exist? Of what sort? Is the outcome consistent with your original hypothesis? Discuss any outlying pieces of data and provide explanations for them. Suggest a formula relating the height of the visible region to the distance from the wall. How might you vary this experiment to examine other relationships?);

RE1.06
– communicate the findings of an experiment clearly and concisely, using appropriate mathematical forms (e.g., written explanations, formulas, charts, tables, graphs), and justify the conclusions reached;

RE1.07

– solve and/or pose problems related to an experiment, using the findings of the experiment.

Comparing Linear and Non-linear Relationships

RE2.01

– construct tables of values, graphs, and formulas to represent linear relations derived from descriptions of realistic situations (e.g., the cost of holding a banquet in a rented hall is $25 per person plus $975 for the hall);

RE2.02

– construct tables of values and scatter plots for linearly related data collected from experiments (e.g., the rebound height of a ball versus the height from which it was dropped) or from secondary sources (e.g., the number of calories in fast food versus the number of grams of fat);

RE2.03

– determine the equation of a line of best fit for a scatter plot, using an informal process (e.g., a process of trial and error on a graphing calculator; calculation of the equation of the line joining two carefully chosen points on the scatter plot);

RE2.04

– construct tables of values and graphs to represent non-linear relations derived from descriptions of realistic situations (*Sample problem*: A triangular prism has a height of 20 cm and a square base. Represent the relationship between the volume of the prism and the side length of its base, as the side length varies);

RE2.05

– construct tables of values and scatter plots for non-linearly related data collected from experiments (e.g., the relationship between height and age) or from secondary sources (e.g., the population of Canada over time); sketch a curve of best fit;

RE2.06

– demonstrate an understanding that straight lines represent linear relations and curves represent non-linear relations.

Describing Connections Between Representations of Relations

RE3.01

– determine values of a linear relation by using the formula of the relation and by interpolating or extrapolating from the graph of the relation (e.g., if a student earns $5/h caring for children, determine how long he or she must work to earn $143);

RE3.02

– describe, in written form, a situation that would explain the events illustrated by a given graph of a relationship between two variables (e.g., write a story that matches the events shown in the graph);

RE3.03

– identify, by calculating finite differences in its table of values, whether a relation is linear or non-linear;

RE3.04

– describe the effect on the graph and the formula of a relation of varying the conditions of a situation they represent (e.g., if a graph showing partial variation represents the cost of producing a yearbook, describe how the appearance of the graph changes if the cost per book is altered; describe how it

changes if the fixed costs are altered).

Analytic Geometry

Overall Expectations

AGV.01B

– determine, through investigation, the relationships between the form of an equation and the shape of its graph with respect to linearity and non-linearity;

AGV.02B

– determine, through investigation, the properties of the slope and *y*-intercept of a linear relation;

AGV.03D

– solve problems, using the properties of linear relations.

Specific Expectations

Investigating the Relationship Between the Equation of a Relation and the Shape of Its Graph

AG1.01

– determine, through investigations, the characteristics that distinguish the equation of a straight line from the equations of non-linear relations (e.g., use graphing software to obtain the graphs of a variety of linear and non-linear relations from their equations; classify the relations according to the shapes of their graphs; focus on the characteristics of the equations of linear relations and how they differ from the characteristics of the equations of non-linear relations);

AG1.02

– select the equations of straight lines from a given set of equations of linear and non-linear relations;

AG1.03

– identify the equation of a line in any of the forms $y = mx + b$, $Ax + By + C = 0$, $x = a$, $y = b$;

AG1.04

– rearrange the equation of a line from the form $y = mx + b$ to the form $Ax + By + C = 0$, and vice versa.

Investigating the Properties of Slope

AG2.01

– determine the slope of a line segment, using various formulas

(e.g., $m = \frac{\text{rise}}{\text{run}}, m = \frac{\Delta y}{\Delta x}, m = \frac{y_2 - y_1}{x_2 - x_1}, m = -\frac{A}{B}$);

AG2.02

– identify the slope of a linear relation as representing a constant rate of change;

AG2.03

– calculate the finite differences in the table of values of a linear relation and relate the result to the slope of the relation;

AG2.04

– identify the geometric significance of m and b in the equation $y=mx + b$ through investigation;

AG2.05

– identify the properties of the slopes of line segments (e.g., direction, positive or negative rate of change, steepness, parallelism, perpendicularity) through investigations facilitated by graphing technology, where appropriate.

Using the Properties of Linear Relations to Solve Problems

AG3.01

– plot points on the *xy*-plane and use the terminology and notation of the *xy*-plane correctly;

AG3.02

– graph lines by hand, using a variety of techniques (e.g., making a table of values, using intercepts, using the slope and *y*-intercept);

AG3.03

– graph lines, using graphing calculators or graphing software;

AG3.04

– determine the equation of a line, given information about the line (e.g., the slope and *y*-intercept, the slope and a point, two points, a line parallel to a given line and having the same *x*-intercept as another given line);

AG3.05

– communicate solutions to multi-step problems in established mathematical form, with clear reasons given for the steps taken;

AG3.06

– describe the meaning of the slope and *y*-intercept for a linear relation arising from a realistic situation, interpolate and extrapolate from the graph and the equation of the relation, and identify and explain any restrictions on the variables in the relation;

AG3.07

– describe a situation that would be modelled by a given linear equation;

AG3.08

– determine the point of intersection of two linear relations, by hand for simple examples, and using graphing calculators or graphing software for more complex examples; interpret the intersection point in the context of an application.

Measurement and Geometry

Overall Expectations

MGV.01B

– determine the optimal values of various measurements through investigations facilitated, where appropriate, by the use of concrete materials, diagrams, and calculators or computer software;

MGV.02B

– solve problems involving the surface area and the volume of three-dimensional objects;

MGV.03B

– formulate conjectures and generalizations about geometric relationships involving two-dimensional figures, through investigations facilitated by dynamic geometry software, where appropriate.

Specific Expectations

Investigating the Optimal Value of Measurements

MG1.01

– identify, through investigation, the effect of varying the dimensions of a rectangular prism or cylinder on the volume or surface area of the object;

MG1.02

– identify, through investigation, the relationships between the volume and surface area of a given rectangular prism or cylinder;

MG1.03

– explain the significance of optimal surface area or volume in various applications (e.g., packaging; the relationship between surface area and heat loss);

MG1.04

– pose and solve a problem involving the relationship between the perimeter and the area of a figure when one of the measures is fixed.

Solving Problems Involving Surface Area and Volume

MG2.01

– solve simple problems, using the formulas for the surface area and the volume of prisms, pyramids, cylinders, cones, and spheres;

MG2.02

– solve multi-step problems involving the volume and the surface area of prisms, cylinders, pyramids, cones, and spheres;

MG2.03

– judge the reasonableness of answers to measurement problems by considering likely results within the situation described in the problem;

MG2.04

– judge the reasonableness of answers produced by a calculator, a computer, or pencil and paper, using mental mathematics and estimation.

Investigating Geometric Relationships

MG3.01

– illustrate and explain the properties of the interior and the exterior angles of triangles and quadrilaterals, and of angles related to parallel lines;

MG3.02

– determine the properties of angle bisectors, medians, andaltitudes in various types of triangles through investigation;

MG3.03

– determine the properties of the sides and the diagonals of polygons (e.g., the diagonals in quadrilaterals, the diagonals of regular pentagons, the figure that results from joining the midpoints of sides of quadrilaterals) through investigation;

MG3.04

– pose questions about geometric relationships, test them, and communicate the findings, using appropriate language and mathematical forms (e.g., written explanations, diagrams, formulas, tables);

MG3.05

– confirm a statement about the relationships between geometric properties by illustrating the statement with examples, or deny the statement on the basis of a counter-example (e.g., confirm or deny the following statement: If a quadrilateral has perpendicular diagonals, then it is a square).