Catholic Curriculum Cooperative Writing Partnership

Course Profile

Principles of Mathematics

Grade 9
Academic

• *for teachers by teachers*

Acknowledgements

Public District School Board Writing Team - Mathematics- Academic

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Development Date: February/March 1999

Course Revisors:

Revision Date: March/April 1999.

Additional Codes:

Eastern Ontario Catholic Curriculum Cooperative

Institute for Catholic Education

Unit #1: Exploring Relationships

Time: 20 Hours
Unit Developer(s)

Arlene Corrigan, Dominique Levac Maureen Vincentine, Linda Sloan, Carolyn Boyer, Tom Steinke, Len St. Clair, Nora Buckley, Sue Trew, Brian McCudden, Margaret Sinclair, David Kurzinger, Paul Costa

Development Date: February/March, 1999.

Unit Description

In this unit, students and teachers will begin to explore both linear and non-linear relationships arising from meaningful problems. Students will develop numeric, graphic and algebraic skills as needed in the context of the activity. Various forms of assessment are built into all the activities.

Strand(s) & Expectations

Ontario Catholic School Graduate Expectations: CGE 3c, 4b, 5a, 7j

Strand(s): Number Sense and Algebra, Relationships.

Overall Expectations: NAV.01, NAV.02, NAV.04, REV.01, REV.02, REV.03.

Specific Expectations: NA1.01, NA1.02, NA1.03, NA1.04, NA1.05, NA1.06, NA2.01, NA2.03, NA2.04, NA2.05, NA2.06, NA4.03, RE1.01, RE1.03, RE1.04, RE1.05, RE1.06, RE1.07, RE2.01, RE2.02, RE2.04, RE2.05, RE2.06, RE3.02, RE3.03, RE3.04.

Activity Titles (Time + Sequence)

Activity 1	Exploring Linear Relationships – Bouncing Balls	7 hours
Activity 2	Exploring Non-Linear Relationships – Mathematical Marathon	7 hours
Activity 3	Exploring Motion	6 hours

Unit Planning Notes

In this unit, students will be actively gathering and analyzing data. Manipulatives are required for activities 1 and 2 (balls, metre sticks, compasses, rulers,...). Graphing calculators and motion detectors are necessary for Activity 3, which involves a comparison of liner and non-linear relationships between distance and time. For schools in which this technology is not yet readily available, Activity 3 might be postponed until a later time in the course.

> Look for text boxes like this one for points at which skill development can be done as needed in the context of the activity.

Sufficient time has been allotted for each activity to include time that is available for skill development.

Prior Knowledge Required

Students should have some facility with numeric, graphing and algebraic skills. When direct instruction is required, this should occur as needed within the context of the activities. All students should be able to engage fully in all of the activities.

Teaching/Learning Strategies

Students will:

Hypothesize – formulate hypotheses associated with linear and non-linear relationships.
Explore/Investigate – through hands-on investigations of linear and non-linear relationships.
Model/Formulate – develop numeric, graphic and algebraic models for exploring linear and non-linear relationships, dependencies and constraints.
Transform/Manipulate – develop numeric, graphical and algebraic skills as needed in the context of their investigations to allow them to move within and between representations.
Infer/Conclude – re-evaluate their hypotheses in light of their learning and make inferences to extend their learning.
Communicate – individually and in groups, orally and in writing, communicate the findings of their investigations by defending their numeric and graphic mathematical models and explaining their reasoning.

Assessment/Evaluation

- performance tasks
- paper and pencil tasks (e.g., quizzes, worksheets, small assignments)
- written reports
- oral presentations
- observation

Resources

Graphing Calculators *(e.g., TI82/83/83Plus)*
Motion Sensors *(e.g., Calculator-Based Ranger)*
Spreadsheet *(e.g., Quattro Pro or Excel)*
Internet
Manipulatives *(e.g., balls, metre sticks, compasses,...)*
Atlas
Student Textbook

Activity #1: Exploring Linear Relationships – Bouncing Balls

Time: 7 hours

Description

In this activity students will explore the relationship between the drop and rebound height of a ball. They will represent the data numerically and graphically. They will analyze the data to determine any patterns in the relationship being modeled.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations:
The graduate is expected to be:

- an effective communicator who presents information and ideas clearly and honestly and with sensitivity to others.
- a reflective, creative and holistic thinker who demonstrates flexibility and adaptability.
- a collaborative contributor who works effectively as an interdependent team member.

Strands: Relationships

Overall Expectations
By the end of this course, students will:

- determine relationships between two variables by collecting and analyzing data. ✪
- describe the connections between various representations of relations.

Specific Expectations
By the end of this course, students will:

- pose problems, identify variables, and formulate hypotheses associated with relationships.
- collect data, using appropriate equipment and/or technology. ✪
- organize and analyze data, using appropriate techniques and technology. ✪
- describe trends and relationships observed in data, make inferences from data, compare the inferences with hypotheses about the data, and explain the differences between the inferences and hypotheses.
- construct tables of values and scatter plots for linearly related data collected from experiments or from secondary sources. ✪

Planning Notes

Prior to beginning the activity:

- place the students in groups of 4
- provide each group with materials: (a metre stick, masking tape, 2.5 m of blank cash register tape, a ball - tennis or rubber, copies of student handout, paper for recording purposes)

Prior Knowledge Required

- ratio (proportional reasoning)
- representing data in charts
- graphing ordered pairs
- choosing appropriate scales
- measurement skills

Teaching/Learning Strategies

Getting Ready

- Students will be placed in groups of four.
- Teacher will demonstrate a sample ball bounce so that students are clear of what drop height and rebound height are.
- Explain that to do the experiment efficiently and accurately, each member of the group must choose and perform a specific task. Each student should record the names of their group members along with the task that each member is to perform.
- Each group will need the following materials: metre stick, masking tape (several strips), 2.5 m of blank cash register tape paper, a ball, 4 copies of the "Student Handout" (Appendix A) and several blank sheets of paper to record the experiences of the group.
- Before the students begin, it is crucial that they know that this is not just a "fill in the missing numbers" activity. Like with any experiment, they must carefully observe and record the procedures and data. This will be crucial when they write their report.

Beginning the Activity

- Students can work in the class or hall.

- Ask probing questions to each group as you circulate through the hallway:
  - "Where does the ball dropper hold the ball relative to the marked height?"
  - "Where does the ball bounce height recorder mark the rebound height of the ball?" (Note that ball height can be marked at the bottom, top, or middle of the ball. It is therefore important that the drop height and rebound height are marked in a consistent fashion.)
  - "Are all the group members contributing to the best of their ability?"
  - "What is the role of each group member?"

- As the groups complete their experiments have them share the group results and observations. Have the students begin to do a rough copy of their rebound height versus drop height graph. The groups will undoubtedly call you over to ensure they are setting up their axes and graphing their data points properly.

> This is an appropriate time to ensure that the students' graphing abilities are adequate. Direct instruction may be required.

Ball Bounce Report

- Each student is now responsible for preparing a **Ball Bounce Report.** The report can be very similar to a science lab report, which includes:
  - Title Page
  - Materials
  - Group Members and Roles
  - Procedure *(should be a detailed, one page description of how your group went about doing the experiment)*
  - Observations *(the completed chart along with any other interesting observations you and your group may have noted)*
  - Discussion *(the rebound height versus drop height graph of your groups data along with a visual line of fit through your data points)*
  - Conclusion *(describe in your own words, the relationship between the drop height and rebound heights of each of the three balls)*

- Assess the **Ball Bounce Report** using the rubric (Appendix B). The key parts of the report are the procedure, graphs and conclusions. Be sure to post the Math Reports around your room to celebrate the work of your students.

Follow-Up

- Have students hypothesize what a table of values and graph for a ball which rebounds three quarters its drop height would look like.
- What would a graph for a superball look like?
- Could you predict how high a ball would rebound if it were dropped from the top of the C.N. Tower?

Analyzing Data Using Technology

At this point you may wish to show students how to input data into lists, setup a scatter plot and perform a linear regression on a graphing calculator *(TI82/83/83Plus)*.

- Have your students input their data from the bouncing ball experiment, into lists in a graphing calculator.
- Have your students construct a scatter plot using the graphing calculator.
- Have your students perform a linear regression.
- Allow the students to critique the resulting line of best fit generated by the calculator.
- The line will probably not pass through the origin. It will make sense to students that if you don't drop a ball, it won't bounce! This should allow students to select the origin as a carefully selected point through which a line of best fit should pass.
- In groups allow students to share strategies to select a second point through which a line of best fit might pass.

You may wish to provide scatter plots for your students, where students carefully select two points through which they can draw a line of best fit. They should defend their choice of points based on the context from which the data points were derived.

Assessment/Evaluation

1. Observational rubric for group data collection (Appendix C)
2. Rubric for the individual written report (Appendix B)

Resources

1. manipulatives (e.g., bouncing balls of various sizes, metre sticks, ...)
2. class set of graphing calculators (e.g., TI82/83/83Plus)
3. http://www.ti.com/calcs/doc
4. Textbook

Accommodations

1. Students should be given the option of doing an oral presentation in place of, or to complement a written report.
2. When assigning roles to members, be sure to assign a role to students that is not an area of limitation.
3. Steps and procedures for using graphing calculators should be provided in written form as well as orally.

Activity #2: Exploring Non-Linear Relationships - Mathematical Marathon

Time: 7 hours

Description

In the spirit of the Terry Fox Marathon can we create a fund-raiser to raise billions of dollars to help the plight of the homeless in Canada and the U.S.? If we do a marathon along the border of Canada and the U.S. how much can we expect an individual participant to raise?

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations:
The graduate is expected to be:

- an effective communicator who presents information and ideas clearly and honestly and with sensitivity to others.
- a reflective, creative and holistic thinker who demonstrates flexibility and adaptability.

Strands: Relationships

Overall Expectations
By the end of this course, students will:

- determine relationships between two variables by collecting and analysing data. ✪
- compare the graphs and formulas of linear and non-linear relations. ✪
- demonstrate understanding of the three basic exponent rules and apply them to simplify expressions.

Specific Expectations
By the end of this course, students will:

- collect data, using appropriate equipment and/or technology. ✪
- organize and analyse data, using appropriate techniques (e.g., making tables and graphs, calculating measures of central tendency) and technology. ✪
- construct tables of values and graphs to represent non-linear relations derived from descriptions of realistic situations. ✪
- construct tables of values and scatter plots for non-linearly related data collected from experiments or from secondary sources; sketch a curve of best fit. ✪
- identify, by calculating finite differences in its table of values, whether a relation is linear or non-linear. ✪
- determine the meaning of negative exponents and of zero as an exponent from activities involving graphing, using technology, and from activities involving patterning. ✪
- represent very large and very small numbers, using scientific notation. ✪
- enter and interpret exponential notation on a scientific calculator, as necessary in calculations involving very large and very small numbers.
- determine, from the examination of patterns, the exponent rules for multiplying and dividing monomials and the exponent rule for the power of a power, and apply these rules in expressions involving one and two variables.

Planning Notes

- Use the Mandelbrot's story "The Length of the British Coastline" as an introduction. (Appendix D)
- Each student requires a map of North America with the Canada/U.S. border clearly defined and a pair of compasses.
- Students work in pairs.
- Spreadsheets/charting software or graphing calculators will be helpful.

Prior Knowledge Required

- measurement skills
- organizing data in charts
- graphing ordered pairs

Teaching/Learning Strategies

"The length depends on the step size !"

- Begin with a brief whole class discussion of what information is required to answer the question posed. When the question of border length emerges, introduce the Mandelbrot story, "How long is the British coastline?"(Appendix D)

- In pairs, students use the map of the Canada/US border and instructions for "How to do a Structured Walk" (Appendix D) to collect and record measurements in columns with headings, "step size", "number of steps", "remaining distance". Each pair should do at least 6 structured walks (3 each) using a range of step sizes from 0.4 cm to 3 cm.

- Students calculate distance estimates (using a formula) for each step size, and plot the ordered pairs (step size, distance), using a spreadsheet or graphing calculator if available.

> You may wish to ensure all students are able to substitute into a formula so that they are able to calculate the perimeter.

- All students make a paper and pencil version of the plot and describe it in words. They will notice that the points do *not* lie approximately on a straight line.

- The teacher will ensure that students make the connection between this and the non-linear nature of the plot. Students use the regression capabilities of a calculator to investigate possible curves of best fit, and make the connection with the exponential model.

> Lead into a discussion about the meaning of negative exponents in this context.

- In groups of 4, students have discussions to consider the bigger problems: "How long is the border really?" and "How much money could one person raise?"

> Ensure students are able to make the conversion of scale from cm to km, and can use scientific notation to represent the larger distances.

- Students need to consider factors such as method of travel along the border over land/water, distance covered in a day, costs incurred per day/period.

- Students now consolidate and enhance their understanding of the three basic exponent rules by completing assignments from the textbook. Include questions with the exponent rule for the power of a power.

- This would also be a good time to enter and interpret exponential notation on a scientific calculator, since some distances will be quite large. Again, use textbook assignments to involve applications with very small numbers.

Report

- Students write a report which includes:
  - an explanation of the problem in their own words;
  - a chart and graph of the data with a discussion of results
  - their estimate of the amount to be raised with a complete justification including assumptions and calculations.

Assessment/Evaluation

1. Observe students for learning and for evidence of their problem solving and inquiry skills as they proceed through the activity. (Appendix C)
2. Students write a brief paragraph, describing how they decided that the relationship between estimate of distance vs. step size is non-linear, followed by a reflection of their ideas, discoveries and concerns/difficulties that arose from the activity. This can be assessed for clarity in communicating mathematical ideas.
3. Teacher evaluates final written report. (Appendix B)

Resources

1. World atlas
2. Lewis, Ron. "Fractals in Your Future", (http://www.eureka.ca/resources/fiyf/chapter1.html)
3. Benoit Mandelbrot website
4. Spreadsheet (computer lab) and/or Graphing Calculators (class set)
5. Compasses, ruler, graph paper

Accommodations

1. Students should be given the option of doing an oral report on tape in place of, or to complement, a written report.
2. The pacing of the activity and complexity of the procedures can be adjusted as required.

Activity #3: Exploring Motion

Time: 6 hours

Description

In this activity, students will explore the concept of rate (relationship between distance and time) by moving in front of a motion sensor. They will develop a sense of what type of motion leads to a linear relation versus a non-linear relation. The instantaneous graphic representation provided by the technology is a powerful tool that allows all students to develop a graphical model from their own motion. This activity is ideal in forming students understanding of predicting the graphical outcome of an event.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations:
The graduate is expected to be:
- a reflective and creative thinker who thinks reflectively and creatively to evaluate situations and solve problems.
- a self-directed, responsible, life-long learner who applies effective communication, decision-making, problem-solving and resource management skills.
- a collaborative contributor who works effectively as an interdependent team member.

Strands: Relationships, Number Sense and Algebra

Overall Expectations
By the end of this course, students will:
- determine relationships between two variables by collecting and analyzing data.
- compare the graphs and formulas of linear and non-linear relations. ✪
- describe the connections between various representations of relations.

Specific Expectations
By the end of this course, students will:
- demonstrate facility with critical numerical skills, including mental mathematics, estimation, operations with integers, and operations with rational numbers. ✪
- distinguish between exact and approximate representations of the same quantity and choose appropriately between them in given situations.
- collect data, using appropriate equipment and/or technology. ✪
- organize and analyse data, using appropriate techniques and technology. ✪
- describe trends and relationships observed in data, make inferences from data, compare the inferences with hypotheses about the data, explain the differences between the inferences and the hypotheses. ✪
- communicate the findings of an experiment clearly and concisely, using appropriate mathematical forms and justify the conclusions reached. ✪
- construct tables of values and graphs to represent non-linear relations derived from descriptions of realistic situations.
- demonstrate an understanding that straight lines represent linear relations and curves represent non-linear relations. ✪
- describe, in written form, a situation that would explain the events illustrated by a given graph or the relationship between two variables. ✪
- describe the effect on the graph and the formula of a relation of varying the conditions of a situation they represent.
- identify the slope of a linear relation as representing a constant rate of change.

Planning Notes

Equipment required:
- a class set of graphing calculators
- one motion sensor for each group of students
- one projection unit and compatible graphing calculator

Prior Knowledge Required

- collecting, organizing and analyzing data
- recognizing relationships as being linear or non-linear numerically and graphically

Teaching/Learning Strategies

- Students observe a teacher and/or student walking demonstration using a motion sensor, graphing calculator and projection unit. You may wish to show students how you would walk in front of a motion sensor to generate the letter "V".
- Students, in small groups (ideally in pairs), explore different types of motion by walking back and forth in front of their motion sensors. You may wish to challenge the students to generate the first letter of their first name by walking in front of the motion sensor. Using technology, students will create several distance/time graphs.

> Have the students explore where the graphing calculator has stored the data represented on the scatter plot. When the students have located the lists, have them determine what each list represents. Students can also trace along the scatter plot and discuss the significance of the ordered pairs in the context of their motion.

- Students observe a teacher and/or student walking demonstration of linear and non-linear motion.
- Students, in small groups (ideally in pairs), practise and explore linear and non-linear motion using their motion sensors. Using technology, students will create several distance/time graphs of linear motion and several distance/time graphs of non-linear motion. Students should record their graphs with a description of their motion.
- Students can design their walk to create a graph that is a) a straight line with a positive slope; b) a straight line with a negative slope; c) several lines with a combination of positive and negative slopes.
- Have students walk at different speeds and in different directions so that they not only investigate positive and negative slopes, but different ratios as well. (Refer to "Explorations, Modelling Motions: High School Activities with the CBR™."

> Discuss with students the meaning of positive and negative integers in this context.

Oral Presentation

- Students make oral presentations of their group results.

Report

- Each student submits a written report of their graphs and a description of their motion that gave rise to their graphs.

Paper and Pencil Assessment Tasks

- Students observe a given motion and then predict and defend the nature of the resulting distance/time graph.
- Students to describe the type of motion that would result from a given distance/time graph.

Performance Assessment Task

- Using the "Distance Match" application on the Ranger program, students walk in front of a motion sensor so as to imitate a given distance/time graph.

Assessment/Evaluation

1. Teacher Observation (Appendix C)
2. Oral Presentation
3. Written Report (Appendix B)
4. Performance Assessment Task
5. Paper and Pencil Tasks

Resources

1. "Getting Started with CBR", Texas Instruments
2. "Explorations, Modelling Motion: High School Activities with the CBR", Texas Instruments
3. http://www.ti.com/calc/docs
4. "Life by the Numbers”, Video Number 7**,** PBS 1998

Accommodation

Provide clean written instructions and steps as needed. Adjust the number of comments required to allow students to fully participate.

Unit #2: Modelling Linear Relationships

Time: 40 Hours

Unit Developer(s)

Arlene Corrigan, Dominique Levac Maureen Vincentine, Linda Sloan, Carolyn Boyer, Tom Steinke, Len St. Clair, Nora Buckley, Sue Trew, Brian McCudden, Margaret Sinclair, David Kurzinger, Paul Costa

Development Date: February/March, 1999.

Unit Description

In this unit, students and teachers will explore numerical, graphical and algebraic models (tables, graphs and equations) of linear relationships arising from meaningful problems. Students will develop numeric, graphic and algebraic skills as needed in the context of the activity. Various forms of assessment are built into all the activities.

Strand(s) & Expectations

Ontario Catholic School Graduate Expectations: CGE 2b, 3c, 3e, 4f, 5a, 5g.

Strands: Number Sense and Algebra, Relationships, Analytic Geometry.

Overall Expectations: NAV.01, NAV.03, NAV.04, REV.01, REV.03, AGV.01, AGV.02, AGV.03.

Specific Expectations: NA1.01, NA1.02, NA1.03, NA1.04, NA1.05, NA1.06, NA2.01, NA2.05, NA2.06, NA3.01, NA3.02, NA3.03, NA3.04, NA3.06, NA4.01, NA4.02, NA4.03, RE1.01, RE1.02, RE1.03, RE1.04, RE1.05, RE1.06, RE1.07, RE2.01, RE2.02, RE2.03, RE3.01, RE3.02, RE3.04, AG1.01, AG1.02, AG1.03, AG1.04, AG2.01, AG2.02, AG2.03, AG2.04, AG2.05, AG3.01, AG3.02, AG3.03, AG3.04, AG3.05, AG3.06, AG3.07, AG3.08.

Activity Titles (Time + Sequence)

Activity 1	Modelling Motion – Walking the Line	8 hours
Activity 2	Modelling Linear Relationships – The Help Line	10 hours
Activity 3	Modelling Linear Relationships – Environmental Issues	9 hours
Activity 4	Modelling Linear Relationships – Bouncing Balls	6 hours
Activity 5	Modelling Intersection of Lines – Athletic Performance	7 hours

Unit Planning Notes

The activities in this unit embed the use of technology, in particular graphing calculators. For schools in which this technology is not yet readily available, teachers may adapt the activities. With the exception of the first, all activities can be accomplished without the use of graphing calculators, if some changes are made. Bear in mind that, without the use of graphing calculators, some of the activities are likely to take more class time.

The use of graphing calculators is essential for Activity 1. The key ideas introduced in this activity include the use of finite differences to describe a constant rate of change, and the introduction of the significance of m and $b$in the equation $y=mx+b$. These concepts do arise in later activities, and teachers are advised to make adjustments, as necessary. Activities 2 and 3 involve the use of a computer lab for spreadsheet and Internet activities. Manipulatives must be available for Activity 4. The student textbook should be used for numeric, graphic, and algebraic skill development at appropriate points in the activities.

Sufficient time has been allotted for each activity to ensure time is available for skill development.

> Look for text boxes like this one for points at which skill development can be done as needed in the context of the activity.

Prior Knowledge Required

Students should be able to comfortably model linear and non-linear relationships numerically and graphically. Should direct instruction be required, this should occur as needed within the context of the activities as opposed to before the activities. All students should be able to engage fully in all of the activities.

Teaching/Learning Strategies

Students will:
Hypothesize – formulate hypotheses associated with linear relationships.
Explore/Investigate – through hands-on investigations of linear relationships.
Model/Formulate – develop numeric, graphic, algebraic and geometric models for exploring linear relationships, dependencies and constraints.
Transform/Manipulate – develop numeric, graphical, algebraic and geometric skills as needed in the context of their investigations to allow them to move within and between representations.
Infer/Conclude – re-evaluate their hypotheses in light of their learning and apply their learning.
Communicate – individually and in groups, orally and in writing, communicate the findings of their investigations, defending their numeric, graphic, algebraic and geometric mathematical models and explaining their reasoning.

Assessment/Evaluation

- performance tasks
- paper and pencil tasks
- written reports
- oral presentations
- observation

Resources

Graphing Calculators *(e.g., TI82/83/83Plus)*
Graphing Software *(e.g., Graphmatica or Zap-A-Graph)*
Motion Sensors *(e.g., Calculator-Based Ranger)*
Spreadsheet *(e.g., Quattro Pro or Excel)*
Internet
Manipulatives *(e.g., balls, metre sticks, compasses,...)*
Dynamic Geometry Software *(e.g., Geometer's SketchPad, Cabri, TI92, Java SketchPad)*
Student Textbook

Activity #1: Modelling Motion – Walking the Line

Time: 8 hours

Description

In this activity, concepts of slope and y-intercept will be addressed formally by linking the characteristics of a linear graph, and its algebraic representation $y=mx+b$, to the motion of the students. Students will explore the concept of rate (relationship between distance and time) by moving in front of a motion sensor *(e.g., CBR)*.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations:
The graduate is expected to be:

- a reflective and creative thinker who thinks reflectively and creatively to evaluate situations and solve problems.
- a collaborative contributor who achieves excellence, originality, and integrity in one's own work and supports these qualities in the work of others.
- a collaborative contributor who works effectively as an interdependent team member.

Strands: Number Sense and Algebra, Relationships, Analytic Geometry

Overall Expectations
At the end of Grade 9, students will:

- determine, through investigation, the relationships between the form of an equation and the shape of its graph with respect to linearity an non-linearity. ✪
- determine, through investigation, the properties of the slope and y-intercept of a linear relation. ✪

Specific Expectations
Students will:

- collect data, using appropriate equipment and/or technology. ✪
- organize and analyse data, using appropriate techniques and technology.
- communicate findings of an experiment clearly and concisely, using appropriate mathematical forms and justify the conclusions reached. ✪
- construct tables of values, graphs, and formulas to represent the linear relations derived from descriptions of realistic situations. ✪
- identify the slope of a linear relation as representing a constant rate of change.
- identify the geometric significance of m and b in the equation $y = mx + b$ through investigation. ✪
- describe the meaning of the slope and y intercept for a linear relation arising from a realistic situation, interpolate and extrapolate from the graph and the equation of the relation, and identify and explain any restrictions on the variables in the relation. ✪
- solve multi-step problems involving applications of percent, ratio, and rate as they arise throughout the course.

Planning Notes

Equipment required:

- a class set of graphing calculators
- one motion sensor *(e.g., CBR)* for each group of students
- one projection unit with compatible graphing calculator

Prior Knowledge Required

- collecting, organizing and analyzing data using technology
- modelling relations numerically and graphically
- concept of rate (relationship between distance and time)

Teaching/Learning Strategies

1. Students observe a teacher and/or student walking demonstration of a constant rate of change using a motion sensor *(e.g., CBR)*, graphing calculator and projection unit. (Note that it may take practice to model a constant walking speed.).

2. Students, in small groups (ideally in pairs), explore constant rates of change by walking back and forth in front of a motion sensor. Using technology, they will create and record distance/time graphs for various constant rates of walking using different starting points.

3. Have students hypothesize about the type of motion that would lead to a horizontal line *(x=a)* and the type of motion that would lead to a vertical line *(y=b)*. Allow the students to test their hypotheses using the motion sensors.

> Have students explore finite differences using the data stored in the lists. This can serve as a numeric link to the concept of a constant rate of change. This may be an opportunity to consolidate students' skills in performing operations with rational numbers.

4. Students prepare a written report describing how a constant rate of change is represented by the slope of a linear relation. Students must be able to defend their results and predictions.

Report

Students present their results in the form of a written report. The report should show the students' ability to communicate their ideas clearly and concisely.

> The geometric significance of m and b in the equation $y = mx + b$ will also be investigated in the context of walking in front of a motion sensor.

Paper and Pencil Task

Students match linear graphs with their algebraic representation in the form $y = mx + b$.

Performance Assessment Task

Students will model, by walking in front of a motion sensor, a linear relation expressed algebraically in the form $y = mx + b$.

Assessment/Evaluation

1. Observe students for learning and for evidence of their problem solving and inquiry skills as they proceed through the activity (Appendix C)
2. Written Report (Appendix B)
3. Performance Assessment Task
4. Paper and Pencil Task

Resources

1. "Getting Started with CBR", Texas Instruments
2. "Explorations, Modelling Motion: High School Activities with the CBR", Texas Instruments
3. www.ti.com/calc/docs
4. "Life by the Numbers", Video Number 7, PBS, 1998

Accommodation

The teacher can control conditions of the group assignment and roles to face life involvement of all students.

Activity #2: Modelling Linear Relationships – The Help Line

Time: 10 hours

Description

The students of your school plan to sponsor a school in the Dominican Republic by sending computers along with some students who will provide instruction while they experience the culture. A car wash (in the school parking lot) will be held, as one of the fundraising activities, to pay for the air fares and to purchase the computers. How much money is likely to be raised by the car wash?

The goal is to raise $50 000 in total. If this goal is reached, how many students and how much equipment are you likely to be able to send?

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations:
The graduate is expected to be:
- a responsible citizen who witnesses Catholic social teaching by promoting equality, democracy, and solidarity for a just, peaceful and compassionate society.

Strands: Number Sense and Algebra, Relationships, Analytic Geometry

Overall Expectations
At the end of Grade 9, students will:
- solve multi-step problems requiring numerical answers, using a variety of strategies and tools. ✪
- manipulate first-degree polynomial expressions to solve first-degree equations. ✪
- solve problems, using the strategy of algebraic modelling. ✪
- describe the connections between various representations of relations.
- determine, through investigation, the properties of the slope and *y*-intercept of a linear relation.
- demonstrate understanding of the three basic exponent rules and apply them to simplify expressions.

Specific Expectations
Students will:
- solve multi-step problems involving applications of percent, ratio, and rate as they arise throughout the course. ✪
- rearrange formulas involving variables in the first degree, with and without substitution, as they arise in topics throughout the course. ✪

- use algebraic modelling as one of several problem-solving strategies in various topics of the course.
- communicate solutions to problems in approximate mathematical forms and justify the reasoning used in solving the problems. ✪
- construct tables of values, graphs, and formulas to represent the linear relations derived from descriptions of realistic situations. ✪
- describe the effect on the graph and the formula of a relation of varying the conditions of a situation they represent. ✪
- identify the equation of a line in any of the forms $y = mx + b$, $Ax + By + C = 0$, $x = a$, $y = b$. ✪
- rearrange the equation of a line from the form $y = mx + b$ to the form $Ax + \underline{B}y + C = 0$, and vise versa. ✪
- identify the slope of a linear relation as representing a constant rate of change.
- describe the meaning of the slope and y intercept for a linear relation arising from a realistic situation, interpolate and extrapolate from the graph and the equation of the relation, and identify and explain any restrictions on the variables in the relation. ✪
- demonstrate facility with critical numerical skills, including mental mathematics, estimation, operations with integers (as necessary for working with equations and analytical geometry), and operations with rational numbers (as necessary in analytic geometry, measurement, and equation solving).
- determine, from the examination of patterns, the exponent rules for multiplying and dividing monomials and the exponent rule for the power of a power, and apply these rules in expressions involving one and two variables.
- add and subtract polynomials.
- multiply a polynomial by a monomial, and factor a polynomial by removing a common factor.
- expand and simplify polynomial expressions involving one variable.

Planning Notes

- This task will **require** the use of a spreadsheet.
- Book a computer lab and ensure you have a spreadsheet application.
- This task provides an excellent opportunity for students to explore the capabilities of a spreadsheet program for tables and graphs.
- When chart is set up for the spreadsheet, select new window and arrange windows vertically, side-by-side, to manipulate tables and graphs simultaneously (refer to software help for spreadsheet instructions).

Prior Knowledge Required

- Some facility with spreadsheet applications.
- Facility modelling linear relationships numerically, graphically.
- Some facility modelling linear relationships algebraically.

Teaching/Learning Strategies

Part 1: $y = mx$

- Begin with a simple investigation of how the amount raised depends on the number of cars washed. Students generate table with headings "number of cars washed", "$ amount raised". Each students decides how much to charge for a car wash and how many cars they can reasonably expect to wash in a day.

Teacher ensures students enter the correct formula for the second column.

- Students look at each other's plots and discuss what is the same, what is different, what makes the difference. They need to make the connection that the more they charge per car, the steeper the plot.

- Suppose y represents the amount of money raised in dollars and x represents the number of cars. Write the equation describing y in terms of x for their graph. Students share the results by writing them on the board.

General equation $y = mx$ is developed in the context of this activity.

- At this point students start a new spreadsheet and graph $y = mx$ for various m-values beginning with $m = 1$. Whole class discussion of whether a line is appropriate for the car wash data. Ask students to come up with an example where a continuous line is appropriate.

Explore situations which give rise to data that could reasonably be modeled with a continuous versus a discrete graphical model.

- Suppose you raise $48 washing 8 cars. Enter these values in your spreadsheet to plot a point to correspond to this situation. Then change m so that the line $y=mx$ goes through this point.

Teacher provides several more points for students to repeat/practise this process.

- Students tabulate $y = mx$ for these different m-values in the spreadsheet and look for patterns in the tables. Teacher ensures students make the connections between and among the finite differences, the m-value, and the slope of the line.

- Ask students what m represents in the context of the problem. Make connection that slope is a rate.

Part 2: $y = mx + b$

- Revisit the initial problem and introduce the fixed cost of supplies for the car wash. Adjust the formula and re-evaluate the amount raised. Plot the pairs relating number of cars washed to amount raised. Compare graphs to the original graphs and discuss what is the same, what is different, and what makes them different. (This is an opportunity to introduce the words direct or partial variations).

- On a new spreadsheet, introduce b into the equation $y = x + b$ or $y = mx + b$, $m = 1$, $b = 0$, and graph it for various b values.

As an exercise, have students change b, so that the line passes through previously selected points. Have students change m or b, so that the line passes through previously selected points.

- Teachers should be prepared to discuss why there are different lines that may pass through any one point.

This may also be an opportune time to discuss negative, positive, and zero slopes. Also, introduce horizontal and vertical lines.

- Revisiting the initial problem and starting a new spreadsheet, students should re-plot their data for *x*-values of 3 rather than 1. Have students calculate the finite differences in the next column over. Class discussion should raise points that differences are all the same, all are 3 times the m value, and why 3 times? (i.e., make the connection with size of increment). Have students change the *x*-values to 5, 10, 12, 16, 22, and 30, observing the changes to the finite differences note any patterns that occur.

- Challenge the students with data from a previous car wash event to see if they can determine the slope from the *x* and *y*-values. Then write a formula to find the slope from a list of *x* and *y*-values. Include in your discussion whether it matters which pairs of x and y-values are used. Help students convert their spreadsheet formula into the mathematical slope formula.

- In answering the question: “How many students and how much equipment are you likely to send?”, students need to research air fares and price of computers. Investigating the possible combinations of the number of computers to send and the number of students to accompany them, students can answer questions like: "If you wanted to send 15 computers at $2000 each, how many students would be able to accompany them at a cost of $1250 each? If you only send 8 students, how many computers can you send?"

Part 3: Assume students have reached the $50 000 goal

- What are other possible combinations that enable you to spend the entire $50 000?

- Letting *x* represent the number of computers and y represent the number of students, write an equation that connects *x* and *y*. Have students start a new spreadsheet, entering the *x* and *y*-values they have found, and graph the line using the ordered pairs. Whole class discussion would include questions like: What is the slope of the line? What is the *y* -intercept? What is the equation in the form $y=mx+b$? We also have $2000x + 1250y = 50\ 000$. Are these the same? How can you show this?

Teacher should ensure that students can manipulate one ($y = mx + b$) into the other ($Ax + By + C = 0$).

- In discussing how useful the line is in this problem, students should consider questions like do all solutions to the problem lie on the line? Do all points on the line represent solutions to the problem? Students should follow up their research by writing a report to answer the question: “How many students and how much equipment are you likely to send?”, giving supporting arguments.

Part 4: Possible Extension

- Now would be a good time for the teacher to diagnose students' ability to work with integers and remediate as necessary. This could then be extended to lessons on manipulating polynomial expressions, supported by textbook resources. When multiplying and dividing monomials, highlight the exponent rules covered in Activity 1. Include the exponent rule for the power of a power.

Assessment/Evaluation

- Observation of students working with spreadsheet and solving problems. (Appendix C)
- At the end of the second day a written reflection on the characteristics of the graph with equation $y = mx + b$.
- Cumulative quiz to assess acquisition of basic skills in calculating slopes, and rearranging equations.
- Written submission with individual answers to question posed, in the description, with justifications.

Resources

1. Spreadsheet software
2. Textbooks
3. Newspapers and advertising flyers
4. Internet

Accommodations

1. Students can be given options to a written report.
2. A symbolic algebraic manipulator *(e.g., TI89, TI92, Maple)* could be used as a compensatory tool for any students for whom algebraic manipulation is a learning obstacle.

Activity #3: Modelling Linear Relationships – Environmental Issues

Time: 9 hours

Description

The students will collect data on the use of energy – specifically electricity. They will use the properties of linear relations and their numeric, graphic and algebraic skills to examine trends and solve problems related to environmental data. This activity gives students and teacher opportunities to examine our lifestyle and understand our responsibilities of Christian stewardship. Students collect data to investigate a multi-step problem which leads to a graphical solution of two linear equations.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations:
The graduate is expected to be:

- a reflective and creative thinker who examines, evaluates and applies knowledge of interdependent systems (physical, political, ethical, socio-economic and ecological) for the development of a just and compassionate society.
- a responsible citizen who respects the environment and uses resources wisely.
- a reflective and creative thinker who thinks reflectively and creatively to evaluate situations and solve problems.

Strands: Number Sense and Algebra, Relationships, Analytic Geometry.

Overall Expectations
At the end of Grade 9, students will:

- describe the connections between various representations of relations. ✪
- determine, through investigation, the properties of the slope and y-intercept of a linear relation. ✪

Specific Expectations

Students will:

- pose problems, identify variables, and formulate hypotheses associated with relationships.
- collect data, using appropriate equipment and/or technology. ✪
- organize and analyse data, using appropriate techniques and technology.
- describe trends and relationships observed in data, make inferences from data, compare the inferences with hypotheses about the data, and explain the differences between the inferences and the hypotheses. ✪
- construct tables of values, graphs, and formulas to represent the linear relations derived from descriptions of realistic situations.
- determine the equation of a line of best fit for a scatter plot, using an informal process. ✪
- determine values of a linear relations be using the formula of the relations and by interpolating or extrapolating from the graph of the relation. ✪
- determine the equation of a line, given information about the line. ✪
- communicate solutions to multi-step problems in established mathematical form, with clear reasons given for the steps taken. ✪
- describe the meaning of the slope and y intercept for a linear relation arising from a realistic situation, interpolate and extrapolate from the graph and the equation of the relation, and identify and explain any restrictions on the variables in the relation. ✪
- determine the point of intersection of two linear relations, by hand for simple examples, and using graphing calculators or graphing software for more complex examples; interpret the intersection point in the context of an application. ✪

Planning

- Students will require access to the Internet or other source of up-to-date data.
- Book a computer lab with Internet access.
- This activity does not require graphing software, however spreadsheets or graphing calculators could be used to advantage.

Prior Knowledge Required

- Plotting data using scaled axes.
- Fitting a visual line of best fit.
- Finding slope between two points.
- Writing equations of lines given slope and intercept.

Teaching/Learning Strategies

Collecting and Displaying Data

- From Environment Canada and statistics sites for world data, or from an encyclopedia, students collect the following data for at least 15 countries over approximately 20 years.
- Statistics on electricity consumption by country
- Population
- Average monthly temperatures

- Plot total electricity used in Canada against years. The teacher should emphasize the importance of choosing a suitable scale since the issue of scale will be an important one in graphing any real life statistics.

- Ask students to consider why the amount of electricity used has increased. Can it be something other than each person using more? Students should notice that there has been an increase in population. How could we remove the population effect from the graph to see if each person is using more?

- Calculate per capita electricity use for the given years.

- Discuss students' predictions of a possible relationship between the variables of per capita use and time (measured over 20 years).

- Plot electricity used per capita in Canada against time. Assign a particular year to be 0 and then use an increment of 1 for each year thereafter.

Analyzing the Data

- In pairs, students fit a visual line of best fit using the convention: a line that passes through as many points as possible and has approximately half the remaining data above the line and half below.
- Calculate the slope by selecting two points on the line, find the intercept and develop an equation for the line of best fit.

> At this time teachers may want to give extra practice finding an equation of a line using the slope and intercept form.

Analyzing the Data (cont'd)
- Discuss the meanings of m, and b in relation to the particular data.
- Slope is the rate of change of electricity use per year.
- The intercept will be the amount used per capita in the starting year.
- Calculate interpolated and extrapolated values by substituting into the equation.

> Teachers may want to take time here to ensure that students have adequate practice at substituting in simple algebraic equations. A context should be provided, so that students can determine if their answer is reasonable. Example: Let $y = 20x + 12$. If $x = 25$, find the value of y.

- Students analyze the predicted values and discuss whether they are realistic. For example, can per capita use of electricity continue to grow infinitely at a constant rate? What limitations exist?
- In this context compare the domain and range of the plotted data to that of the line of best fit.
- Also at this time emphasize the distinction between discrete data as this plot displays (the only values known are at particular points) and the continuous values shown by the line of best fit.
- In groups of approximately four, students repeat the process above for two other countries and then present their graphs, defend the calculations of *m, b,* and the equation and give their interpretation of slope and intercept.

Reporting

- Based on their results and the results of the other groups students write a report to include:
  - a description of the trends noticed.
  - a comparison of Canada's place in the world regarding consumption of electricity and recommendations for the future.

• Students investigate alternate sources of energy. At present much electricity production is reliant on coal or nuclear plants. New technology is emerging that should bring the advantages of electricity to everyone while protecting the environment.

• In pairs, students use the Internet or recent articles to gather the following information about a renewable source of energy such as wind or solar power.
  Consider such things as:
  - the advantages of one source of power over another.
  - the initial cost for a Canadian house that has this type of power source.
  - the quantity of additional back up energy required to supplement the energy provided by traditional sources.

• Students share their findings and discuss the advantages of renewable energy from the position of our responsibility to the world's people and to our planet.

Multi-Step Problems

Students in small groups (2–3) investigate a multi-step problem using the renewable energy context. Consider the two families below :

• Fred and Jane have decided to switch to solar energy. They have researched their options and believe that it will cost them $15 000 to install the required hardware. They plan to have a back up energy source which they believe will cost an additional $600 per year.
• Gus and Paula, who have an identical house, have decided to continue with their present setup. They estimate that their total energy bills are $3500 per year.

• By graphing the total costs for each family against time, investigate whether it makes sense for Fred and Jane to switch.

• Consider that Fred and Jane borrowed the $15 000. If they paid $300 interest per year, how does this affect the answer to the question?

• What other important issue is involved in this question? Can you find a method to evaluate the impact on the earth of Fred and Jane's switch? What if 20 000 households switched?

• For the two lines in the graph find the point at which Gus and Paula's total costs exceed Fred and Jane's.

> Spend time here investigating the meaning of the intersection point of two lines in a meaningful context, and give practice in using the graphical method to solve them. This is an opportunity to use graphing software and use a trace function to find the intersection point. There should be an emphasis on the idea that the intersection point satisfies both equations. This offers another opportunity to substitute into algebraic expressions.

Assessment/Evaluation

1. Student collecting, plotting and analyzing of data will be assessed with an observation checklist. (Appendix C)
2. For the group assignment on two other countries: plots, lines of best fit, slopes, intercepts, equations and interpretation of these will be handed in and assessed for completion and accuracy.
3. Written report on the results of the country comparisons to be handed in and assessed for communication and application of procedures.

4. A set of questions on finding unit rates, calculating slopes, finding an equation based on slope and intercept or two points, and finding values by substituting in simple algebraic formulas will be marked for knowledge and understanding.
5. The solution to the multi-step problem will be marked for correct mathematical form and clear explanations and problem solving and inquiry.

Resources

1. http://www.energy.ca/ELECTRIC.html
2. http://www.statcan.ca

Accommodations

Reports can vary by mode of presentation and by level of complexity.

Activity #4: Modelling Linear Relationships - Bouncing Balls

Time: 6 hours

Description

Students will explore the linear relationship between a ball's drop height and rebound height in a technology rich environment. Students, in Unit one, will have developed a sense of the difficulty of accurately collecting data by hand for this activity. Here, they will use graphing calculators and motion sensors to collect the data accurately and efficiently. They will develop and explore numeric, graphic, algebraic and geometric models of the resulting linear relationship.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations:
The graduate is expected to be:

- an effective communicator who presents information and ideas clearly and honestly and with sensitivity to others.
- a reflective, creative and holistic thinker who demonstrates flexibility and adaptability.

Strands: Number Sense and Algebra, Relationships, Analytic Geometry, Measurement and Geometry

Overall Expectations
At the end of grade 9, students will:

- solve problems, using the strategy of algebraic modelling. ✪
- determine, through investigation, the properties of the slope and y-intercept of a linear relation.
- formulate conjectures and generalizations about geometric relationships involving two-dimensional figures, through investigations facilitated by dynamic geometry software, where appropriate. ✪

Specific Expectations
Students will:

- use algebraic modelling as one of several problem-solving strategies in various topics of the course.
- compare algebraic modelling with other strategies used for solving the same problem;

- communicate solutions to problems in approximate mathematical forms and justify the reasoning used in solving the problems.✪
- collect data, using appropriate equipment and/or technology. ✪
- construct tables of values and scatter plots for linearly related data collected from experiments of from secondary sources. ✪
- determine the equation of a line of best fit for a scatter plot, using an informal process. ✪
- identify the geometric significance of *m* and *b* in the equation $y = mx + b$ through investigation.
- determine the equation of a line, given information about the line.✪

Planning Notes

The teacher must have several balls and a class set of graphing calculators and motion sensors. The teacher should book time in the computer lab for the dynamic geometry exploration.

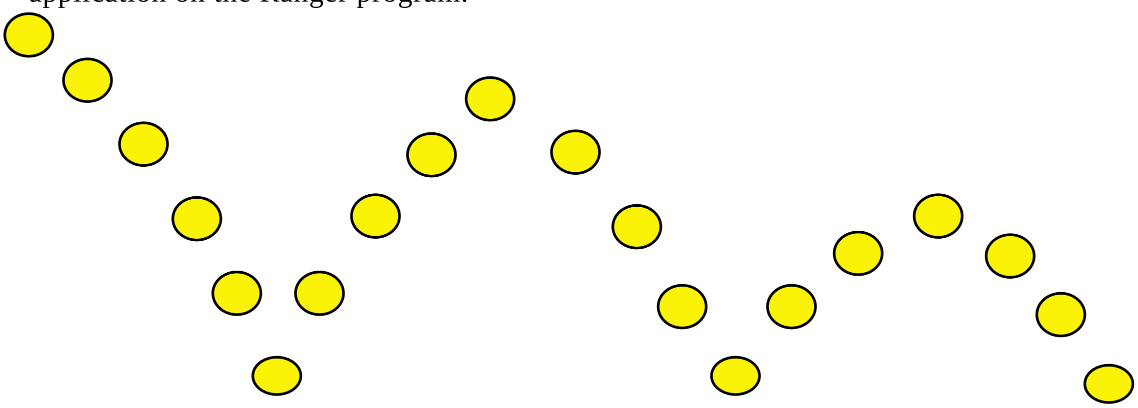
Prior Knowledge Required

- Lists, scatterplots, linear regressions on a graphing calculator
- Visual line of best fit

Teaching/Learning Strategies

Collecting Data With Technology

- Teacher demonstrates ball bounce using a graphing calculator, motion sensor and the Ball Bounce application on the Ranger program.



- Students collect data from ball bounce experiment in pairs with graphing calculator and motion sensor.

Analyzing the Data With Technology

- Teacher demonstrates the Trace, List, Scatter Plot, Regression analysis sequence on the graphing calculator. The process involves tracing along the graph to read, interpret record the peak heights as drop height and rebound height (see Unit 1, Activity 1). The drop heights and rebound heights will be input into lists. Set up a scatter plot. Perform a linear regression on the data.
- Students, in pairs, use the Trace, List, Scatter Plot, Regression sequence to analyze their data with the graphing calculator.
- Teacher demonstrates the Least Squares Fit on Dynamic Geometry Software.
- Students, in pairs, critique the algebraic model given by the graphing calculator and refine their algebraic model by exploring other, potentially better lines of fit, with the Least Squares Fit geometric model using Dynamic Geometry Software.

Discuss different points one could choose through which a line of best fit might be drawn. The justification for the selection of points must be firmly grounded in the context of the situation which gave rise to the data. Different scenarios should be explored.

Report

- Each student prepares a written report which they defend their algebraic model of a line of best fit.

Assessment/Evaluation

1. Observation Rubric (Appendix C)
2. Paper and Pencil Task
3. Written Report (Appendix B)

Resources

1. "TI Explorations: Modelling with CBR", Texas Instruments
2. http://www.ti.com/calc/docs

Accommodations

Students for whom the display on the graphing calculator is too small, should be provided with a viewscreen and a fluorescent palette or they should work on a computer using a spreadsheet application.

Activity #5: Modelling Intersection of Lines – Athletic Performance

Time: 7 hours

Description

In this activity, students collect men's and women's world record times in swimming and running events using secondary sources such as the Internet. Students use scatter plots and lines of best fit to model their results. Students interpret the resultant linear-regression equations as well as any restrictions encountered. By discovering the intersection point of their lines, students can predict when and/or if women will match or outperform men in the events.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations:

The graduate is expected to be:

- a reflective and creative thinker who thinks reflectively and creatively to evaluate situations and solve problems.
- a self-directed, responsible, life-long learner who applies effective communication, decision-making, problem-solving and resource management skills.

Strands: Number Sense and Algebra, Relationships, Analytic Geometry

Overall Expectations

At the end of Grade 9, students will:

- graph lines, using graphing calculators or graphing software. ✪

Specific Expectations

Students will:

- determine the equation of a line of best fit for a scatter plot, using an informal process. ✪
- construct tables of values and scatter plots for linearly related data collected from experiments of from secondary sources. ✪
- calculate the finite differences in the table of values of a linear relation and relate the result to the slope of the relation. ✪
- identify properties of the slopes of line segments through investigations facilitated by graphing technology, where appropriate. ✪
- describe the meaning of the slope and *y*-intercept for a linear relation arising from a realistic situation, interpolate and extrapolate from the graph and the equation of the relation, and identify and explain any restrictions on the variables in the relation. ✪
- determine the point of intersection of two linear relations, by hand for simple examples, and using graphing calculators or graphing software for more complex examples; interpret the intersection point in the context of an application. ✪

Planning Notes

Equipment required:

- a class set of graphing calculators
- Internet access (book computer lab time)
- Textbook

Prior Knowledge Required

- Collecting, organizing and analyzing data using technology.
- Recognizing relationships as being linear or non-linear.

Teaching/Learning Strategies

The Scenario

- Students are grouped in pairs.
- Each pair is assigned an event to research world record times over the past 50 years for both men and women.
- Students may use the Internet and/or library for research purposes.

Events:	**Track**	**Swimming**
	100 m	50-m freestyle
	200 m	100-m freestyle
	400 m	200-m freestyle
	800 m	400-m freestyle
	1 000 m	
	1 500 m	
	2 000 m	
	3 000 m	
	5 000 m	
	10 000 m	
	marathon	

- Using graphing technology, students produce scatter plots for the data collected: one scatter plot for the men’s results and another for the women’s results.

- Using the linear-regression capabilities of the technology, students determine a line of best fit and plot it through each scatter plot. Students can use the line of best fit to predict world record times in 5 years time, 10 years time and 20 years time from now for each graph.

- Using graphing technology, students plot both linear-regression equations on the same set of axes and use the resulting point of intersection to predict when and/or if the women's record will equal the men's record.

> Teachers may wish to allow the students to explore different ways of finding the point of intersection (numerically, graphically, algebraically).

Reporting

- Students prepare to present their results to the class by considering:
  - What assumptions are built into the graphs?
  - What factors could change the answers ?
  - Any restrictions on extrapolating from their lines of best fit?
  - How does the slope of the men's line of best fit compares with that of the women's line of best fit?

- Students prepare a written report of the activity in which they compare their particular event's results to that of their classmates and discuss the similarities and differences.

> A paper and pencil assessment task provides students with data from 2 new sources. Students use graphing technology to produce scatterplots and linear-regression equations to determine when and /or if the lines intersect. Students explain their results in context.

Assessment/Evaluation

1. Teacher Observation of Activities Checklist (Appendix C)
2. Oral Presentations
3. Written Report (Appendix B)
4. Paper and Pencil Assessment Task

Resources

1. Angier, Natalie. "Two Experts Say Women Who Run May Overtake Men." New York Times, 7 Jan., 1992.
2. Whipp, Brian J., & Susan A. Ward. "Will Women Soon Outrun Men?" Nature 355 (Jan. 1992): 25
3. Internet
4. http://infoplease.com
5. www.hkkk.fi/~niininen/links.html
6. www.netutah.com/swimlinx

Accommodations

Students can be given detailed directions and procedures or supported with scribed notes.

Appendix A: STUDENT HANDOUT – BALL BOUNCE

Materials:

✓ Ball (tennis, golf, table tennis, crazy...)
✓ Masking tape
✓ Metre stick
✓ Roll of blank cash register tape (or other paper to tape on wall)

Problem:

What is the relationship between the height that a ball is dropped and the height that the ball bounces?

Directions:

In groups of four, choose a role for each group member (*material collector, ball dropper, ball bounce height recorder, recorder*).
✓ Drop the ball from the first specified height, recording the bounce height.
✓ Drop the ball from the same height two times to get an average drop height.
✓ Repeat the process for the other specified drop heights.
✓ Be sure to keep a detailed record of how your group goes about carrying out this investigation.

Data:

Drop Height (cm)	Ball Type Bounce Height
200	
175	
150	
125	
100	
75	
50	
25	

Analyzing Your Data & Presenting Your Findings

Each member of the group is required to prepare a **BALL BOUNCE REPORT** that should include:
✓ list of group members and their roles;
✓ detailed description of how your group carried out this investigation;
✓ complete data chart (above);
✓ graph of the bounce height versus the drop height for each of the balls (the vertical axis is the bounce height and the horizontal axis is the drop height);
✓ visual line of fit through the data points for each ball;
✓ description in words of the relationship between the drop height and bounce height for each of the balls.

Appendix B: SAMPLE RUBRIC - WRITTEN REPORT

(Note: This Rubric is designed specifically for Unit 1, Activity 2. It can be used as a model for other activities)

		Level 1	Level 2	Level 3	Level 4
Model/ Formulate	Measure-ment.	Incomplete and/or step sizes not appropriate.	Measurements complete but some step sizes not appropriately chosen.	Measurements complete with appropriate choice of step sizes.	Student shows deeper understanding of problem by using creative means to generate additional measurements.
Transform/ Manipulate	Calculating Distance Chart/ Plotting Scale Conversion	Many errors in distance calculations. Many errors in scales, labels, titles and/or many points plotted incorrectly. Many errors in use of scale and/or use of scientific notation.	Some errors in distance calculations. Some errors in scales, labels, titles and/or some points plotted incorrectly. Many errors in use of scale and/or use of scientific notation.	Almost all distances calculated correctly. Charts appropriately titled. Appropriate scales and labels on graphs. Points accurately plotted. Uses map scale to convert distance with a few minor errors.	All distances calculated correctly. Uses map scale to accurately convert distance.
Infer/ Conclude	Analysis of Finite Differences Regression Analysis Deciding on distance along the border, and calculation of costs, etc.	Finite difference calculations missing or contain many errors. Understanding of difference between graphs of linear and non-linear relationships not demonstrated. Analysis not done. Calculations incomplete.	Some finite differences not calculated correctly. Infers correctly that relationship is non-linear. Recognizes that linear model does not fit but does not identify exponential model as correct. Cost calculations contain errors and/or no justification for quantities used and/or no explanation of assumptions made.	Calculates finite differences correctly and infers correctly that relationship is non-linear. Correctly identifies the exponential model as the closest fit for the data. Calculations complete and correct. Student clearly states all assumptions made and justifies all quantities used.	As level 3 but also looks for patterns in the differences calculated. As level 3 but also discusses the limitations of the exponential model for this problem. Student extends problem in some way, e.g., compares several schedules to maximize profits from marathon, or includes a detailed discussion of the notion of an infinite border length.
Communicate	Communi-cation	Poor use of mathematical language and/or arguments weak.	Does not use complete sentences and/or mathematical language. Arguments may not be fully developed or may be unclear.	Communicates clearly and effectively using mathematical language. Develops arguments fully, clearly stating all assumptions and considerations involved.	Communicates persuasively and effectively using mathematical language. Develops arguments fully, clearly stating all assumptions and considerations involved.

Appendix C: SAMPLE OBSERVATIONAL RUBRIC

Criteria	Level 1	Level 2	Level 3	Level 4
Engages in task	Begins task but with need of considerable prompting	Begins task with some prompting	Begins task without need of prompting	Begins task promptly and encourages others to begin
Applies appropriate strategies	Requires consistent support and prompting to pursue alternate strategies	Pursues alternate strategies with frequent assistance and limited prompting	Pursues alternate strategies with only limited assistance	Actively pursues alternate strategies independently
Uses resource materials effectively and independently	Does not refer to notes, text or other resources before seeking assistance	Rarely refers to notes, text or other resources before seeking assistance	Frequently refers to notes, text or other resources before seeking assistance	Consistently refers to notes, text or other resources before seeking assistance
Works effectively with others in the group	Assumes passive role and contributes infrequently and often in a limited way	Assumes passive role and contributes usually only when prompted	Assumes active role and contributes freely to the group	Assumes leadership role and tries to encourage all to contribute
Contributes effectively to the work of the group	Contribution is limited and only when prompted	Contribution is infrequent but often will volunteer ideas	Contribution is frequent and does not require prompting to share ideas	Contribution is frequent and builds on others ideas
Uses the materials and resources effectively	Requires frequent support in finding **and** using the materials.	Will often need support in finding **or** in using the materials	Needs only limited and infrequent support in finding or in using the materials	Needs little or no support to find and use materials, and consistently assists others to find and use
Is an active problem solver	Will explore very few possibilities and often stops before it is solved Rarely seeks alternate solutions	Will explore some possibilities but may stop before problem is solved Sometimes seeks alternate solutions	Will explore a few possibilities until the problem is solved Often seeks alternate solutions	Will explore many possibilities until the problem is solved Will seek alternate solutions

Choose the criteria that you can effectively and efficiently assess in the time you have available. Do not assess all criteria at one time. All criteria can be assessed as students work in groups, but some criteria can also be assessed as students work independently.

Appendix D: Supporting Materials for Unit 1, Activity 2

1. Mandelbrot Story

Benoit Mandelbrot, a famous mathematician (who is still living!) was commissioned to find the length of the British coastline. After months of working on this problem, he came back with his response. His response was not particularly well received by those who had paid him to give them an "answer". His "answer" was: "It depends!!!"

This activity is designed to allow students to develop, through a hands-on investigation, an appreciation for Mandelbrot's puzzling answer. The "dependency" is a rich avenue for exploration and discussion. The mathematical models students will construct to represent this scenario generate a non-linear relationships (exponential). The "dependency" arises from the step size one chooses.

2. Structured Walk

(a) Set your pair of compasses to your chosen step size (e.g. 5 cm) (L).
(b) "Walk" along the borderline with your compass, counting the number of "steps" (n).
(c) Measure the remainder with your ruler (r)
(d) Record your L, n and r values in a chart (see below) and calculate the length (perimeter P) using the formula $P=nL+r$.
(e) Repeat steps (a) - (d) for your next chosen step size (e.g. 4 cm).

Step Size (L in cm)	Number of Steps (n)	Remainder (r in cm)	Perimeter/Length ($P = nL + r$) (in cm)
5			
4			
3 ...			