Catholic District School Board Writing Partnership

Course Profile

Principles of Mathematics

Grade 9
Academic

- *for teachers by teachers*

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Preface

Special Note

In Units 1 and 2 additions have been made to the activities to reflect the curriculum expectations for the Number Sense and Algebra Strand.

Please note that Appendix B and C of Units 1 and 2 are referenced in Unit 3.

These insertions to Units 1 and 2 of the Grade 9 Academic Mathematics profile incorporate the following expectations:
Overall Expectations: NAV.01, NAV.02, NAV.03, NAV.04.
Specific Expectations: NA1.01, NA2.05, NA2.06, NA3.01, NA3.02, NA3.03.

Unit 1, Activity 2 "Mathematical Marathon"

Overall Expectations: NAV.02.
Specific Expectations: NA2.05, NA2.06.

Teaching /Learning Strategies

- Students now consolidate and enhance their understanding of the three basic exponent rules by completing assignments from the textbook. Include questions with the exponent rule for the power of a power.
- This would also be a good time to enter and interpret exponential notation on a scientific calculator, since some distances will be quite large. Again, use textbook assignments to involve applications with very small numbers.

Unit 1, Activity 3 "Exploring Motion"

Specific Expectations: NA1.01.

Teaching/Learning Strategies

- Students can design their walk to create a graph that is a) a straight line with a positive slope; b) a straight line with a negative slope; c) several lines with a combination of positive and negative slopes.
- Have students walk at different speeds and in different directions so that they not only investigate positive and negative slopes, but different ratios as well (refer to "Explorations, Modelling Motions: High School Activities with the CBR™").

Discuss with students the meaning of positive and negative integers in this context.

Unit 2, Activity 1 "Walking the Line"

Specific Expectations: NA1.03.

Teaching/Learning Strategies

On p. Unit 2-4, add to the box in #3:

This may be an opportunity to consolidate students' skills in performing operations with rational numbers.

Unit 2, Activity 2 "The Help Line"

Overall Expectations: NAV.01, NAV.02, NAV.03, NAV.04.

Specific Expectations: NA1.01, NA2.06, NA3.01, NA3.02, NA3.03.

Part 4: Possible Extension

Now would be a good time for the teacher to diagnose students' ability to work with integers and provide remediation as necessary. This could then be extended to lessons on manipulating polynomial expressions, supported by textbook resources. When multiplying and dividing monomials, highlight the exponent rules covered in Activity 1. Include the exponent rule for the power of a power.

Unit 3: Measurement and Geometry

Time: 40 hours
Unit Developer(s): Carolyn Boyer, Arlene Corrigan, Paul Costa, Anne Delahunt, Lori Goodfriend, Dominique Levac, Brian McCudden, Catherine Rea, Len St. Clair, Margaret Sinclair
Development Date: July - September 1999

Unit Description

The unit is divided into three sub-units.

Unit 3A	Solving Problems Involving Measurement	20 hours
Unit 3B	Optimization of Measurement	10 hours
Unit 3C	Exploring Geometric Properties of Plane Figures	10 hours

In this unit, skills such as mental mathematics, estimation, approximating, and solving multi-step problems are consolidated. Students extend their skills with manipulating polynomial expressions to solve first-degree equations and then apply these to measurement problems. Students determine the optimal values of various measurements, solve problems involving surface area and volume of three-dimensional objects, and use dynamic geometry software to make generalizations about geometric relationships.

Unit 3A: Solving Problems Involving Measurement

Time: 20 hours

Unit Description

Students develop formulas for the surface area and volume of prisms, pyramids, cones, cylinders, and spheres. They apply the formulas to solve multi-step problems. Within the context of measurement, students solve linear equations, rearrange formulas, and evaluate numerical expressions involving exponents. They consolidate skills of mental mathematics and estimation, demonstrate the effective use of a scientific calculator, and judge the reasonableness of answers to problems.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE 2b, CGE 4f, CGE 5a, CGE 5b, CGE 5g, CGE 7j.
Strand(s): Measurement and Geometry, Number Sense and Algebra
Overall Expectations: MGV.02, NAV.01, NAV.03.
Specific Expectations: MG2.01, MG2.02, MG 2.03, MG2.04, NA1.01, NA1.02, NA1.03, NA1.04, NA1.05, NA1.06, NA2.01, NA2.02, NA3.04, NA3.05, NA3.06.

Activity Titles (Time and Sequence)

Activity 1	Measurement and Geometry: Get Ready!	75 minutes
Activity 2	Surface Area and Volume of a Prism: The Prisms Around Us	75 minutes
Activity 3	Surface Area and Volume of a Cylinder	150 minutes
Activity 4	Assessment Activity	75 minutes
Activity 5	Volume of pyramid and cone	75 minutes
Activity 6	Surface area of pyramid and cone	150 minutes
Activity 7	Surface area and volume of sphere	150 minutes
Activity 8	Review, problems assignment, paper/pencil test	150 minutes
Activity 9	Solving First Degree Equations: A Balancing Act	300 minutes

Unit Planning Notes

The ability to solve multi-step problems becomes more important as students advance in their study of mathematics. This unit provides an opportunity for students to focus on that skill within the context of problems involving surface area and volume.

Many expectations of Number Sense and Algebra are also an essential part of the learning. This unit suggests that teachers take advantage of every opportunity to assist students in consolidating their understanding of exact versus approximate values, effective use of scientific calculators, and use of estimation in judging reasonableness of answers. The integration of percent, ratio, and rate within the problems to be solved provides opportunities for students to consolidate those important numeric skills. When planning lessons in this unit, it is important to keep in mind the mosaic of expectations to be achieved.

The problem solving assignment included in Activity 8 takes the form of a story and consists of a set of multi-step problems that may require estimation as part of the solution. The teacher should hand out the assignment at the beginning of the unit and encourage students to complete questions as they acquire the knowledge while working through the unit.

In solving problems involving measurement, it is frequently necessary to rearrange formulas and solve equations. Students' skills in solving equations are consolidated and extended in this unit.

Prior Knowledge Required

- perimeter and area of rectangles, triangles, parallelograms, trapezoids, and circles
- experience with solving multi-step arithmetic problems, including the importance of communication
- understanding of the concepts of percent, ratio, and rate; skills in applying percent, ratio, and rate
- skills and strategies in mental mathematics and estimation
- solution of simple equations (to the level of $ax + b = c$)

Teaching/Learning Strategies

- Use whole group instruction for things, such as: reviews of formulas from Grade 8; concepts and units of perimeter, area, surface area, volume, capacity; demonstrating development of formulas; teaching of skills in using scientific calculators, skills in solving equations; fostering development of good habits in problem solving.
- Students work individually and in pairs to solve problems.
- Students work in groups of three or four in developing some formulas.

Assessment/Evaluation

- periodic small quizzes to check understanding and progress (one opportunity is embedded in the activities; other, smaller quizzes may be inserted, as necessary)
- problem-solving assignments, including problem posing
- pencil and paper test

Resources

The main resource is the core program mathematics textbook.

Accommodations

This unit primarily involves the solution of multi-step problems and related calculations. Communication, meaning the form of solutions, is an important consideration.

The following accommodations might be considered:

- Have supportive materials available for students who demonstrate a lack of the required prior knowledge during the first activity.
- Allow students to work at some problems in pairs, to assist in developing initial understandings. Students must be able to solve multi-step problems independently to meet the curriculum expectations.

Activity 1: Measurement and Geometry: Get Ready!

Time: 75 minutes

Description

Students consolidate their mental mathematics and estimation skills, and judge the reasonableness of answers within the context of the perimeter and area of composite figures. Students consolidate their skills with using a scientific calculator effectively in working with formulas that involve exponents, and rational and irrational numbers.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE 4f, 5a.
Overall Expectations: NAV.01.
Specific Expectations: NA1.01, NA1.02, NA1.03, NA1.04, NA1.05, NA1.06, NA2.04, NA3.05.

Planning Notes

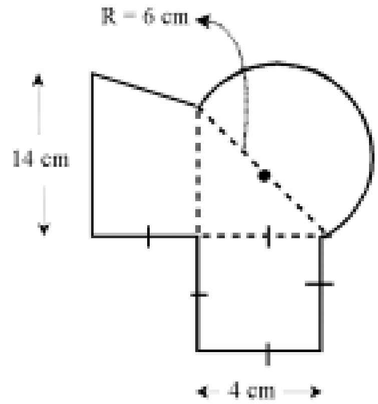
This introductory lesson is an opportunity to recap students' prior knowledge, to judge experience with multi-step problem solving, and to open the discussion of the expectations involving exact value, use of a scientific calculator, and judging reasonableness of answers.

Prior Knowledge Required

- concept of perimeter; units in which perimeter is commonly measured
- concept of area; units in which area is commonly measured
- calculation of perimeter of a figure bounded by straight lines; formula for the circumference of a circle
- formulas for the area of a rectangle, square, triangle, parallelogram, trapezoid, and circle
- determining the length of the side of a right triangle, using the Pythagorean theorem

Teaching/Learning Strategies

- In a whole group discussion, quickly review the elements listed under Prior Knowledge Required. Carry out an example involving a composite figure, such as:
  Determine the perimeter and area of the figure below:



- Emphasize the importance of communication in problem solving – writing a solution so that someone else can understand the thinking process involved. Remind students about the syntax involved in substituting into a formula. From the above example,
  Area of half-circle
  = ½πr^2
  = (½)(π)$(6)^2$
  = 56.5 cm^2
- Discuss how to handle fractions in a formula when using a calculator. In the example, most of the calculation can be done mentally (½ x 36), with the result multiplied by the π-button on the calculator. With fractions that would lead to *repeating decimals*, students should take advantage of the full decimal accuracy available on the calculator by punching the fraction in (e.g., if the fraction ⅔ were involved, students would punch in 2 ÷ 3).
- Discuss how to handle exponents on a scientific calculator.
- Discuss order of operations on a scientific calculator.
- Discuss the value π:
  - where it comes from (ratio of circumference to diameter for any circle)
  - its nature as an *irrational number* (Rational numbers have been discussed in Units 1 and 2. This would be a good opportunity to explain irrational numbers by contrast to rational.)
  - the use of the π symbol in substitution
  - the approximate nature of π when used in calculation, and the advantage of the π -button on the calculator over the value 3.14.
- Discuss the rounding of answers to measurement problems – when to round and what type of rounding to use.
- Model the use of estimation to judge the reasonableness of the answer produced by a calculator – estimate the answer before doing the calculation. Discuss estimation by rounding to compatible numbers (e.g., B is about 3), by operating with compatible numbers (e.g., in estimating the calculation (½)(π)$(6)^2$, it makes sense to square 6 and multiply by ½, then multiply by 3 as the estimation of π), multiplying and dividing numbers ending in zero. Encourage students to make estimation a regular part of their calculation procedure. Model estimation frequently. Encourage students to share the different methods by which they carry out a particular estimation.

- Discuss *when* and *when not* to use a calculator. Some calculations can be done much more quickly mentally, with the results incorporated into a larger calculation (e.g., as in $(½)(\pi)(6)^2$ in the example above).
- Introduce the students to an example of a word problem that involves an application of area or perimeter. Focus on expected form in the solution, the importance of drawing a diagram, and variation in rounding rules in relation to a context (e.g., dollars and cents). Discuss and model judging the reasonableness of an answer in relation to the context of the problem.
  Example:
  A circular garden has a radius of 3.2 m. It is surrounded by a pathway of width 0.8 m.
  a) To fertilize the garden, the owner mixes a powdered fertilizer with water and then sprays it on. The directions require that 25 mL of the fertilizer be mixed with 4 L of water. This covers 10 m^2 of garden. How much of the powdered fertilizer will be needed for one application on the garden?
  b) The pathway is made up of interlocking bricks. Each brick covers an area of 300 cm^2 and costs $1.25. Determine the value of the bricks on the pathway.
  c) What percent does the area of the pathway represent of the total area of garden and pathway combined?
- Select a homework assignment from the student text that involves determining the perimeter and/or area of composite figures, and solving problems involving applications of perimeter and area. Supplement as necessary with problems that integrate ratio, rate, and percent. Include problems in which the perimeter or area is known and a dimension must be found. Use these for the purpose of discussing rearrangement of formulas by substitution. (This continues throughout the unit.)

Activity 2: Surface Area and Volume of a Prism: The Prisms Around Us

Time: 75 minutes

Description

In this activity, students generalize their knowledge of the surface area and volume of rectangular and triangular prisms to include the surface area and volume of any prism. They discuss prisms in their environment as models.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE 3c.
Overall Expectations: MGV .02, NAV.01.
Specific Expectations: MG2.01, MG2.02, MG2.03, MG2.04.

Prior Knowledge Required

- Characteristics of rectangular and triangular prisms, and the similarity between them
- Concepts and units of measurement for surface area and volume
- Surface area and volume of rectangular and triangular prisms

Planning Notes

Have the following available for demonstration purposes:

- objects in the shape of a rectangular prism, a triangular prism, some other prism
- a series of congruent rectangles constructed from interlocking blocks to demonstrate the formula Volume = Area of base x height

During this lesson, continue to model and emphasize the following embedded learnings:

- using a scientific calculator effectively, including:
  - knowing *when* and *when not* to use it
  - how to handle fractions and exponents
  - considerations of order of operations
- using rounding appropriately in solutions to problems
- using mental mathematics and estimation to judge the reasonableness of answers produced by a calculator
- judging reasonableness of answers in the context of a problem
- observing correct form in communicating the solution to a problem

Teaching/Learning Strategies

- In a whole group presentation, use a model to elicit from students the characteristics of rectangular and triangular prisms. Discuss what is in common in their characteristics to identify a general definition for a prism (faces are rectangles, top and bottom are congruent, parallel polygons). Describe other possible prisms and where students may have seen them in the environment around them. Have models available for demonstration (cereal boxes, candy bar boxes, the "big box" approach in the design of shopping malls, any other unusual packaging in the shape of a prism)
- Review the meaning of surface area and the units in which it is typically measured. Ask students to suggest examples of areas for which each unit would be used. Elicit from students a method for calculating the surface area of *any* prism (sum of the areas of all its faces).
  Introduce and explain the term *lateral surface area* (e.g., the sum of the areas of all the side faces of a prism) and ask students to describe situations in which the lateral surface area would be needed instead of the *total* surface area.
  Do a sample problem involving the calculation of the surface area of a prism.
- Review the meaning of the volume and the units in which it is measured. Ask students to suggest examples of objects for which each unit would be used to describe the volume.
- Elicit from students the formula for calculating the volume of a rectangular prism (V=lwh) and a triangular prism (V = Area of base x height), and an explanation of their origin. Be prepared to model using interlocking blocks, if necessary. (Have several rectangles built, each having the same area. Stack them one on top of another. Since the layers are identical, the volume is the Area of the base x Height.)
- Discuss the relationship between capacity and volume and identify units of capacity. Ask students to identify quantities that are measured in units of capacity instead of units of volume.
- Identify the relationship between units of volume and units of capacity, (e.g., 1 mL of water occupies 1 cm^3 of space). Extrapolate this relationship to determine the number of litres in 1 m^3 of space (1 kL = 1000 L). Ask students to identify something in their surroundings, at school or at home, that would hold 1 kL of water.
- Do a sample problem involving the volume of a prism that is neither rectangular nor triangular. Include a reference to capacity. For example:
  A water trough is in the shape of a trapezoidal prism. Its base has internal side lengths of 85 cm and 60 cm and an internal height of 50 cm. The total internal length of the trough is 1.2 m. The trough is filled to 45% of its capacity. How many litres of water does it contain?
- Select a homework assignment from the student textbook that involves determining the surface area and/or volume of prisms. Supplement as necessary with problems that integrate ratio, rate, and percent.

Activity 3: Surface Area and Volume of a Cylinder

Time: 150 minutes

Description

Students apply their knowledge of the surface area and volume of rectangular prisms to develop formulas for the surface area and volume of cylinders.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE 3c.
Overall Expectations: MGV.02, NAV.01.
Specific Expectations: MG2.02, MG2.03, MG2.04, MG2.05.

Planning Notes

During this lesson, continue to model and emphasize the following embedded learnings:

- using a scientific calculator effectively, including:
  - knowing *when* and *when not* to use it
  - how to handle fractions and exponents
  - considerations of order of operations
- using rounding appropriately in solutions to problems
- using mental mathematics and estimation to judge the reasonableness of answers produced by a calculator
- judging reasonableness of answers in the context of a problem
- observing correct form in communicating the solution to a problem

Prior Knowledge Required

- surface area and volume of rectangular and triangular prisms and the origin of their formulas (Measurement, Grade 8)

Teaching/Learning Strategies

- Compare the structure of a cylinder to that of a rectangular prism, noting similarities, (e.g., in both the top and bottom are congruent, parallel faces; in both the "sides" are perpendicular to the base). Note differences (e.g., the sides of a rectangular prism are rectangles; a cylinder has only one continuous "side").
- Use the similarity between rectangular prisms and cylinders to suggest a method for determining the volume of a cylinder: Volume = Area of Base x Height. As a model, use a cylindrical package of cookies to illustrate further. Complete the process by substituting the formula for the area of the base, which is a circle.
  So, Volume of a cylinder = $\pi r^2 h$.
- Do one or two sample problems involving calculation of the volume of a cylinder. Include a composite figure and a word problem.
- Select a homework assignment from the student textbook that involves determining the volume of cylinders. Supplement as necessary with problems that integrate ratio, rate, and percent. Include problems that involve compositions of cylinders and prisms.

- To develop the formula for the surface area of a cylinder, ask each student to roll a piece of paper into a tube. Then, identify the shapes that make up the tube. The circle for top and bottom are obvious – but what shape is the side? It came from the piece of paper, so it must be a rectangle. Ask students to determine the height of the rectangle (same as the height of the tube). What is the length of the rectangle? (the circumference of the base). Additional models might include the labels on soup or fruit cans, which are easily removed.
  The rectangle has width h and length $2\pi r$. What is its area?
  Putting the pieces together, the formula for total surface area is:
  TSA of a cylinder = $2\pi rh + 2\pi r^2$
  The formula for lateral surface area will be: LSA of a cylinder = $2\pi rh$
- Do one or two sample problems involving calculation of the surface area of a cylinder. Include a composite figure and a word problem.
- Select a homework assignment from the student textbook that involves determining the surface area of cylinders. Supplement as necessary with problems that integrate ratio, rate, and percent. Include problems in which the volume or lateral surface area of a cylinder is known, and one dimension must be found.

Activity 4: Assessment Activity

Time: 75 minutes

Description

The following assessment is designed in two parts, a pencil/paper assessment and a problem-posing assignment.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE 2b, CGE4f, CGE5g.
Overall Expectations: MGV.02, NAV.01.
Specific Expectations: MG2.01, MG2.02, MG2.03, MG2.04, NA1.01, NA1.02, NA1.03, NA1.04, NA1.05, NA1.06, NA2.01, NA2.02, NA3.04, NA3.05, NA3.06.

Planning Notes

- Create a pencil/paper quiz on perimeter and area of plane figures; surface area and volume of prisms and cylinders.

Teaching/Learning Strategies

- Have students complete the pencil-paper quiz and then begin work on the following assignment, finishing it for homework:
  Pose and solve two problems, one involving surface area and one involving volume/capacity. Involve percent, rate, or ratio in at least one of them.
  Each problem is marked out of 7, based on the following set of criteria:
  - The problem requires a multi-step solution.
  - The problem involves an interesting application.
  - The problem involves realistic measurements.
  - The problem is worded clearly.
  - The final answer is correct.
  - The solution is presented in correct form, including use of English, proper formulas and correct units.
  - Rounding is used correctly in the solution.

Activity 5: Volume of Pyramids and Cones

Time: 75 minutes

Description

Students develop and apply the formulas for the volume of a square-based pyramid and the volume of a cone.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE 3c.
Strand(s): Measurement and Geometry
Overall Expectations: MGV.02.
Specific Expectations: MG2.02, MG2.03.

Prior Knowledge Required

- formulas for the volume of a rectangular prism and the volume of a cylinder

Planning Notes

Obtain a volume set. This is a commercially available resource that contains a plastic model of a rectangular prism, a pyramid, a cylinder, a cone, and a sphere. The models have compatible dimensions (i.e., the same base and height); they are hollow, so that they can fit within one another. If a volume set is not available, make models from cardstock that will hold its shape. You need a rectangular prism and a pyramid, having the same base and height; a cone and a cylinder, having the same base and height.

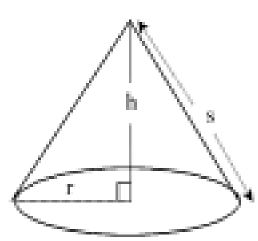
Make available material with which to fill the models (e.g., rice, sand, small plastic pellets), in a quantity sufficient to fill the rectangular prism and the cylinder.

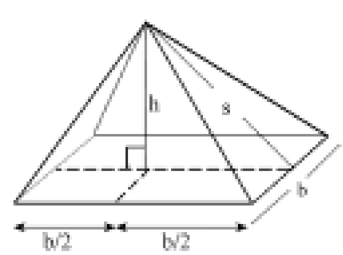
During this lesson, continue to model and emphasize the following embedded learnings:

- using a scientific calculator effectively, including:
  - knowing *when* and *when not* to use it
  - how to handle fractions and exponents
  - considerations of order of operations
- using rounding appropriately in solutions to problems
- using mental mathematics and estimation to judge the reasonableness of answers produced by a calculator
- judging reasonableness of answers in the context of a problem
- observing correct form in communicating the solution to a problem

Teaching/Learning Strategies

- In a whole group presentation using the models and diagrams, illustrate the features and dimensions of a cone and a pyramid.





- Using the models of the rectangular prism and the square-based pyramid, demonstrate that these prisms have the same base and height. (With the volume set, the pyramid fits inside the rectangular prism.)
- Ask students how they think the volumes would compare (i.e., Would there be a relationship between the volumes?) Students will likely guess that the volume of the rectangular prism is somewhere between two and four times the volume of the pyramid.
  Test the relationship by filling the pyramid with the material chosen and pouring into the prism. Count the number of times that this can be done (3). You might have a student do the demonstration. The conclusion reached is that the volume of a square-based pyramid is one-third the volume of a rectangular prism *having the same base and height*. A formula might be written as:
  $V = \frac{1}{3}b^2h$, where b is the side length of the base and h is the interior height of the pyramid.
- Repeat the activity, this time using the cone and the cylinder that have the same base and height. A similar conclusion is reached: the volume of a cone is one-third the volume of a cylinder having the same base and height. Then, a formula might be written as:
  $V = \frac{1}{3}\pi r^2h$, where r is the radius of the base and h is the interior height of the cone.
- Do sample problems involving the volume of cones and square-based pyramids. Include composite figures that involve, not only pyramids and cones, but also rectangular prisms and cylinders. Also include application problems.
- Select a homework assignment from the student textbook that are problems involving volume of pyramids and cones. Include problems in which the volume is known, and one dimension must be solved for.

Activity 6: Surface Area of a Pyramid and a Cone

Time: 150 minutes

Description

Students develop and apply the formulas for the volume of a square-based pyramid and the volume of a cone.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE 3c.
Strand(s): Measurement and Geometry
Overall Expectations: MGV.02.
Specific Expectations: MG2.01, MG2.02, MG2.03, MG2.04.

Prior Knowledge Required

- formulas for the circumference of a circle and the area of a rectangle, a triangle, and a circle
- solution of problems involving proportion
- exponent rule for division

Planning Notes

Have on hand blank paper, tape, scissors, rulers, compasses, and protractors for students to use in building models of square-based pyramids and prisms.

During this lesson, continue to model and emphasize the following embedded learnings:

- using a scientific calculator effectively, including:
  - knowing *when* and *when not* to use it
  - how to handle fractions and exponents
  - considerations of order of operations

- using rounding appropriately in solutions to problems
- using mental mathematics and estimation to judge the reasonableness of answers produced by a calculator
- judging reasonableness of answers in the context of a problem
- observing correct form in communicating the solution to a problem

Teaching/Learning Strategies

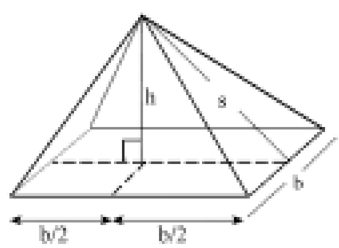
The Square-based Pyramid

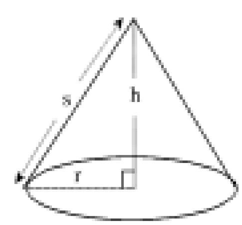
- Have students work in pairs. Each pair is to build a model of a square-based pyramid, having a base length of 6 cm and a slant height of 5 cm. They determine the interior height of the pyramid, both by measurement and by calculation.
- Ask students to report the interior heights of their pyramids. Discuss the actual calculation of the interior height, using the Pythagorean theorem.
  Use a diagram of a pyramid showing right triangle relationship among semi-base, height, and slant height.

  From the diagram: $s^2 = h^2 + (\frac{b}{2})^2$
- Ask students to describe the way in which they would determine the total surface area and the lateral surface area of a square-based pyramid:
  Total surface area = area of square based + 4(area of one triangular face)
  Lateral surface area = 4(area of one triangular face)
- Do sample problems involving the total and lateral surface area of square-based pyramids. Include a composite figure that may involve, not only a square-based pyramid, but also rectangular prisms or cylinders. Also include an application problem. Give the pyramid height in one problem and the slant height in the other.
- Select a homework assignment from the student textbook. Include problems involving composite figures and problems involving applications. Also include problems in which the lateral surface area is known, and one dimension must be solved for.

The Cone

- The development of the formula for the surface area of a cone is similar to that of the square-based pyramid, but is complicated by the continuous lateral surface of the cone. Begin by comparing a cone and a square-based pyramid having the same height, and for which the diameter of the cone is equal to the base length of the pyramid. Identify the fact that a similar Pythagorean relationship will exist in the cone to that of the pyramid.





In the pyramid, $s^2 = h^2 + (\frac{b}{2})^2$

In the cone, $s^2 = h^2 + r^2$

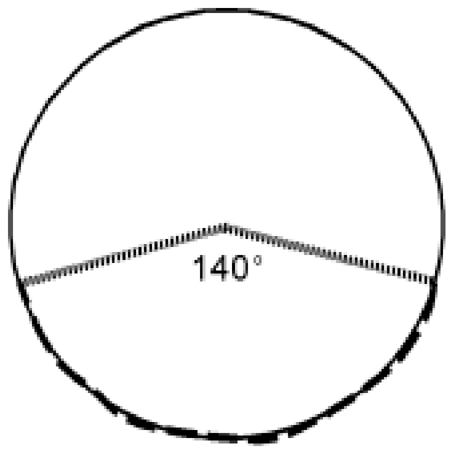
In the pyramid, the area of the base was given by the formula for the area of a rectangle; in a cone, it will be given by the formula for the area of a circle.
In the pyramid, the lateral surface area was given by determining the area of one triangular face, and then multiplying by 4.

- Pose this problem: How will we determine the lateral surface area of the cone, which is a continuous surface? Students work in pairs to build a model of a cone by cutting a sector from a circle and highlighting key parts using the worksheet Surface Area of a Cone: Setting the Stage. Then, they identify where the important parts of the cone came from on the original circle, and attempt to develop a formula for the surface area of the sector (or the lateral surface area of the cone).
- See the teacher worksheet Developing the Formula for the Surface Area of a Cone. After students have built their models and attempted to create a formula, walk through the development with them.
- Do sample problems involving the total and lateral surface area of cones. Include a composite figure that may involve, not only a square-based pyramid, but also rectangular prisms or cylinders. Also include an application problem. Give the cone height in one example and the slant height in another.
- Select a homework assignment from the student textbook. Include problems in which the lateral surface area is known, and one dimension must be solved for. Include application problems and those involving composite figures.

Student Worksheet: Surface Area of a Cone: Setting the Stage

Work with a partner on this activity.

1. a) Using a ruler and compass, construct a circle having a radius of 8 cm.
   b) Mark the centre of the circle. Construct an angle of 140E at the centre of the circle, creating a sector, as shown in the diagram.



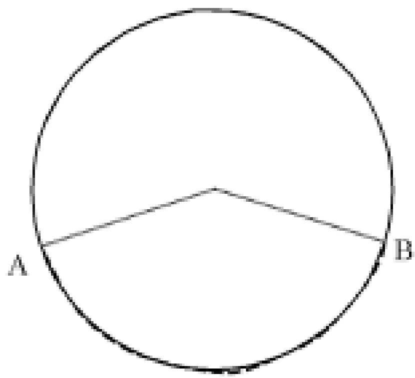
   c) Using a coloured marker, trace heavily over the arc at the bottom of the circle. Using a different coloured marker, trace heavily over the two radii of the circle.
   d) Cut out the sector and use it to make a cone. Tape the edges together.

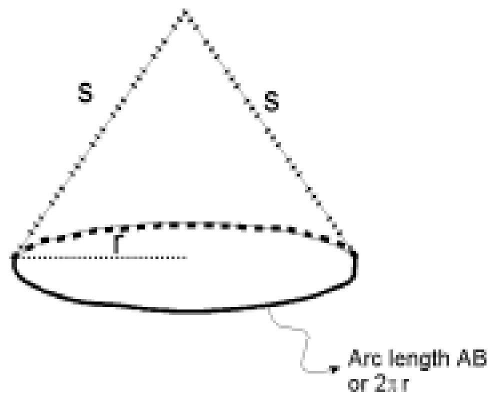
2. Examine your cone.
   a) Which part of the original sector has created the base circle of the cone?
   b) Where on the cone do the radii of the original semi-circle appear?
   c) Draw diagrams on the bottom of this page to illustrate what you have found.

3. How could you determine the surface area of the cone?

Teacher Worksheet: Developing the Formula for the Surface Area of a Cone

Given: A circle having radius *s*, standing on *Arc AB*. Cut out the sector and use it to make a cone.





The lateral surface area of the cone is the same as the area of the sector.

Let the radius of the base of the cone be *r*.

We have:

$$\frac{\text{Area of sector}}{\text{Area of original circle}} = \frac{\text{Arc length } AB}{\text{Circumference of original circle}}$$

$$\frac{\text{Area of sector}}{\pi s^2} = \frac{2\pi r}{2\pi s}$$

$$\text{Area of sector} = \frac{\pi r s^2}{s}$$

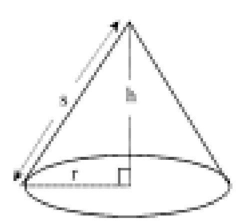
$$\text{Area of sector} = \pi r s$$

Therefore, *the Lateral Surface Area of a Cone Is Given By:*

LSA = πrs, where r is the *radius of the base* of the cone, and s is the *slant height* of the cone.

NOTE that if h is the *height of the cone*, then:

$s^2 = \sqrt{h^2 + r^2}$



Activity 7: Surface Area and Volume of a Sphere

Time: 150 minutes

Description

Students develop and apply the formulas for the volume and surface area of a sphere.

Strand(s) and Expectations

See the Unit overview for a list of the expectations involved in each activity of Unit 3A.

Prior Knowledge Required

- formula for the area of a circle
- Archimedes Principle (When an object is submerged in water, it displaces an amount of water equal to its own volume.)

Planning Notes

Measurement activities are used to develop the formulas for surface area and volume. In the activity involving the formula for surface area, students work in groups of four, with each group needing an orange (as spherical and easy to peel as possible), a piece of graph paper, and a ruler. To save class time and increase accuracy, it might be worthwhile to score the peel of the oranges ahead of time.

As a second demonstration of the surface area of a sphere, use a baseball (one that you won't mind undoing the stitching on).

The development of the formula for volume of a sphere involves a demonstration of water displacement. A submersible sphere, such as a baseball is needed, along with a clear, graduated container and sufficient coloured water to submerge the baseball.

During this lesson, continue to model and emphasize the following embedded learnings:

- using a scientific calculator effectively, including:
  - knowing *when* and *when not* to use it
  - how to handle fractions and exponents
  - considerations of order of operations
- using rounding appropriately in solutions to problems
- using mental mathematics and estimation to judge the reasonableness of answers produced by a calculator
- judging reasonableness of answers in the context of a problem
- observing correct form in communicating the solution to a problem

Teaching/Learning Strategies

<u>The Surface Area of a Sphere</u>

- In this developmental activity, students work in groups of four. Supply each group with an orange (with the peel scored), a piece of graph paper, and a ruler. They peel the orange and estimate the area of the peel by spreading it out on graph paper. Students divide the orange itself into two halves, as carefully as possible. The cross-section of the orange should approximate a circle; students carefully measure its radius and calculate its area.
  Have each group record its data on the board (total area of peel, cross-sectional area of orange). Ask students if they see any constant relationship between the pieces of data, or how they might test for one (calculate ratio of total peel area to cross-sectional area). The ratios should be reasonably constant, and within the neighbourhood of 4:1. Discuss data that might not fit the trend and why it may have occurred.

Translate the result to a formula: the total area of the peel represents the total surface area of the sphere (orange). The cross-sectional area of the orange was calculated using the formula $A = \pi r^2$. So, the total surface area of the sphere must be given by: TSA = $4\pi r^2$

Another illustration of the formula can be found by undoing the stitching on a baseball and laying the leather flat. The resulting shape approximates four circles.

- Do sample problems involving the total surface area of spheres. Include a composite figure that may combine spheres or hemispheres with other objects. Also include an application problem.
- Select a homework assignment from the student textbook. Include problems involving composite figures and problems involving applications. Also include problems in which the total surface area is known, and *r* must be found.

The Volume of a Sphere

- Begin with a discussion of what students think the formula for the volume of a sphere should look like. Since a sphere is circular, π is likely to be involved and since the only dimension on a sphere is its radius, *r* must be involved also. For volume, the third dimension must be involved. Students should guess that the formula would involve πr^3. The displacement activity that follows is intended to determine by what factor πr^3 would be multiplied.
- The activity can be done as a teacher demonstration or as a student group activity. The advantage of students carrying out the activity is that data is gathered for a variety of spherical objects; in the teacher demonstration, a result is drawn from the data for only one object.
- The activity:
    - Estimate as accurately as possible the radius of a spherical object that will not float, such as a baseball or a billiard ball. Calculate the value πr^3.
    - Fill a graduated beaker with coloured water to a level that allows the spherical object to be completely submerged. Remind students of Archimedes principle, that is, that the amount of water displaced by the sphere will be equal to the volume of the sphere. Record the starting level of the water. Submerge the ball and record the resulting level. Subtract the two water level figures. The result is the volume of the sphere.
    - Now, compare the volume estimate by displacement to the calculated value of πr^3, by ratio. The result should be around 1.3. The formula, in fact, is

$$V = \frac{4}{3}\pi r^3.$$

- Do sample problems involving the volume of spheres. Include a composite figure that may combine spheres or hemispheres with other objects. Also include an application problem.
- Select a homework assignment from the student textbook. Include problems involving composite figures and problems involving applications.

Activity 8: Review and Problem Assignment; Test
Time: 75 minutes

Description
Students prepare for a pencil/paper test of applications of the formulas for the surface area and volume of prisms, square-based pyramids, cylinders, cones, and spheres.

Strand(s) and Expectations
Ontario Catholic School Graduate Expectations: CGE 2b, CGE 4f, CGE 5a, CGE 5b, CGE 5g, CGE 7j.
Strand(s): Measurement and Geometry, Number Sense and Algebra
Overall Expectations: MGV.02, NAV.01.
Specific Expectations: MG2.01, MG2.02, MG 2.03, MG2.04, NA1.01, NA1.02, NA1.03, NA1.04, NA1.05, NA1.06, NA2.01, NA2.02, NA3.05

Teaching/Learning Strategies
- Prepare a review lesson as appropriate for the students in the classroom. Assign appropriate questions from the textbook as review.
- A problems assignment is included as student worksheets Student Activity – The Question Game and Student Activity – Answering the Questions! Hand out this assignment at the beginning of the unit and encourage students to work at it during the unit. The assignment involves some questions that require students to estimate key information, along the line of Fermi problems.
- Create a pencil/paper test reflecting the work of the unit.

Student Activity - The Question Game

In the story below, a number of questions are posed by two boys as they shop in a grocery store. Your job is to answer the questions. The numbers throughout the story refer to those on the Answering the Questions worksheet, where you will find more detailed statements of the problems.

John is a 17-year-old high school student who volunteered to care for Al, the 7-year-old son of a neighbour, every Saturday and to do the family shopping. On one particular Saturday, the two boys set off for the local mall, shopping list in hand.

Al was particularly inquisitive and seemed able to find mathematics at every turn. As he and John entered the grocery store, they walked past a row of shopping carts. John pulled one out of a row. "Hey, John!" said Al, "Have you ever wondered what total volume of groceries is wheeled out of here in shopping carts on a typical Saturday?" John answered: "That thought hasn't crossed my mind, but it's certainly an intriguing one." **[1]**

The boys walked on in silence, and eventually passed the deli counter. Al spied a counter full of cheese cut into triangular wedges and wrapped in foil. "Hey, John! I've got another question for you," he said. "How many rolls of tin foil do you think it would take to wrap all the cheese wedges in that display?" John pondered momentarily, then answered, "Hmmmm, I'll have to think about that one!" **[2]**

Continuing their shopping, Al and John entered the aisle where the soup was kept – chicken noodle was on the list. "I feel another question coming on," chirped Al. "Oh, great," responded John, "Let's hear it." Al proceeded, "I was just wondering – if you cut off the labels from all the cans of soup in this counter and laid them out on the floor, would they cover the entire aisle?" **[3(a)]** John responded, "That's a great question. And I have one too: If you emptied the soup from all those cans, how many bathtubs would it fill?" **[3(b)]** Al laughed happily, "Now you're getting into the game!"

And so the rest of the shopping trip went – Al and John taking turns posing grocery store problems. In the ice cream aisle, John reached for a package of waffle cones and Al asked, "I wonder how much waffle was needed to make all 12 cones in that box?" **[4(a)]** John responded, "I don't know, but I bet that it would take more than 4 litres of ice cream to fill all the cones." **[4(b) (c)]**

In the fruit department, John stood looking at a display of oranges. They were all quite round and about the same size. The oranges were stacked in a pile with a 5x5 square on the bottom row, 4x4 square on top of that, then a 3x3 square, a 2x2 square, and, at the top of the display, there was one orange. John asked "I wonder what the total surface area of all the peel on all those oranges is?" **[5(a)]** Al responded, "I've got a good one too! I wonder how much air there is between all those oranges in that stack? **[5(b)]** Suppose you had to build a container to hold them all in that shape – how much cardboard would you need?" **[5(c)]**

The boys went through the check-out, paid for the groceries, and started homeward. "I kind of like this question game," said Al. "Me, too," answered John. "But you know, we didn't answer any of those questions, we only asked them. I wonder what we need to know in order to find the answers?"

Student Activity - Answering the Questions!

Communication is important in your solution to these problems. Be sure to use good form and precise language to identify the steps you are taking, and justify all estimates used.

The solution to each question will be marked out of 5 according to the criteria below:

- Correct procedures and formulas are used in calculating measurements.
- Estimates are reasonable and are justified or explained.
- The overall method used for solving the problem would lead to a correct solution.
- The final answer is correct, based on the estimates used.
- Good form is used in communicating the solution, including correct use of language, proper substitution, correct units.

1. The buggy of a shopping cart is a trapezoidal prism. The trapezoids have a height of 50 cm and base lengths of 110 cm and 80 cm. A grocery store has 200 carts and they are used repeatedly on a Saturday. Estimate the total volume of groceries wheeled out of the grocery store on a typical Saturday.
2. Each wedge of cheese in a display is a triangular prism, having approximately the following dimensions: height of wedge is 3 cm; bottom and top of wedge are triangles having base 6 cm and height 5 cm. The pile of wedges on the counter contains about 30 wedges. How many rolls of aluminum foil would be needed to wrap all the wedges?
3. A soup counter has two shelves, each measuring 1.5 m long. Each shelf contains three rows of cans stacked one on top of another. The cans, which are closely packed on the shelves, are all a standard soup can size.
   a) If the labels were cut off of all the cans and laid on the floor, would they cover the aisle?
   b) If you emptied the soup from all those cans, how many bathtubs would it fill?
4. A waffle cone measures about 15 cm high with a diameter of 6 cm.
   a) How much "waffle" is needed to make a box of 12 cones?
   b) What percentage of a 4 L ice cream carton would be needed to fill all the cones, each cone just to the brim?
   c) Would 4 litres of ice cream be enough if each cone was full, and a hemispherical scoop set right on top?
5. A display of oranges was constructed so that they were stacked in a pile with a 5x5 square on the bottom row, 4x4 square on top of that, then a 3x3 square, a 2x2 square, and, at the top of the display, there was one orange. The oranges were all very round, and each had an approximate diameter of 6.5 cm.
   a) What is the total surface are of all the oranges?
   b) How much air is between all the oranges in the stack?
   c) How much cardboard would be needed to build a container to house the oranges, stacked as they are?

Activity 9: Solving First-Degree Equations: A Balancing Act

Time: 300 minutes

Description

Students use the balance method to solve equations of the first degree and to rearrange formulas involving variables of the first degree, with and without substitution.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE 2b.
Strand(s): Number Sense and Algebra
Overall Expectations: NAV.03.
Specific Expectations: NA3.04, NA3.06.

Prior Knowledge Required

- solving simple equations, up to the level ax + b = c
  i.e., equations of the form:

  $x + 4 = 10$ $\quad$ $-5 - a = 11$ $\quad$ $-6y = -18$ $\quad$ $\frac{k}{5} = -4$ $\quad$ $3 - 4x = 11$

- rearranging formulas in the context of measurement problems, or in the context of rearranging the equations of straight lines from one form to another

Planning Notes

Students have solved simple equations in Units 1 and 2. If the balance method has already been demonstrated, skip the first step described below. If not, borrow a two-pan balance from the Science department.

Teaching/Learning Strategies

- Demonstrate the use of a two-pan balance – to weigh an item, put it on one side; then add weights to the other side until the pans come in balance.
  The equals sign (=) in an equation is like the balance: to keep the equation in balance, whatever you do to one side, you must do to the other.

  Two Basic Rules for Solving Equations
  - To keep an equation in balance (=), if you do something to one side, you must do the same to the other.
  - To remove a term from an equation, perform the *opposite* or *inverse* operation:

OPERATION	INVERSE
+	-
-	+
x	÷
÷	x

- Use the balance method to model the solutions to some simple equations, such as:

  a) $-3x = 21$ $\quad$ b) $\frac{k}{-3} = 2$ $\quad$ c) $12 = k - 5$ $\quad$ d) $4a + 1 = 9$

  Note that in solving equations in which fractions are involved, it is important to maintain the balance method rather than "cross-multiplying".

- During the first four hours allotted to this activity, extend the students' experience in using the balance method to include equations at each stage shown below:
  - $5 + 9x = 29 - 3x$
  - $5x - 3 + 6x - 7x = 6 + 8 + 9x - 2$
  - $3 + 5(x - 1) = 2(x - 6) + 1$
  - $\frac{b}{6} = \frac{-3}{5}$
  - $\frac{4x - 3}{5} = \frac{2x + 7}{4}$
  - $\frac{y}{2} - 1 = \frac{y}{5} + \frac{1}{4}$

  At each stage, discuss which steps may be done mentally and which should be written down. Encourage students to leave steps out as they are ready.
- Provide ample opportunities for practice at each stage.

Assessment/Evaluation
- Hold periodic small quizzes as necessary during the teaching.
- During the fifth hour allotted, conduct a review and pencil/paper quiz.

Unit 3B: Optimization of Measurement

Time: 10 hours

Unit Description
To develop a sense of the meaning of working with a fixed measure (such as surface area), students design and construct cardboard toy characters as a class project. They take them to the children's wing of the local hospital. Students then investigate some mathematics of optimization of measurement, including the effect that varying dimensions of some three-dimensional shapes has on the surface area and volume of the shapes, the relationship between the surface area and volume of rectangular prisms and of cylinders, and between the perimeter and area of a rectangle.

Strand(s) and Expectations:
Ontario Catholic School Graduate Expectations: CGE3b, CGE4a, CGE7j.
Strand(s): Measurement and Geometry
Overall Expectations: MGV.01, MGV.02, MGV.03, NAV.01.
Specific Expectations: MG1.01, MG1.02, MG1.03, MG1.04, NA1.01, NA1.02.

Activity Titles (Time and Sequence)
Activity 1	Toys for Children	150 minutes
Activity 2	Exploring Surface Area and Volume on Rectangular Prisms	225 minutes
Activity 3	Comparing Surface Area and Volume on a Cylinder	150 minutes
Activity 4	Comparing Perimeter and Area of a Rectangle	75 minutes

Unit Planning Notes

Students work co-operatively, either in pairs or larger groups, throughout the unit. An assessment rubric is provided that tracks observation of student characteristics during group work. To make the tracking possible, it is important to consider the structure of the groups at the beginning of the unit, and keep them constant throughout.

A variety of materials are needed during the unit, including poster boards, scissors, tape, glue, rulers, protractors, compass sets, and styrofoam spheres. Students need access to graphing calculators or spreadsheets during the final three activities of the unit.

Prior Knowledge Required

- construction of prisms, pyramids, cones, and spheres
- formulas for the surface area and volume of prisms, pyramids, cones, and spheres
- using a spreadsheet or graphing calculator for calculation and graphing

Teaching/Learning Strategies

- whole group presentations and class discussions
- students working in pairs and groups
- students carrying out investigations

Assessment/Evaluation

- Assess students using Assessment of Work in a Group." There is space on the chart for the addition of other characteristics.
- Observation and rating by the teacher
  During the time students are working in class on this unit, observe and rate them on some or all of the characteristics.
- Rating by other students at the end of the activity
  Each student chooses two characteristics on which he/she wishes to be rated by the other people in the group.
- Assess written reports of investigations from criteria given to students.

Assessment of Work in a Group

CHARACTERISTIC	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
The student:				
LISTENING	- is easily distracted	- listens intermittently to others	- listens attentively to others	- listens actively and focuses full attention on the speaker
RECEIVING AND USING FEEDBACK	- makes limited use of the suggestions of others	- accepts feedback from others	- uses feedback as a basis for improvement	- builds new ideas from the feedback of others
PROVIDING FEEDBACK TO OTHERS	- provides limited feedback to others	- provides relevant but sometimes fragmented feedback to others	- provides constructive, relevant feedback to others	- provides detailed feedback and creative strategies for improvement
COMMITMENT TO TASK	- pays limited attention to the task	- has occasional lapses in attention to task	- remains on task throughout the activity	- remains on task throughout the activity and effectively encourages others to do so

NAME OF STUDENT BEING RATED: ______________________________

RATING DONE BY: ______________________________

Activity 1: Toys for Children

Time: 150 minutes

Description

Students are given a fixed amount of material from which they must construct a required set of cardboard animals. Students are required to use as much of the provided material as possible, without exceeding what is provided.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE3b, CGE4a, CGE7j
Strand(s): Measurement and Geometry
Overall Expectations: MGV.02
Specific Expectations: MG2.01, MG2.03, MG2.04

Planning Notes

Students need space and flat surfaces to work on for this activity. Materials include: four poster boards per group, scissors, tape, glue, rulers, protractors, compass sets, and styrofoam spheres. Students construct the animals during class, but take them home for decorating.

Prior Knowledge Required

- construction of prisms, pyramids, cones, and spheres
- formulas for the surface area and volume of prisms, pyramids, cones, and spheres

Teaching/Learning Strategies

Students work in groups of four, with each group limited to four poster boards. Each group constructs toys using the following 3-D objects to create and decorate toy characters. The toys can be completed using various materials and imagination:

Example:

Toy Character	Hat	Face	Body
Tin Man	Cone	Cylinder	Cylinder
Clown	Cone	Sphere (styrofoam)	Cube
Dog		Triangular pyramid	Triangular prism
Space Girl		Square pyramid	Rectangular prism

Within their groups, students work together to plan the use of the available material. They assign construction of the various toys to individual members of the group.

After constructing the toys, students estimate, as accurately as possible, the amount of material remaining from their original four poster boards.

Each group submits:

- a list identifying which students are responsible for the construction of each toy
- a written description of the method used to estimate the amount of remaining material.

Assessment/Evaluation

Begin the assessment by observation, using the Assessment of Work in a Group.

Assess the completion of the task out of 10 marks, using the following scale:

- Up to 5 marks to each student for *completion* of their assigned animals
- Up to 5 marks to each student for the written submission, assessed based on the following criteria:
  - the list identifying students' assignments is present
  - students did not exceed the amount of material provided, but created all required animals
  - the written estimate of remaining material includes a reasonable procedure
  - the calculations are accurate
  - the written estimate is clearly presented

Accommodations

- Modify the task as necessary for students who may have difficulty in the constructions.

Activity 2: Exploring Surface Area and Volume on Rectangular Prisms

Time: 225 minutes

Description

Students carry out a guided investigation of the relationship between the surface area and side length of a cube and between the volume and the side length of a cube. Students design and carry out an investigation of the relationship between the surface area of a rectangular prism and its dimensions and between the volume of a rectangular prism and its dimensions.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE3b, CGE4a, CGE7j.
Strand(s): Measurement and Geometry
Overall Expectations: MGV.01.
Specific Expectations: MG1.01, MG1.02.

Prior Knowledge Required

- formulas for the surface area and volume of a rectangular prism
- use of a spreadsheet or graphing calculator for calculation and graphing
- steps involved in carrying out an investigation

Teaching/Learning Strategies

- Have students work in pairs to complete the problems found on student worksheet Growing Cubes, in which they explore the effect on the surface area and volume of a cube by varying its side length. Discuss the results with the whole group.
- Assign the investigation found on student worksheet Growing Rectangular Prisms. Provide students time in class to work on it. An important part of the activity is the realization of the increased complexity added by the presence of three dimensions on a rectangular prism. Circulate, giving guidance as necessary, and help students visualize the complexity.
- Students may wish to make use of graphing calculators or spreadsheets for calculations. Please note that records of the calculations are required for the written report. A spreadsheet may be the better choice, if it is possible to arrange printing.

Assessment/Evaluation

- Use the rubric Assessment of Work in a Group.
- Assess the students' written reports based on the criteria given on the student worksheet Growing Rectangular Prisms. Either design a rubric around the criteria or create a marking scheme to support them. Note that students must hand in individual write-ups, each containing some material the same and some that will differ.

Accommodations

- Students might be given the option of an oral explanation of the report. Students should provide written data as the basis upon which to make the oral report.

Student Worksheet – Growing Cubes

1. Consider a cube that is growing in side length, beginning at 1 cm and increasing in 1 cm increments.
   a) Without making any calculations, sketch the graph that would represent the manner in which its surface area would grow. Label your axes. Explain your choice.
   b) Without making any calculations, sketch the graph that would represent the manner in which its volume would grow. Label your axes. Explain your choice.
2. a) By hand or using a graphing calculator or spreadsheet, set up a table of values for surface area versus side length of the cube and construct the graph. Compare the result to your hypothesis.
   b) Describe in words the relationship between the surface area of a cube and its side length.
   c) Write the equation of the relation.
   d) What is the effect on the surface area of a cube of:
      - doubling its side length?
      - tripling its side length?
      - multiplying its side length by *n*?
   e) Calculate the finite differences in the table of values. Examine the sequence of finite differences and describe anything that you discover.
3. a) By hand or using a graphing calculator or spreadsheet, set up a table of values for volume versus side length of the cube and construct the graph. Compare the result to your hypothesis.
   b) Describe in words the relationship between the volume of a cube and its side length.
   c) Write the equation of the relation.
   d) What is the effect on the volume of a cube of:
      - doubling its side length?
      - tripling its side length?
      - multiplying its side length by *n*?
   e) Calculate the finite differences in the table of values. Examine the sequence of finite differences and describe anything that you discover.
4. Compare the two graphs. Which has the greater rate of change? Justify your answer.

Student Worksheet – Growing Rectangular Prisms

Work with a partner in this investigation. Each partner hands in a written report, one partner on the surface area relationship and the other partner on the volume relationship.

1. Design a method to investigate the effect on the surface area and volume of a rectangular prism by varying its dimensions. One person explores surface area, while the other explores volume. Each partner submits a written report for the investigation, including:
    - an explanation of the design of the investigation;
    - data calculated, graphs constructed;
    - conclusions regarding the relationship between the volume/surface area of a rectangular prism and its dimensions;
    - comparison of this investigation with that of the cube done in class:
      How did the investigation designs compare?
      How did the results compare?

The written report will be assessed on the following criteria:

- the design of the experiment reflects consideration of the dimensions of a rectangular prism
- the data collected are sufficient to draw reasonable conclusions
- the graph(s) are constructed in accordance with the expectations of good form
- sound conclusions are given and are justified, based on the data
- complete comparisons are done with the investigations involving a cube
- the report is clear, well organized, and easy-to-follow

Activity 3: Comparing Surface Area and Volume on a Cylinder

Time: 150 minutes

Description

Students work individually to investigate the surface area of a cylinder whose volume is fixed. Students discuss applications in which knowledge of this relationship would be important.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE3b, CGE4a, CGE7j.
Strand(s): Measurement and Geometry
Overall Expectations: MGV.01.
Specific Expectations: MG1.02, MG1.03.

Planning Notes

Bring tin cans of a variety of sizes and shapes to class.

Students work individually at an investigation.

Prior Knowledge Required

- formulas for surface area and volume of a cylinder
- an understanding of the process of exploring the relationship between the surface area or volume of an object and its dimensions

Teaching/Learning Strategies

- Display the tin cans at the front of the class and ask the question: How does a company choose the shape of the can in which to put its product? Student answers may include: the nature of the product, aesthetics, cost of materials, etc.
- Focus on the cost of materials as an important determining factor. Ask what variables are involved in the cost of materials. Answers may include: type of material, amount of material, etc.
  Ask: What determines the type of material?
  What measure gives the amount of material? (surface area)
  What determines the amount of material? (This is the question in which we are interested.)
  The amount of material is related to the volume of the product to be packaged. What might influence this variable? Answers may include: likely or customary package amounts; decisions perhaps based on market research.
  It is likely that the volume will be fixed by some sort of market-related variable. Pose the challenge for students: "With a fixed volume, how can we use the minimum amount of material?" (See student worksheet, Minimizing Material Costs.)
- Students work individually on the assignment, making use of graphing calculators or spreadsheets for calculations. Please note that records of the calculations are required for the written report (i.e., a spreadsheet may be the better choice, if it is possible to arrange printing).

Assessment/Evaluation

- Use the rubric Assessment of Work in a Group. Since students are working individually on this assignment, focus on the characteristic “commitment to task” or some other individual characteristic that you are tracking.
- Create a marking scheme or rubric to support the criteria described for assessment on student worksheet, Minimizing Material Costs.

Accommodations

- Students might be given the option of an oral explanation of the report. Students should provide written data as the basis upon which to make the oral report.

Student Worksheet – Minimizing Material Costs

A manufacturer wishes to package salt in a cylindrical container that has a volume of 1000 cm3. Your assignment is to determine the dimensions of the package that will require the minimum amount of material in its construction.

1. Design an investigation to determine the dimensions of a cylinder having minimum surface area for a volume of 1000 cm3.
2. In a written report, provide:
   - a description of the design of your investigation
   - a record of all calculations made and graphs drawn
   - conclusions reached, justified by reference to the data
3. For the cylinder of minimum surface area that you identified, what is the relationship between the dimensions?
4. Describe other circumstances in which it would be important to minimize surface area for a fixed volume.

Assessment of the Assignment

Questions 1 - 3 will be assessed on the following criteria:

- the design of the investigation reflects consideration of the dimensions of a cylinder
- the data collected is sufficient to draw reasonable conclusions
- the graph(s) are constructed in accordance with the expectations of good form
- sound conclusions are given and are justified, based on the data
- the correct relationship is identified between the dimensions of the cylinder having minimum surface area
- the report is clear, well organized, and easy to follow

Question 4 will be assessed on the following criteria:

- at least two other circumstances are described
- a clear understanding is indicated of how volume and surface area are related within each circumstance

Activity 4: Comparing Perimeter and Area of a Rectangle

Time: 75 minutes

Description

Students work in pairs to explore the relationships between the perimeter and the area of a rectangle. Students identify examples of situations in which these relationships are important.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE3b, CGE4a, CGE7j.
Strand(s): Measurement and Geometry
Overall Expectations: MGV.01.
Specific Expectations: MG1.04.

Prior Knowledge Required

- formulas for the perimeter and area of a rectangle
- experience of fixing one measure of an object while varying the other

Teaching/Learning Strategies

- Monitor student activity while they complete student worksheet, The Relationship between Perimeter and Area.
- Plan about 15 minutes for a whole-class discussion of the results.
- Focus on the nature of each relationship (when perimeter is fixed, a maximum area is reached when the rectangle is a square; when area is fixed, a minimum perimeter is reached, also for a square). Compare the results to those found in the previous activity involving surface area and volume of a cylinder. Does the nature of the results in the two cases have anything in common?
- Ask students for their ideas about where the relationships between perimeter and area of a rectangle would be important.

Assessment/Evaluation

- Have students complete the worksheet Assessment of Work in a Group, rating their partner from the previous activities.
- Use the rubric Assessment of Work in a Group. Since students are working individually on this assignment, focus on the characteristic “commitment to task” or some other characteristic that you are tracking.

Student Worksheet – The Relationship between Perimeter and Area

Consider the relationship between the perimeter and area of a rectangle.

Suppose perimeter was fixed, how would area vary?
Suppose area was fixed, how would perimeter vary?

1. Working with your partner, design experiments to explore these two questions. Carefully record all calculations made and graphs drawn. Give a precise description of the nature of the relationship that you identify.

2. Describe circumstances in which it might be important to know the nature of these relationships.

Unit 3C: Exploring Geometric Properties of Plane Figures

Time: 10 hours

Unit Description

Students review and apply the angle properties of triangles, quadrilaterals, and parallel lines. They investigate the properties of the medians, angle bisectors, and altitudes in various types of triangles. Students explore the properties of the sides and diagonals of various polynomials, while posing and testing questions about the relationships. Students confirm or deny statements about the relationships between geometric properties.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE 3c, CGE 5e.
Overall Expectations: MGV.03.
Specific Expectations: MG 3.01, MG3.02, MG3.03, MG3.04, MG3.05.

Activity Titles (Time and Sequence)

Activity 1	Quadrilaterals Involving Parallel Lines	150 minutes
Activity 2	Investigating and Applying the Properties of Diagonals in Quadrilaterals and Polygons	150 minutes
Activity 3	Investigating Interior and Exterior Angles: Applications to Patterns	75 minutes
Activity 4	Investigating Geometric Relationships – Properties of Angle Bisectors, Medians, Altitudes, and Perpendicular Bisectors	150 minutes
Activity 5	Summative Assessment Activity	75 minutes

Unit Planning Notes

Dynamic geometry software will be used to identify properties of plane figures and explore relatioships among them. In most activities, alternative approaches are provided that do not include dynamic geometry software.

The resource, *Exploring Geometry with the Geometer's Sketchpad*, is referenced throughout. Published by Key Curriculum Press, this resource is licensed to the Ontario Ministry of Education and is available in ".pdf" form on *The Geometer's Sketchpad*™ CD-ROM being sent to each secondary school.

Prior Knowledge Required

- basic vocabulary of geometry (e.g., types of angles, types of triangles, types of quadrilaterals)
- geometric properties of angles in triangles and in parallel lines

Teaching/Learning Strategies

- Students work individually, in pairs, and in groups.
- Teachers facilitate independent student work.
- Teachers lead whole class discussions.

Assessment/Evaluation

- assessment by observation
- written explanations, journal entries
- diagnostic test
- pencil/paper tests and tasks

Resources

Exploring Geometry with the Geometer's Sketchpad. Key Curriculum Press (licensed to the Ontario Ministry of Education)

Activity 1: Quadrilaterals involving parallel lines

Time: 150 minutes

Description

Students re-examine the properties of corresponding, co-interior, alternate and opposite angles as they are related to parallel lines. This provides an opportunity for teachers to introduce the use of dynamic geometry software in a familiar setting. Students extend their knowledge as they examine quadrilaterals formed by two transversals crossing a pair of parallel lines.

Strand(s) and Expectations

Catholic Graduate Expectations: CGE 3c, CGE 5e.
Strand(s): Measurement and Geometry
Overall Expectations: MGV.03.
Specific Expectations: MG3.01.

Planning Notes

STEP	TITLE	TIME
1	Students complete diagnostic test on grade 8 skills.	10 minutes
2	Activity to re-examine angles and parallel lines either with dynamic software or using hands-on investigations.	45 minutes
3	Students complete additional practice in the textbook.	20 minutes
4	Activity to explore properties of the angles and sides related to quadrilaterals formed by two transversals crossing a pair of parallel lines.	30 minutes
5	Class discussion on the properties of sides and angles in special quadrilaterals.	30 minutes
6	Students write a summary in their journals of the properties of sides and angles of special quadrilaterals.	15 minutes

- Gather the following materials: grid paper, protractor, ruler, dynamic geometry software, if available. If teachers choose to use software for all or part of this activity, students must have facility with dynamic geometry software.

Prior Knowledge Required

Geometry and Spatial Sense (Grade 8): identify and investigate the relationships of angles; identify angle properties of parallel and perpendicular lines (interior, corresponding, opposite, alternate); describe the relationship between pairs of angles within parallel lines and transversals.

Teaching/Learning Strategies

Diagnostic Assessment: Teachers prepare a brief diagnostic assessment based on the Grade 8 expectations and based on the results to make adjustments to this activity.

The teacher facilitates a class discussion to generate examples of parallel lines within the classroom (e.g. edges of window frames, walls,) and in building and decorating (e.g. bricks, rooflines, wallpaper borders). The teacher poses several questions such as:

- How can you tell if lines in a room are parallel? (e.g., the top and bottom edges of the blackboard) Most students will suggest measuring to see if the distance is constant.
- When would this method be difficult? (e.g., lines that are very far apart, lines that are slanted.)
- How can we be sure that the edges of the ceiling and floor are parallel if we don't measure the distance between them?
- Consider a jungle gym. How can the installers be sure that two climbing bars are parallel if they meet the same slanted pole?

Student Activity

A. If *The Geometer's Sketchpad*™ is available:

1. Students examine the angles formed by a transversal by working through Properties of Parallel Lines, p. 17 in *Exploring Geometry with The Geometer's Sketchpad*.
2. Students examine the properties of quadrilaterals formed by two transversals:
    - construct a pair of parallel lines as in the previous work. Then they construct a transversal to intersect these lines at A and B respectively and a second transversal to intersect the lines at D and C respectively
    - measure the angles at the four vertices of the quadrilateral ABCD.
    - try to drag a vertex until ABCD is:
        - a parallelogram
        - a trapezoid
        - a rectangle
    - share their reasons for deciding when ABCD is a particular shape and refer to angle measurements to justify their reasoning.
    - name any other quadrilateral they can create (some may offer square or rhombus) and describe how they could be sure they had constructed one of these (they would need to measure lengths of sides).
    - name any quadrilateral they cannot create by dragging ABCD (all are possible except the general quadrilateral).

B. If dynamic software is not available:

1. Students may do the following:
    - draw a slanted line across a ruled sheet of paper or fold the paper on an angle.
    - measure all angles at the intersection of this transversal and any two of the ruled lines.
    - share their results and describe orally any patterns they notice.
    - draw a slanted line across two lines that are almost parallel. They check whether there are any patterns amongst the angles.
    - share their results and formulate a test for parallel lines.
    - use their test by checking whether lines in patterns around them are parallel.
    - write a summary statement about the angles formed by a transversal intersecting parallel lines and draw their own diagram using different colours or symbols to illustrate alternate, corresponding and co-interior angles.

2. Students then:
    - draw a pair of parallel lines on lined paper. They draw two transversals (non-parallel) and label the intersections with the pair of parallel lines so that the quadrilateral formed is ABCD.
    - measure sides of ABCD and the angles generated at the vertices.
    - repeat the above procedures to draw a parallelogram, rhombus, square, and rectangle by choosing appropriate transversals.

Concluding the Activity

A. The teacher leads a class discussion on the properties of the angles and sides in these special quadrilaterals.

B. In their journals, students write a summary of the angle and side properties of special quadrilaterals.

Possible Extensions

Discuss what is meant by "parallel".

- Look at art and examine how parallel lines are displayed to show perspective.
- How can we extend the word parallel to planes? (e.g. what does it mean to say "the desk top is parallel to the floor.")

Assessment/Evaluation

Formative

- Diagnostic Assessment is used at the beginning of the activity.
- Assess journal entries, using the Communication section of Appendix B: Written Report (from Units 1 and 2).
- Teacher could follow-up with short knowledge/understanding formative assessment at the start of the next class to check that students are able to calculate angle measures in questions involving using parallel lines.

Accommodations

- Students could be paired with another student for the activity.

Resources

Exploring Geometry with The Geometer's Sketchpad. Key Curriculum Press (licensed to the Ontario Ministry of Education)

Activity 2: Investigating and Applying the properties of diagonals in quadrilaterals and polygons

Total: 150 minutes

Description

In this activity, students use dynamic geometry software or geometric instruments to examine the properties of diagonals in quadrilaterals and other polygons. They use their skills to design a colourful kite (or ornament) as a community project.

Strand(s) and Expectations

Catholic Graduate Expectations: CGE2b, CGE3c, CGE5a, CGE7j.
Strand(s): Measurement and Geometry, Relationships
Overall Expectations: MGV.03.
Specific Expectations: MG3.03, MG3.04, MG3.05.

Planning Notes

- Required Materials: Student Worksheets A and B, photocopies of assorted quadrilaterals as described in Part A, protractor, ruler, dynamic geometry software, if available.
- In Student Worksheets A and B, students confirm or deny statements based on geometric properties. The teacher introduces the idea of illustrating with an example or with a counter-example to support their reasoning. This could be introduced by initially providing two or three statements to the class and asking students to label them as true or false. If the statement is true, students illustrate it with an example; if the statement is false, students provide a counter-example.
- If teachers choose to use software for all or part of this activity, students must have facility with dynamic geometry software.

Prior Knowledge Required

- Geometry and Spatial Sense (Grade 8): identify, describe, compare and classify geometric figures; construct and solve problems involving lines and angles;
- Measurement and Geometry: illustrate and explain the properties of …angles related to parallel lines.

Teaching/Learning Strategies

Part A

Prior to starting each of these investigations, the teacher may need to do some skill review (either using dynamic geometry software or geometric instruments).

Step 1

If dynamic geometry software is available:

- Students investigate properties of the diagonals of quadrilaterals using the following: *Exploring Geometry with The Geometer's Sketchpad*, pp. 89-92, 95-96.

If dynamic geometry software is not available:

- The students work in groups of four.
- The teacher prepares photocopies of assorted large scale quadrilaterals. Each group requires one each of: a general quadrilateral, parallelogram, rhombus, square, rectangle, and trapezoid.
  Note: To facilitate discussion there should be several different versions of each figure.
- Students draw diagonals for each of the above shapes and measure and discuss the following:
  - the lengths of the diagonals
  - the measurements of the angles formed at the vertices of the quadrilateral
  - the measurements of the angles formed at the intersection of the diagonals
  - whether the diagonals bisect one another

Step 2

- All students individually complete Student Worksheet A.
- The teacher leads a discussion of the answers to the worksheet questions.

Step 3

If dynamic software is available:

- Students proceed with the activities on pp. 102 and 103, *Exploring Geometry with The Geometer's Sketchpad*.

If software is not available:

- Students draw six different quadrilaterals, join the midpoints of their sides to form another quadrilateral.
- Students examine the new figures and make a conjecture about the shapes.
- Students then test their conjecture and write a report in their journals to summarize their findings.

Part B

Students work in pairs to complete the following two activities.

1. Pentagon/Star Activity

- The teacher provides each pair of students with a photocopy of a large scale regular pentagon.
- Students draw all diagonals and consider the following question:
  What figure is formed by the line segments joining intersections of the diagonals?
- Students then highlight the star shape formed at the vertices of the original pentagon, measure the angles at the points of the star and describe a relationship among the angles.
- Students provide a written summary of their conclusions in their journals.

2. Numbers of Diagonals of a Polygon

- Students draw a quadrilateral, a pentagon, a hexagon, a heptagon and an octagon.
- Students draw all possible diagonals in each case.
- Students complete Student Worksheet B.

Part C

A class of students volunteers to make colourful kites (or polygonal Christmas tree ornaments) to present to a local day care, seniors centre or preschool. If the item is a kite, it should be designed to include all diagonals as braces and be finished in an artistic fashion.

Note: A kite is a formal geometric term describing a quadrilateral with two distinct pairs of consecutive equal sides. It is expected that students will think of a "kite" simply as a lightweight object that flies while pulled by a string. Teachers should clarify this concept at the start of this activity.

This portion of the project provides an excellent opportunity for students to work collaboratively in groups of 2-3 to create a design. The teacher may choose to hold a friendly competition for the best design and display all of the designs around the classroom or in a prominent place in the school. Teachers should review criteria of the assessment rubric to ensure that students understand expectations of a level 3 or 4 project.

This project is started in class but should be completed on the students' own time.

Project Guidelines

The Kite (or ornament) design must be polygonal in shape and include diagonals as braces.
The finished product must include:

- a scale diagram of the kite design on grid paper
- a list of material used to complete the design
- 1-2 paragraphs highlighting aspects of the design

Possible Extension: Students investigate the star formed by the diagonals in a non-regular pentagon and find an explanation for the sum of the angles at the points.

Assessment/Evaluation

- Use the observation rubric for group work, Appendix C (from Units 1 and 2).
- Assess paper and pencil tasks for accuracy and completeness.
- Assess journal entries, using the Communication section of Appendix B: Written Report (from Units 1 and 2).
- Use the Summative Assessment Rubric for the Design Activity for application and communication skills during the kite design project.

Accommodations

- Students who will have difficulty using the toolbar in geometry software could be paired with another student for the activity.
- For enrichment, students could research and then create a more sophisticated, complex design such as the box kite. Internet web sites are an excellent resource.
- Students can work in pairs to complete this activity.

Resources

Exploring Geometry with The Geometer's Sketchpad. Key Curriculum Press (licensed to the Ontario Ministry of Education)

Internet web sites on kites such as the hila science camp site:
http://fox.nstn.ca/~hila/projects/Kite/kite.html

Student Worksheet A

1. Write a summary statement about the diagonals in each of the following:

Quadrilateral	
Square	
Rectangle	
Parallelogram	
Rhombus	
Trapezoid	

2. Using your geometric knowledge, confirm or deny the statement below. Use counter-examples and mathematical language to support or challenge the hypothesis.
   If the diagonals of a quadrilateral bisect each other at right angles,
   then the quadrilateral must be a square.

3. Hypothesize a relationship that you believe to be false about diagonals in quadrilaterals. Use counter-examples and mathematical language to support or challenge the hypothesis.

Student Worksheet B

1. Complete the following chart:

Shape	Number of Vertices	Number of Sides	Number of Diagonals

2. State a formula for the number of diagonals at each vertex of a regular polygon with *n* sides (this will be an algebraic expression).

3. State a formula for the total number of diagonals in a regular polygon with *n* sides will be an algebraic expression).

Summative Assessment Rubric for the Design Activity

SCALE CATEGORY	LEVEL ONE (50 - 59 %)	LEVEL TWO (60 - 69 %)	LEVEL THREE (70 - 79 %)	LEVEL FOUR (80 - 100 %)
• Application • Chooses appropriate materials and tools for task and recognizes limitations	- chooses appropriate materials with considerable teacher assistance	- able to choose appropriate materials with some teacher assistance	- chooses appropriate materials independently and with confidence	- demonstrates innovation, creativity and confidence in choice of materials
• Applies concepts of area, diagonals in a quadrilateral or regular polygon to task	- applies concepts only with considerable assistance - makes several significant errors in calculations	- applies concepts with some assistance· - makes some significant errors in calculations	- applies concepts accurately and consistently, without assistance	- applies concepts accurately and with ease; assists others with applying concepts
• Applies logical, efficient, co-operative procedure to the task and recognizes limitations of design process	- infrequently applies efficient procedure to task and requires considerable assistance in following sequence· - frequent lack of co-operation - rarely recognizes limits in process without considerable assistance	- applies efficient procedure to task some of the time, and requires limited assistance in following sequence· - usually co-operative, may need some prompting· - able to recognize some limits in process with assistance/prompting	- applies efficient procedure to task most of the time, with minimal clarifying questions· - generally co-operative approach to task· - able to recognize and communicate limits in process (e.g., limits of given material)	- routinely applies efficient and innovative procedure to task· - consistently co-operative and encourages co-operation and creativity of others· - recognizes limits in process and suggests alternative solutions
Communication Communicates their reasoning, in writing and with a diagram for, their kite design	- incomplete diagram with limited detail (e.g., few or no measurements)· - explanation unclear and incomplete	- complete diagram with some detail· - explanation attempts to justify some decisions but does not address at least one key area in design (e.g., diagonal braces)	- complete diagram with all important details included· - explanation is clear, complete, logical and provides considerable justification for elements of the design	- complete diagram with sophisticated, creative detail (e.g., consideration of aerodynamics in design)· - explanation is complete, clear and easy to read with elaborate justification for elements of design

Activity 3: Investigating Interior and Exterior angles: Applications to Patterns

Time: 75 minutes

Description

Students explore and analyse the properties of interior and exterior angles of triangles and quadrilaterals, then use these to examine possibilities for design. They find the sum of the interior angles of a triangle, the sum of the angles of a quadrilateral and the sum of exterior angles for triangles and quadrilaterals. Dynamic geometry software may be used.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: 2c, 3c, 4b.
Strand(s): Measurement & Geometry
Overall Expectations: MGV. 03.
Specific Expectations: MG3.01.

Planning Notes

This activity explores the properties of interior and exterior angles.

Step	Activity	Time (approx.)
1	Examine patterns in fabric design and flooring	5 minutes
2	Investigation of the interior and exterior angles of triangles and quadrilaterals	45 minutes
3	Apply the knowledge obtained to re-examine the patterns	10 minutes
4	Do activities from the textbook to reinforce concepts	15 minutes

- Have students work in pairs.
- Provide each group with materials – scissors, protractors.

Prior Knowledge Required

- Geometry and Spatial Sense (Grade 8): identify the angle properties of intersecting, parallel, and perpendicular lines; understand the sum of the interior angles of a triangle.

Teaching/Learning Strategies

Student Activity

Teacher displays patterns in fabric and flooring materials that use triangles and quadrilaterals to tessellate (tile the floor, leaving no gaps). Sources may include flooring brochures, pictures in catalogues, actual fabric. Teacher leads a discussion of the various shapes, (e.g., some slate tiles may be general quadrilaterals, some ceramic tiles may be diamond shaped or triangular). Teacher poses the question: Is there a tile in the shape of a triangle or quadrilateral that would not "work" to create a floor or fabric design without gaps?

Students explore the sum of the interior angles in a triangle and in a quadrilateral through a hands-on approach.

If The Geometer's Sketchpad™ is available:

- Students may investigate the sum through the worksheet Triangle Sum, p. 65 in *Exploring Geometry with The Geometer's Sketchpad*. Then they extend the method used to explore the sum of the interior angles of a quadrilateral.

If dynamic geometry software is not available:

- Each student cuts out a large triangle, measures the angles, and writes these angle measurements on the triangle.
- Students rip off the vertices of the triangles and line them up. The teacher leads a discussion of why this shows that the sum of the angles is 180°.
- Repeat this activity for a general quadrilateral relating the sum of the angles to the fact that a quadrilateral can be cut into two triangles.

To investigate the exterior angles:

- complete the activities Exterior Angles in a Triangle, p. 66 in *Exploring Geometry with The Geometer's Sketchpad*. The activity Exterior Angles in a Polygon, p. 109 can be done to investigate the sum of the exterior angles of quadrilaterals; however, it leads students to explore the results for polygons in general. An alternative is to have students extend the method on page 66 to explore the sum of the exterior angles in quadrilaterals.

If dynamic software is not available,

- Students construct a triangle with pencils so that the sides extend past the vertices and discuss the relationship between each interior angle and its associated exterior angle.
- Students make a very small triangle with the pencils. The teacher asks them to predict the sum of the exterior angles. Students check their prediction by drawing a triangle, measuring the exterior angles and finding the sum.
- The teacher asks students to predict the sum of the exterior angles in a quadrilateral. Each pair of students constructs a quadrilateral and checks the prediction.
- The teacher creates a chart on the board and students enter the results for their quadrilateral. The teacher leads a discussion of how the pencil model followed above helps explain why the sum of the exterior angles is 360°.

Re-examine the flooring and fabric patterns as a whole class. Students write an answer to the question posed in part 1, justifying their answers by referring to the results obtained.

Students practise further by using the material found in the textbook.

Teacher -Facilitation

The teacher asks probing questions of each pair while circulating through the class in order to ensure that they have a grasp of the concepts and verifies that each student has contributed to the best of his/her ability.

Follow-Up

- The teacher checks that the students have a solid grasp of the new concepts as they work on exercises in the student textbook.

Extensions

- Students hypothesize and then investigate the sum of the interior angles in other polygons by dividing them into triangles.
- Students can investigate the size of the angle in each of the regular polygons: (e.g., regular pentagon, regular hexagon.
- Students could extend their knowledge of interior and exterior angles to higher order polygons through further investigation with or without dynamic geometry software.

Assessment/Evaluation

- Use the observational rubric for group work, Appendix C, from Units 1 and 2.
- Journal entry – use criteria for Communication in Appendix B: Written Report
- Paper and pencil tasks.

Accommodations

Students may work in pairs.

Resources

Dynamic geometry software

Exploring Geometry with The Geometer's Sketchpad. Key Curriculum Press (licensed to the Ministry of Education)

Mathematics Teacher, Problem Solvers, October 1994, Volume 87, Number 7, pp. 490-495.

Activity 4: Investigating Geometric Relationships – Properties of Angle Bisectors, Medians, Altitudes, and Perpendicular Bisectors

Time: 150 minutes

Description

Students investigate the medians, altitudes, angle bisectors and the perpendicular bisectors of sides of triangles. They construct four cardboard triangles and find one of the special intersection points for each. Then, they consider several problem scenarios and use their constructions to find the solutions.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE 4b, 2c.
Strand(s): Measurement and Geometry
Overall expectations: MGV.03.
Specific Expectations: MG3.02, MG3.04.

Planning Notes

This activity is divided into four parts:

Step	Activity	Time (approx.)
1	Construct and cut out four cardboard scalene triangles with special intersection points constructed	60 minutes
2	Work on additional questions from the textbook	15 minutes
3	Using the constructions, solve two problem scenarios and communicate the solutions in writing	50 minutes
4	Work on additional questions from the textbook	25 minutes

An alternative approach to the explorations with the cardboard triangles is to use the blackline masters, pp. 71 to 77, *Exploring Geometry with The Geometer's Sketchpad*.

Prior Learning Required

Geometry and Spatial Sense (Grade 8): construct and solve problems involving lines and angles; investigate geometric mathematical theories to solve problems; use mathematical language effectively to describe geometric concepts, reasoning and investigations.

Teaching/Learning Strategies

Students construct four triangles using regular paper. The teacher emphasizes that the students draw large, scalene triangles. After constructing one of the special sets of lines on each triangle, students glue their triangles to cardboard and cut them out. It is important that the teacher demonstrates appropriate construction techniques using ruler and compasses, paper folding or Miras®. The teacher circulates around the classroom, assisting students in completing their activities.

Students (in pairs) investigate the solutions to the following scenarios.

A. Pose the challenge: If equal weights are attached to each vertex of a triangle, find the point at which a support must be placed so that the triangle will balance.

- The students attach 1 large paper clip (or tape a penny) to each vertex of each constructed triangle. They try to balance each triangle on the tip of a sharp pencil.
- Pairs share their results and identify the special point.
- The teacher leads a brief discussion of balance:
  - Demonstrate that a rod with equal weights on each end will balance at the midpoint.
  - If the weight is doubled at one end the balance point divides the rod length in the ratio 2:1.
- Pairs of students use their constructed triangles and the information about balance to explain and justify the location of the centroid and write this explanation in their journals.

B. Pose the challenge: The provincial government is planning to locate a communications tower to serve three communities in the north. Three straight roads link these communities in a triangular shape. If the tower is to be placed at the same perpendicular distance from each road, find its location.

- Students use the constructed triangles to model this scenario. The sides represent the three roads.
- Students examine the four intersection points to discover which has the required property.
- Students explain in writing their reasons for making their choice.

Extension

- Consider the question in B with the following change – The tower is to be located equidistant from each community. Students find the balance point of a pentagon. They can explore this using the weight ideas above.

Assessment/Evaluation

- Use the observational rubric for group work Appendix C (from Units 1 and 2).
- Journal entry – use criteria for Communication in Appendix B: Written Report (from Units 1 and 2)
- Paper and pencil tasks

Accommodations

- All written instructions can be given to students orally.
- Students work in pairs

Resources

Exploring Geometry with The Geometer's Sketchpad. Key Curriculum Press, licensed to the Ontario Ministry of Education.

Resources for use of MIRAs® (available from suppliers of mathematical resources)

Activity 5: Summative Assessment Activity

Time: 75 minutes

Description

Sample Questions for Paper and Pencil Test

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE2b, CGE2c, CGE3c.
Overall Expectations: MGV.03.
Specific Expectations: MG3.01, MG3.02, MG3.03, MG3.04, MG3.05.

Planning Notes

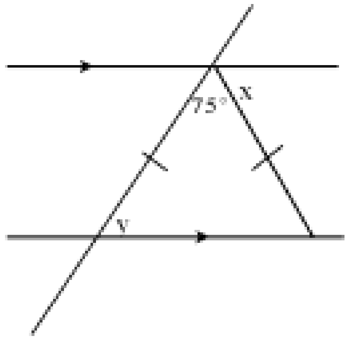
These sample questions are designed to allow students to demonstrate the full range of their learning in this unit. It is important to include a representative collection of these questions in addition to questions that demonstrate knowledge and understanding, in any summative test prepared for this unit.

Sample Questions for Pencil and Paper Test

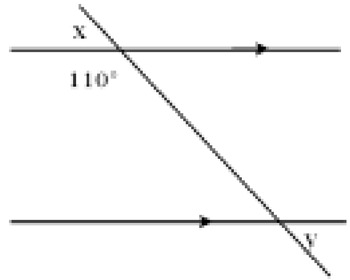
1. A sports engineering firm has designed a very innovative and challenging triangular trampoline with sides measuring 3m each. They wish to mark the balance point of the trampoline with their logo. Where should they place the logo? Justify the method you chose to find this location.
2. a) In which type of triangle (scalene, isosceles, etc.) are the centroid, orthocentre, and incenter the same point? Explain your reasoning.
   b) In which type of triangle do an angle bisector, a median and an altitude coincide? Explain your reasoning.
3. a) Construct a large isosceles triangle. Construct its medians, altitudes and angle bisectors.
   b) What special property does the centroid, orthocentre, and incenter share?
4. What am I?
   a) I have two pairs of equal sides and one of my angles is a right angle. I am not a square.
   b) I have four sides and my angles are not all equal but my diagonals intersect at right angles.
   c) I have 9 diagonals.
   d) When you draw my diagonals you get a smaller copy of me.
5. State whether the following statements are true or false:
   a) A square is a rectangle.
   b) A rectangle is a square.
   c) A pentagon has 8 diagonals.
   d) The exterior angles of a pentagon have a sum of 360°.

6. Calculate the value of x and y in each of the following diagrams.

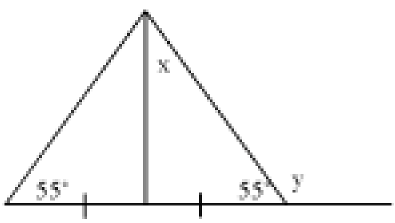
a)



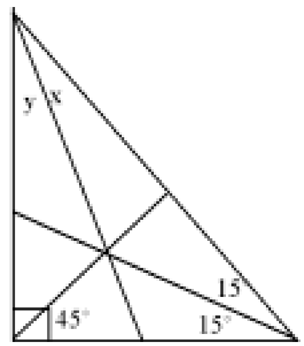
b)



c)



d)



Unit 4: Making Connections

Time: 10 hours

Description

Students engage in activities that reflect the content and procedures of the course, in preparation for final assessment activities which may include a performance assessment and a final exam.

Strand(s) and Expectations

Ontario Catholic School Graduation Expectations: CGE 2b, CGE 5a, CGE 5b.
Overall Expectations: all
Specific Expectations: all

Unit Planning Notes

Ten hours are allotted for preparation and carrying out of the final assessment activities. It is recommended that these activities include both a performance assessment and a final examination.

Four sample activities are included that might be used as part of the preparation for the exam or performance assessment. It is recommended that teachers supplement these activities with material drawn from the student textbook or other sources.

The activities are embedded within the following context:

> *A local organization has donated a piece of land to be used as a community park. The land and a small wading pool have been donated, and the community has come together to do the necessary preparation work. Students from the local schools are very involved.*

The activities are found on the following worksheets.

Student Worksheet – The Paper Chase

You have been put in charge of “Advertising” for the grand opening of the Community Park and have contacted two local newspapers to inquire about their rates for placing an ad. *The Lake Graphit Gazette* charges $ 15 per day for the ad that you want to place, regardless of how many days you wish to place the ad. Another local paper, *The Mount Slope Reporter* charges a flat rate of $ 100 plus $5 per day for the same ad.

1. Write an equation to represent the relationship between total cost (C) and number of days (n) for:
   a) *The Lake Graphit Gazette*
   b) *The Mount Slope Reporter*

2. a) Construct a table of values for each relation. Pay special attention to which variable is the independent variable.
   b) Construct the graph of each relation on the same set of axes.
   c) Which line is steeper? Why?
   d) State the slope of each relation.
   e) Explain the meaning of the slope within this application.

3. a) At what point do the two lines cross?
   b) Interpret the point of intersection in the context of this application.
   c) Under what circumstances would you choose to advertise in each publication? Explain how you reached your conclusions.

4. You discover a third local publication, which would charge $ 137.50 for 5 days and $ 162.50 for 15 days. Assume that there is a linear relation between the cost of the ad and the number of days that it would run.
   a) Determine the cost per day for the ad in this publication.
   b) Determine the flat fee charged.
   c) Write the equation of the linear relation.
   d) Add this line to your graph. Explain which publication is now the better choice.

Student Worksheet – A Pool Is Cool!

In one corner of the park, a section has been fenced off to enclose an in-ground wading pool for younger children. The section is in the shape of a trapezoid, having height 14 m and base lengths 25 m and 18 m. The wading pool is cylindrical in shape, having a diameter of 8 m and a height of 50 cm. The area outside the pool, but inside the fence, is planted in lawn.

A group of students has assumed responsibility for preparing the play area..

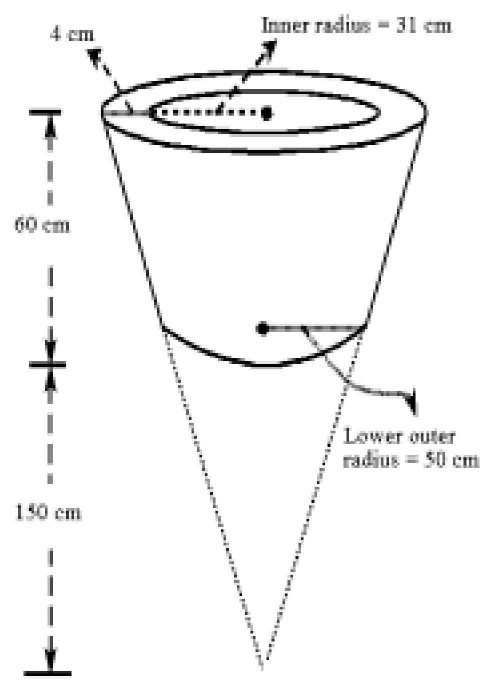
1. Draw a diagram to represent the situation.

2. One part of the preparation is the painting of the fence, which is 1.5 m high and constructed of closely packed boards. A local merchant is donating paint that requires 4 L for every 65 m2 of coverage. To put two coats of paint on both sides of the fence, how many 4L cans are needed?

3. The wading pool is to be filled to a depth of 30 cm. A hose is available that flows water at a rate of 9 L/min. How long will it take to fill the pool to the required height?

4. Before opening the area, the lawn is to be topped with top soil and then re-seeded. If the top soil is to be applied at a constant depth of 2 cm, how many m^3 of top soil are required?

5. There is a planter in the shape of a “frustum” on each side of the entrance to the play area. As shown in the diagram, a frustum is a cone that has been truncated (cut) parallel to its base. The dimensions of the planter are shown in the diagram. If each planter is filled to the brim with potting soil, how much soil is needed?



Student Worksheet: Something for the Gardeners

Another group of students has been assigned the task of designing a garden area for the park. It is to be surrounded by a low hedging plant and enough plants are available to create a perimeter of 60 m.
The group has been asked to identify the shape (e.g., triangle, quadrilateral, pentagon, ...) that would provide the maximum enclosed area for a perimeter of 60 m.

Design an investigation to identify the required shape. Prepare a written report that includes:

- an explanation of the design process
- all diagrams, calculations, tables, graphs used as part of the investigation
- a statement of the final shape identified, along with the dimensions that yield maximum area
- an explanation of how you reached your conclusion and your thoughts on why the identified shape yields the maximum area
- a description of other factors that might influence the choice of shape for the garden

Student Worksheet: Water, Water, Everywhere!

All of the students brought along water because the day was warm and the work quite strenuous. One student had a sealed plastic bottle filled with water. A leak opened in the bottle and the water drained out at a constant rate.

Time *t* (seconds)	**Height (*h*) of water in bottle (cm)**
0	25
10	22
20	19
30	16
40	14
50	12
60	8
70	6
80	4
90	2
100	0

The table above identifies the height (in cm) of water in the bottle at time t seconds after the draining began.

a) Recopy the table and extend to calculate the finite differences.

b) Is this relation linear or non-linear? Explain.

c) Construct a graph for the data.

d) Using the shape of the graph and values of the finite differences, sketch a possible shape for the bottle. Explain your reasoning.