Catholic District School Board Writing Partnership

Course Profile

Foundations of Mathematics

Grade 9
Applied

• *for teachers by teachers*

Acknowledgments

Catholic District School Board Writing Team – Mathematics - Applied

Lead Board
  Ottawa-Carleton Catholic School Board
  Sean Kelly, Manager

Course Developers
  Arlene Corrigan, Renfrew County Catholic District School Board
  Dominique Levac, Catholic District School Board of Eastern Ontario
  Carolyn Boyer, Ottawa Carleton Catholic School Board
  Len St.Clair, Catholic District School Board of Eastern Ontario
  Brian McGudden, Toronto Catholic District School Board
  Margaret Sinclair, Toronto Catholic District School Board
  Paul Costa, Toronto Catholic District School Board
  Lori Goodfriend, Catholic District School Board of Eastern Ontario
  Catherine Rea, Ottawa-Carleton Catholic School Board
  Anne Delahunt, Ottawa-Carleton Catholic School Board

Eastern Ontario Catholic Curriculum Cooperative

Institute for Catholic Education

Preface

Special Note

In Units 1 and 2 additions have been made to the activities to reflect the curriculum expectations for the Number Sense and Algebra Strand.

Please note that Appendix B and C of Units 1 and 2 are referenced in Unit 3.

These insertions to Units 1 and 2 of the Grade 9 Applied Mathematics profile incorporate the following expectations:
Overall Expectations: NAV.01, NAV.02, NAV.03, NAV.04.
Specific Expectations: NA1.02, NA1.03, NA2.05, NA2.06, NA3.01, NA3.02.

Unit 1, Activity 2 “Mathematical Marathon”

Overall Expectations: NAV.02.
Specific Expectations: NA2.05, NA2.06.

Teaching /Learning Strategies

- Students now consolidate and enhance their understanding of the three basic exponent rules by completing assignments from the textbook. Include questions with the exponent rule for the power of a power.
- This would also be a good time to enter and interpret exponential notation on a scientific calculator, since some distances will be quite large. Again, use textbook assignments to involve applications with very small numbers.

Unit 1, Activity 3 “Exploring Motion”

Specific Expectations: NA1.02.

Teaching/Learning Strategies

- Students can design their walk to create a graph that is a) a straight line with a positive slope; b) a straight line with a negative slope; c) several lines with a combination of positive and negative slopes.
- Have students walk at different speeds and in different directions so that they not only investigate positive and negative slopes, but different ratios as well (refer to “Explorations, Modelling Motions: High School Activities with the CBR™”).

Discuss with students the meaning of positive and negative integers in this context.

Unit 2, Activity 1 “Walking the Line”

Specific Expectations: NA1.03.

Teaching/Learning Strategies

On p. Unit 2-4, add to the box in #3:
This may be an opportunity to consolidate students’ skills in performing operations with rational numbers.

Unit 2, Activity 2 “The Help Line”

Overall Expectations: NAV.01, NAV.02, NAV.03, NAV.04.
Specific Expectations: NA1.02, NA2.06, NA3.01, NA3.02.

Part 4: Possible Extension
Now would be a good time for the teacher to diagnose students’ ability to work with integers and provide remediation as necessary. This could then be extended to lessons on manipulating polynomial expressions, supported by textbook resources. When multiplying and dividing monomials, highlight the exponent rules covered in Activity 1. Include the exponent rule for the power of a power.

Coding of Expectations

Remove the coded expectation indicated below, which is not a part of the Applied course:
NA3.06
- rearrange formulas involving variables in the first degree, with and without substitution, as they arise in topics throughout the course (e.g., analytic geometry, measurement)

Unit 3: Measurement and Geometry

Time: 40 hours
Unit Developer(s): Carolyn Boyer, Arlene Corrigan, Paul Costa, Anne Delahunt, Lori Goodfriend, Dominique Levac, Brian McCudden, Catherine Rea, Len St. Clair, Margaret Sinclair
Development Date: July - September 1999

Unit Description

The unit is divided into 3 sub units.

Unit 3A	Solving Problems Involving Measurement	23.75 hours
Unit 3B	Optimization of Measurement	6.75 hours
Unit 3C	Exploring Geometric Properties of Plane Figures	10 hours

In this unit, skills such as mental mathematics, estimation, approximating, and solving problems are consolidated. Students will solve problems involving the perimeter and area of composite plane figures and the surface area and volume of three-dimensional objects; they will determine the optimal values of various measurements and use dynamic geometry software to make generalizations about geometric relationships. Students will extend their skills with manipulating polynomial expressions to solve first-degree equations.

3A: Solving Problems Involving Measurement

Time: 23.75 hours

Unit Description

Students solve problems involving the perimeter and area of composite plane figures and develop formulas for the surface area of prisms and cylinders and for the volume of prisms, cylinders, cones, and spheres. They apply the formulas to solve problems. Within the context of measurement, students solve linear equations, rearrange formulas, and evaluate numerical expressions involving exponents. They consolidate skills of mental mathematics and estimation, demonstrate the effective use of a scientific calculator, and judge the reasonableness of answers to problems.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE 2b, CGE 4f, CGE 5a, CGE 5b, CGE 5g, CGE 7j.
Strand(s): Measurement and Geometry, Number Sense and Algebra
Overall Expectations: MGV.02, NAV.01, NAV.03.
Specific Expectations: MG2.01, MG2.02, MG 2.03, MG2.04, MG2.05, NA1.01, NA1.02, NA1.03, NA1.04, NA1.05, NA1.06, NA2.01, NA2.02, NA3.03, NA3.04, NA3.05.

Activity Titles (Time and Sequence)

Activity 1	Perimeter and Area of Composite Plane Figures	225 minutes
Activity 2	Surface Area and Volume of a Prism: The Prisms Around Us	75 minutes
Activity 3	Surface Area and Volume of a Cylinder	150 minutes
Activity 4	Assessment Activity	75 minutes
Activity 5	Volume of a Cone	75 minutes
Activity 6	Volume of Sphere	75 minutes
Activity 7	Review and Problems Assignment; Test	300 minutes
Activity 8	Solving First Degree Equations: A Balancing Act	450 minutes

Unit Planning Notes

The ability to solve multi-step problems becomes more important as students advance in their study of mathematics. This unit provides an opportunity to focus on that skill, within the context of problems involving surface area and volume.

Many expectations of Number Sense and Algebra are also an essential part of the learning. This unit suggests that teachers take advantage of every opportunity to assist students in consolidating their understanding of the effective use of scientific calculators and the use of estimation in judging reasonableness of answers. The integration of percent, ratio, and rate within the problems to be solved provides opportunities for students to consolidate those important numeric skills. When planning lessons in this unit, it is important to keep in mind the mosaic of expectations to be achieved.

The problem-solving assignment included in Activity 7 takes the form of a story and consists of a set of multi-step problems that may require estimation as part of the solution. Hand out the assignment at the beginning of the unit. Encourage students to complete questions as they acquire the knowledge while working through the unit.

In solving problems involving measurement, it is frequently necessary to rearrange formulas and solve equations; students' skills in solving equations are consolidated and extended in this unit.

Prior Knowledge Required

- perimeter and area of rectangles, triangles, parallelograms, trapezoids, and circles
- experience with solving arithmetic problems, including the importance of communication in problem solving
- understanding of the concepts of percent, ratio, and rate; skills in applying percent, ratio, and rate
- skills and strategies in mental mathematics and estimation
- solution of simple equations (to the level of $ax + b = c$)

Teaching/Learning Strategies

- Teachers use whole group instruction for things, such as: reviews of formulas from Grade 8; concepts and units of perimeter, area, surface area, volume, capacity; demonstrating the development of formulas; teaching of the skills in using scientific calculators, skills in solving equations; fostering development of good habits in problem solving.
- Students work individually and in pairs to solve problems.
- Students work in groups of three or four in developing some formulas.

Assessment/Evaluation

- periodic small quizzes to check understanding and progress (one opportunity is embedded in the activities; other, smaller quizzes may be inserted, as necessary)
- problem-solving assignments, including problem posing
- pencil and paper test

Accommodations

- Allow students to work at some problems in pairs, to assist in developing initial understandings. Bear in mind that the ability to solve problems as an individual is what is required to meet expectations.

Resources

The main resource is the core program mathematics textbook.

Activity 1: Perimeter and Area of Composite Plane Figures

Time: 225 minutes

Description

Within the context of the perimeter and area of composite figures, students consolidate their mental mathematics and estimation skills, and judge the reasonableness of answers. Students consolidate their skills with using a scientific calculator effectively in working with formulas that involve exponents and rational numbers.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE 4f, 5a.
Overall Expectations: NAV.01.
Specific Expectations: NA1.01, NA1.02, NA1.03, NA1.04, NA1.05, NA1.06, NA2.04, NA3.05.

Prior Knowledge Required

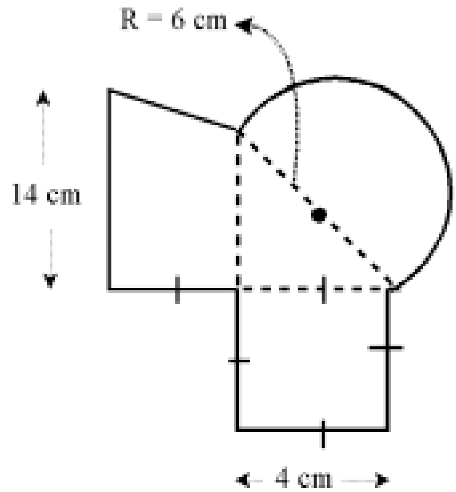
- concept of perimeter; units in which perimeter is commonly measured
- concept of area; units in which area is commonly measured
- calculation of perimeter of a figure bounded by straight lines; formula for the circumference of a circle
- formulas for the area of a rectangle, square, triangle, parallelogram, trapezoid, and circle
- determining the length of the side of a right triangle, using the Pythagorean theorem

Planning Notes

This introductory lesson is an opportunity to review students' prior knowledge, to judge experience with problem solving, and to open the discussion of the expectations involving the effective use of a scientific calculator and judging the reasonableness of answers.

Teaching/Learning Strategies

- In a whole group discussion, review the elements listed under Prior Knowledge Required. Carry out an example involving a composite figure, such as:
  Determine the perimeter and area of the figure below:



- Emphasize the importance of communication in problem solving – writing a solution so that someone else can understand the thinking process involved. Remind students about the syntax involved in substituting into a formula. From the above example:
  Area of half-circle
  $= \frac{1}{2}\pi r^2$
  $= (\frac{1}{2})(\pi)(6)^2$
  $= 56.5 \text{ cm}^2$
- Discuss how to handle fractions in a formula when using a calculator. In the example, most of the calculation can be done mentally left ($\frac{1}{2} \times 36$), with the result multiplied by the π-key on the calculator. With fractions that would lead to repeating decimals, students should take advantage of the full decimal accuracy available on the calculator by punching the fraction in (e.g., if the fraction ⅔ were involved, students would key in 2 ÷ 3).
- Discuss how to handle exponents on a scientific calculator.
- Discuss order of operations on a scientific calculator.
- Discuss the value π:
  - where it comes from (ratio of circumference to diameter for any circle)
  - the use of the π symbol in substitution
  - the approximate nature of π when used in calculation, and the advantage of the π-key on the calculator over the value 3.14.
- Discuss the rounding of answers to measurement problems – when to round and what type of rounding to use.
- Model the use of estimation to judge the reasonableness of the answer produced by a calculator – estimate the answer before doing the calculation. Discuss estimation by rounding to compatible numbers (e.g., π is about 3), by operating with compatible numbers (e.g., in estimating the calculation $(\frac{1}{2})(\pi)(6)^2$, it makes sense to square 6 and multiply by ½, then multiply by 3 as the estimation of π), multiplying and dividing numbers ending in zero. Encourage students to make estimation a regular part of their calculation procedure. Model estimation frequently. Encourage students to share the different methods by which they carry out a particular estimation.

- Discuss *when* and *when not* to use a calculator. Some calculations can be done much more quickly mentally, with the results incorporated into a larger calculation (e.g., as in $(½)(\pi)(6)^2$ in the example above).
- Introduce the assignment found on Student Worksheet: Application of Area and Perimeter within a Scale Diagram (two pages). Review the calculations involved in interpreting a scale diagram. Emphasize that dimensions should be converted *before* any area or perimeter calculations are made, not after. (Why? In area, if the calculation is done using the measurement directly from the scale diagram, then the resulting area must be multiplied by the *square* of the scale factor. Most students have difficulty in understanding this.)
- Pose and solve a problem based on the scale diagram. Model the form of solution expected.
- Monitor student progress closely, while they are working on the activity. Periodically, check their measurements from the scale diagram and their conversions to actual dimensions. Check student solutions to word problems for form and correct calculations. Sit with each student and "pseudo-mark" one solution using the five-criterion marking scheme given on page two of the worksheet.

Assessment/Evaluation

- Assess the word problems in the assignment, using the five-criterion marking scheme given on the worksheet (page 2)

Accommodations

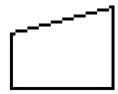
Extension: Have students create a scale diagram of an area of their choice and pose and solve three perimeter/area problems based on it.

Resources

Use the core student textbook for additional practice in calculating perimeter and area, as necessary.

Student Worksheet: Application of Area and Perimeter Within a Scale Diagram
(page 1 of 2)

The diagram on the following page is drawn to a scale of 1 cm represents 1.5 m. The diagram represents the landscaping around a house. In the diagram, anything that is not shaded in is grass, except the area around the pool shaped like the diagram below. This area is cement. Use the diagram and the scale to answer the questions below.



*As you are doing questions 1-7, use the scale to determine the **actual** dimensions of the objects. Use the actual dimensions in your calculations.*

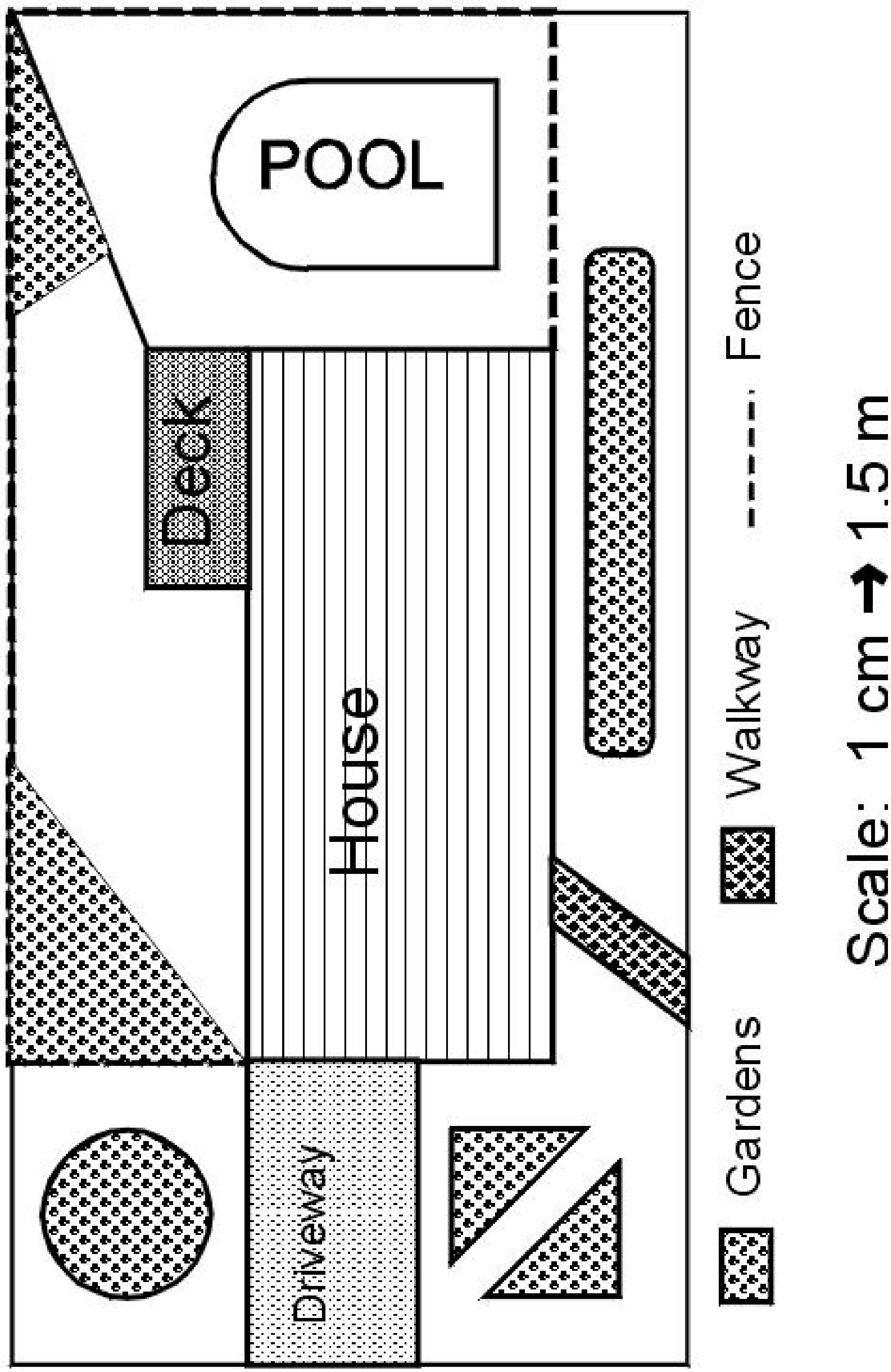
1. What percent of the lot is covered by the house?
2. a) Determine the length of the fence that surrounds the backyard and the pool area.
   b) The owner plans to replace the fence this year. The cost will be $15 per fence post and $3 per m of fencing needed. Assume that there will be a fence post at every corner and that the posts are placed approximately every 2 m. Determine the cost of the fence.
3. a) Determine the total area of all the gardens.
   b) To fertilize the gardens, the owner mixes a powdered fertilizer with water and then sprays it on. The directions require that 25 mL of the fertilizer be mixed with 4 L of water. This will then cover 10 m^2 of garden. How much of the powdered fertilizer will be needed for one application on all the gardens?
4. The owner plans to put a decorative fence around the circular garden at the side of the house. Determine the length of the fence.
5. Determine the area of the cement surrounding the pool.
6. The walkway at the front of the house is made up of interlocking bricks. Each brick covers an area of 300 cm^2 and costs $1.25. Determine the value of the brick on the walkway.
7. The owner estimates that it takes 90 minutes to cut all the grass on the lot. Determine the rate of grass cutting in m^2 per minute.

ASSESSMENT
Each question is assessed on a five-point basis:

- A genuine effort has been made to answer the question.
- The actual dimensions used are accurate.
- The method used to solve the problem is correct.
- The calculations are correct.
- Good form is used in the solution.

[35] TOTAL MARKS

Student Worksheet: Application of Area and Perimeter Within a Scale Diagram
(page 2 of 2)



Activity 2: Surface Area and Volume of a Prism: The Prisms Around Us

Time: 75 minutes

Description

In this activity, students generalize their knowledge of the surface area and volume of rectangular and triangular prisms to include the surface area and volume of any prism. They discuss prisms in the environment as models.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE 3c.
Overall Expectations: MGV .02, NAV.01.
Specific Expectations: MG2.02, MG2.03, MG2.04, 2.05.

Prior Knowledge Required

- characteristics of rectangular and triangular prisms and the similarity between them
- concepts and units of measurement for surface area and volume
- surface area and volume of rectangular and triangular prisms

Planning Notes

Have the following available for demonstration purposes:

- objects in the shape of a rectangular prism, a triangular prism, some other prism
- a series of congruent rectangles constructed from interlocking blocks, to demonstrate the formula Volume = Area of base x height

During this lesson, continue to model and emphasize the following embedded learnings:

- using a scientific calculator effectively, including:
  - knowing *when* and *when not* to use it
  - how to handle fractions and exponents
  - considerations of order of operations
- using rounding appropriately in solutions to problems
- using mental mathematics and estimation to judge the reasonableness of answers produced by a calculator
- judging reasonableness of answers in the context of a problem
- observing correct form in communicating the solution to a problem

Teaching/Learning Strategies

- In a whole group presentation, use a model to elicit from students the characteristics of rectangular and triangular prisms. Discuss what is in common in their characteristics to identify a general definition for a prism (faces are rectangles, top and bottom are congruent, parallel polygons). Describe other possible prisms and where students may have seen them in the environment around them. Have models available for demonstration (cereal boxes, candy bar boxes, any other unusual packaging in the shape of a prism)
- Review the meaning of surface area and the units in which it is typically measured. Ask students to suggest examples of areas for which each unit would be used. Elicit from students a method for calculating the surface area of *any* prism (sum of the areas of all its faces).

- Introduce and explain the term *lateral surface area* (e.g., the sum of the areas of all the side faces of a prism) and ask students to describe situations in which the lateral surface area would be needed instead of the *total* surface area.
  Do a sample problem involving the calculation of the surface area of a prism.
- Review the meaning of the volume and the units in which it is measured. Ask students to suggest examples of objects for which each unit would be used to describe the volume. Elicit from students the formula for calculating the volume of a rectangular prism (V=lwh) and a triangular prism (V = Area of base × height) and an explanation of their origin. Be prepared to model using interlocking blocks, if necessary. (Have several rectangles built, each having the same area. Stack them one on top of another. Since the layers are identical, the volume is the Area of the base × Height.)
- Discuss the relationship between capacity and volume and identify units of capacity. Ask students to identify quantities that are measured in units of capacity instead of units of volume.
- Identify the relationship between units of volume and units of capacity, (e.g., 1 mL of water occupies 1 cm^3 of space.) Extrapolate this relationship to determine the number of litres in 1 m^3 of space (1 kL = 1000 L). Ask students to identify something in their surroundings, at school, or at home that would hold 1 kL of water.
- Do a sample problem involving the volume of a prism that is neither rectangular nor triangular. Include a reference to capacity. For example:
  A water trough is in the shape of a trapezoidal prism. Its base has internal side lengths of 85 cm and 60 cm and an internal height of 50 cm. The total internal length of the trough is 1.2 m.
  a) What is the capacity of the trough?
  b) If the trough is filled to 45% of its capacity, how many litres of water does it contain?
- Select a homework assignment from the student textbook that involves determining the surface area and volume of prisms. Include a problem that integrates ratio, rate, or percent.

Accommodations
- Present multi-step problems in parts, as necessary to build the problem-solving skills of some students.

Activity 3: Surface Area and Volume of a Cylinder
Time: 150 minutes

Description
Students apply their knowledge of the surface area and volume of rectangular prisms to develop formulas for the surface area and volume of cylinders.

Strand(s) and Expectations
Ontario Catholic School Graduate Expectations: CGE 3c.
Overall Expectations: MGV.02.
Specific Expectations: MG2.02, MG2.03, MG2.04, MG2.05.

Prior Knowledge Required
- surface area and volume of rectangular and triangular prisms and the origin of their formulas

Planning Notes

During this lesson, continue to model and emphasize the following embedded learnings:

- using a scientific calculator effectively, including:
  - knowing *when* and *when not* to use it
  - how to handle fractions and exponents
  - considerations of order of operations
- using rounding appropriately in solutions to problems
- using mental mathematics and estimation to judge the reasonableness of answers produced by a calculator
- judging reasonableness of answers in the context of a problem
- observing correct form in communicating the solution to a problem

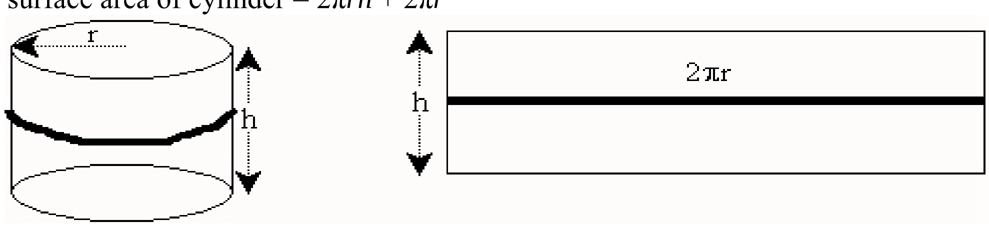
Teaching/Learning Strategies

Volume of a Cylinder

- Compare the structure of a cylinder to that of a rectangular prism, noting similarities (e.g., in both, the top and bottom are congruent, parallel faces; in both the "sides" are perpendicular to the base). Note differences (e.g., the "sides" of a rectangular prism are rectangles; a cylinder has only one continuous side).
- Use the similarity between rectangular prisms and cylinders to suggest a method for determining the volume of a cylinder: Volume = Area of Base × Height. As a model, use a cylindrical package of cookies to illustrate further. Complete the process by substituting the formula for the area of the base, which is a circle.
  So, Volume of a cylinder = $\pi r^2 h$.
- Do sample problems involving calculation of the volume of a cylinder. Include a composite figure and a word problem.
- Select a homework assignment from the student textbook that involves determining the volume of cylinders. Include:
  - a problem that integrates ratio, rate, or percent;
  - problems that involve compositions of cylinders and prisms;
  - a problem in which the volume is known and one dimension must be found.

Surface Area of a Cylinder

- To develop the formula for the surface area of a cylinder, ask each student to roll a piece of paper into a tube. Then identify the shapes that make up the tube. The circle for top and bottom are obvious – but what shape is the side? It came from the piece of paper, so it must be a rectangle. Ask students to determine the height of the rectangle (same as the height of the tube). What about the width of the rectangle? (Ask students to draw a line around the circumference of the tube. Then open the tube. The circumference line has become the width of the rectangle.) Additional models might include the labels on soup or fruit cans, which are easily removed.
  So the rectangle has width h and length $2\pi r$. What is its area?
  Area of rectangle = lw Lateral surface area of cylinder = $2\pi rh$
  and total surface area of cylinder = $2\pi rh + 2\pi r^2$



- Do sample problems involving calculation of the surface area of a cylinder, including a composite object and a word problem.
- Select a homework assignment from the student textbook that involves determining the surface area of cylinders. Include:
  - a problem that integrates ratio, rate, or percent;
  - problems that involve compositions of cylinders and prisms;
  - a problem in which the lateral surface area is known and one dimension must be found

Activity 4: Assessment Activity

Time: 75 minutes

Description

The following assessment is designed in two parts, a pencil and paper assessment and a problem-posing assignment.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE 2b, CGE4f, CGE5G.
Overall Expectations: MGV.02, NAV.01.
Specific Expectations: MG2.01, MG2.02, MG 2.03, MG2.04, MG2.05, NA1.01, NA1.03, NA1.04, NA1.05, NA1.06, NA2.01, NA2.02.

Teaching/Learning Strategies

- Create a pencil and paper quiz on perimeter and area of plane figures; surface area and volume of prisms and cylinders.
- Upon completion of the quiz, instruct students to work on the following assignment, finishing it for homework:
  Pose and solve two problems, one involving surface area and one involving volume/capacity. Involve percent, rate, or ratio in at least one of them.
- Each problem is to be marked out of 7, based on the following set of criteria:
  - The problem requires a multi-step solution.
  - The problem involves an interesting application.
  - The problem involves realistic measurements.
  - The problem is worded clearly.
  - The final answer is correct.
  - The solution is presented in correct form, including use of English, proper formulas, and correct units.
  - Rounding is used correctly in the solution.

Activity 5: Volume of a Cone

Time: 75 minutes

Description

Students develop and apply the formula for the volume of a cone.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE 3c.
Strand(s): Measurement and Geometry
Overall Expectations: MGV.02.
Specific Expectations: MG2.02, MG2.03, MG2.04, MG2.05.

Prior Knowledge Required

- formula for the volume of a cylinder

Planning Notes

Obtain a volume set. This is a commercially available resource that contains a plastic model of a rectangular prism, a pyramid, a cylinder, a cone, and a sphere. The models have compatible dimensions (i.e., the same base and height); they are hollow, so that they can fit within one another. If a volume set is not available, make models from Bristol board or stiff cardstock that holds its shape. You need a cone and a cylinder, having the same base and height.

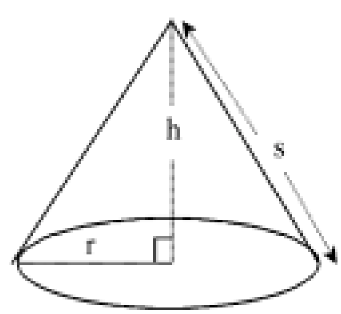
Make available material with which to fill the models (e.g., rice, sand, small plastic pellets), in a quantity sufficient to fill the cylinder.

During this lesson, continue to model and emphasize the following embedded learnings:

- using a scientific calculator effectively, including:
  - knowing *when* and *when not* to use it
  - how to handle fractions and exponents
  - considerations of order of operations
- using rounding appropriately in solutions to problems
- using mental mathematics and estimation to judge the reasonableness of answers produced by a calculator
- judging reasonableness of answers in the context of a problem
- observing correct form in communicating the solution to a problem

Teaching/Learning Strategies

- In a whole group presentation using the models and diagrams, illustrate the features and dimensions of a cone.



- Using the models of the cylinder and cone, demonstrate that these prisms have the same base and height. (With the volume set, the cone fits exactly inside the cylinder.)
  Ask students how they think the volumes would compare (i.e., Would there be a relationship between the volumes?) Students will likely guess that the volume of the cylinder is somewhere between two and four times the volume of the cone.
  Test the relationship by filling the cone with the material chosen and pouring into the prism. Count the number of times that this can be done (3). You might have a student do the demonstration.
  The conclusion reached is that the volume of a cone is one-third the volume of a cylinder *having the same base and height*. The formula for the volume of cone, then, is:
  $V = \frac{1}{3}\pi r^2 h$, where r is the radius of the base and h is the interior height of the cone.
- Do sample problems involving the volumes of cones. Include:
  - composite figures that involve, not only cones, but also rectangular prisms and cylinders;
  - application problems;
- Select a homework assignment from the student textbook that involves volumes of cones.
  - a problem that integrates ratio, rate, or percent;
  - problems that involve compositions of cylinders and prisms;
  - a problem in which the volume is known and one dimension must be found

Resources
Volume set (available from commercial sources of mathematics resources and materials)

Activity 6: Volume of a Sphere
Time: 75 minutes

Description
Students develop and apply the formulas for the volume of a sphere.

Strand(s) and Expectations
Ontario Catholic School Graduate Expectations: CGE 3c.
Strand(s): Measurement and Geometry
Overall Expectations: MGV.02.
Specific Expectations: MG2.02, MG2.03, MG2.04, MG2.05.

Prior Knowledge Required
- formula for the area of a circle
- Archimedes Principle (When an object is submerged in water, it displaces an amount of water equal to its own volume.)

Planning Notes
The development of the formula for volume of a sphere involves a demonstration of water displacement. A submersible sphere, such as a baseball or a billiard ball, is needed, along with a clear, graduated container and sufficient coloured water for submersion of the spherical object.

During this lesson, continue to model and emphasize the following embedded learnings:
- using a scientific calculator effectively, including:
  - knowing *when* and *when not* to use it
  - how to handle fractions and exponents
  - considerations of order of operations

- using rounding appropriately in solutions to problems
- using mental mathematics and estimation to judge the reasonableness of answers produced by a calculator
- judging reasonableness of answers in the context of a problem
- observing correct form in communicating the solution to a problem

Teaching/Learning Strategies

<u>The Volume of a Sphere</u>

- Begin with a discussion of what students think the formula for the volume of a sphere should look like. Since a sphere is circular, π is likely to be involved. Since the only dimension on a sphere is its radius, *r* must be involved also. Since volume is what we want, thinking in three dimensions must be involved. A likely guess at the formula is that it would involve πr^3. The displacement activity that follows is intended to determine by what factor πr^3 would be multiplied.
- The activity can be done as a teacher demo or as a student group activity. The advantage of having students carrying out the activity is that data is gathered for a variety of spherical objects; in the teacher demonstration, a result is drawn from the data for only one object. The teacher demonstration, however, is more time-efficient.
- The activity:
  - Estimate as accurately as possible the radius of a spherical object that will not float, such as a baseball or a billiard ball. Calculate the value πr^3.
  - Fill a graduated beaker with coloured water to a level that would allow the complete submersion of the spherical object. Remind students of Archimedes principle, that is, that the amount of water displaced by the sphere will be equal to the volume of the sphere. Record the starting level of the water. Submerge the ball and record the resulting level. Subtract the two water level figures. The result is the volume of the sphere.
  - Now, compare the volume estimate by displacement to the calculated value of πr^3, by ratio. The result should be around 1.3. The formula, in fact, is

    $$V = \frac{4}{3}\pi r^3.$$
- Do sample problems involving the volume of spheres. Include a composite figure that may combine spheres or hemispheres with other objects. Also include an application problem.
- Select a homework assignment from the student textbook. Include problems involving composite figures and problems involving applications.

Activity 7: Review and Problems Assignment; Test

Time: 300 minutes

Description

Students prepare for a pencil and paper test of applications of the formulas for the perimeter and area of composite plane figures, the surface area of prisms and cylinders, and the volume of prisms, cylinders, cones, and spheres.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE 2b, CGE 4f, CGE 5a, CGE 5b, CGE 5g, CGE 7j.
Strand(s): Measurement and Geometry, Number Sense and Algebra
Overall Expectations: MGV.02, NAV.01.
Specific Expectations: MG2.01, MG2.02, MG 2.03, MG2.04, MG2.05, NA1.01, NA1.02, NA1.03, NA1.04, NA1.05, NA1.06, NA2.01, NA2.02, NA3.04, NA3.05.

Teaching/Learning Strategies

- Create a review lesson as appropriate to the students in the classroom. Assign appropriate questions from the textbook as review.
- A problems assignment is included as a student worksheet under the headings Student Activity – The Question Game and Student Activity – Answering the Questions! Hand this assignment out at the beginning of the unit and encourage students to work at it during the unit. The assignment involves some questions that require students to estimate key information, along the line of Fermi problems.
- Create a pencil and paper test reflecting the work of the unit.

Accommodations

- Assist students as necessary in breaking complex questions into steps.

Student Activity – The Question Game

In the story below, a number of questions are posed by two boys as they shop in a grocery store. Your job is to answer the questions. The numbers throughout the story refer to those on the Answering the Questions! worksheet, where you will find more detailed statements of the problems.

John is a 17-year-old high school student who volunteered to care for Al, the 7-year-old son of a neighbour, every Saturday and to do the family shopping. On one particular Saturday, the two boys set off for the local mall, shopping list in hand.

Al was particularly inquisitive and seemed able to find mathematics at every turn. As he and John entered the grocery store, they walked past a row of shopping carts. John pulled one out of a row. “Hey, John!” said Al, “Have you ever wondered what total volume of groceries is wheeled out of here in shopping carts on a typical Saturday?” John answered: “That thought hasn’t crossed my mind, but it’s certainly an intriguing one.” **[1]**

The boys walked on in silence, and eventually passed the deli counter. Al spied a counter full of cheese cut into triangular wedges and wrapped in foil. “Hey, John! I’ve got another question for you,” he said. “How many rolls of tin foil do you think it would take to wrap all the cheese wedges in that display?” John pondered momentarily, then answered, “Hmmmm, I’ll have to think about that one!” **[2]**

Continuing their shopping, Al and John entered the aisle where the soup was kept – chicken noodle was on the list. “I feel another question coming on,” chirped Al. “Oh, great,” responded John, “Let’s hear it.” Al proceeded, “I was just wondering – if you cut off the labels from all the cans of soup in this counter and laid them out on the floor, would they cover the entire aisle?” **[3(a)]** John responded, “That’s a great question. And I have one too: If you emptied the soup from all those cans, how many bathtubs would it fill?” **[3(b)]** Al laughed happily, “Now you’re getting into the game!”

And so the rest of the shopping trip went – Al and John taking turns posing grocery store problems. In the ice cream aisle, John reached for a package of waffle cones and Al asked, “I wonder if a 4-litre carton of ice cream would fill all these cones?” **[4(a) (b)]**

In the fruit department, John stood looking at a display of oranges. They were all quite round, and about the same size. The oranges were stacked in a pile with a 5x5 square on the bottom row, 4x4 square on top of that, then a 3x3 square, a 2x2 square, and, at the top of the display, there was one orange. Al exclaimed, “I’ve got a great question! Suppose you had to build a cardboard box to hold all those oranges so that they would just fit into the box. What would its dimensions be?” **[5(a)]** John responded, “Great! – and how much empty space would be inside the box?” **[5(b)]**

The boys went through the check-out, paid for the groceries, and started homeward. “I kind of like this question game,” said Al. “Me, too,” answered John. “But you know, we didn’t answer any of those questions, we only asked them. I wonder what we need to know in order to find the answers?”

Student Activity – Answering the Questions!

Communication is important in your solution to these problems. Be sure to use good form and precise language to identify the steps you are taking, and justify all estimates used.

The solution to each question will be marked out of 5 according to the criteria below:

- Correct procedures and formulas are used in calculating measurements.
- Estimates are reasonable and are justified or explained.
- The overall method used for solving the problem would lead to a correct solution.
- The final answer is correct, based on the estimates used.
- Good form is used in communicating the solution, including correct use of language, proper substitution, and correct units.

1. The buggy of a shopping cart is a trapezoidal prism. The trapezoids on the ends have a height of 50 cm and base lengths of 110 cm and 80 cm. The buggies are 60 cm wide. A grocery store has 200 carts and they are used repeatedly on a Saturday. Estimate the total volume of groceries wheeled out of the grocery store on a typical Saturday.
2. Each wedge of cheese in a display is a triangular prism, having approximately the following dimensions: height of wedge is 3 cm; bottom and top of wedge are triangles having base 6 cm and height 5 cm. The pile of wedges on the counter contains about 30 wedges. How many rolls of aluminum foil would be needed to wrap all the wedges?
3. A soup counter has two shelves, each measuring 1.5 m long. Each shelf contains three rows of cans stacked one on top of another. The cans, which are closely packed on the shelves, are all a standard soup can size.
   a) If the labels were cut off of all the cans and laid on the floor, would they cover the aisle?
   b) If you emptied the soup from all those cans, how many bathtubs would it fill?
4. A waffle cone measures about 15 cm high with a diameter of 6 cm.
   a) What percentage of a 4 L ice cream carton would be needed to fill all the cones, each cone just to the brim?
   b) Would 4 litres of ice cream be enough if each cone was filled and then a hemispherical scoop set right on top?
5. A display of oranges was constructed so that they were stacked in a pile with a 5x5 square on the bottom row, 4x4 square on top of that, then a 3x3 square, a 2x2 square, and, at the top of the display, there was one orange. The oranges were all very round, and each had an approximate diameter of 6.5 cm.
   a) What would be the dimensions of one cardboard box (rectangular prism) into which the oranges would just fit?
   b) How much unused space would be left inside the cardboard box that you designed?

Activity 8: Solving First-Degree Equations: A Balancing Act

Time: 450 minutes

Description

Students use the balance method to solve equations of the first degree.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE 2b.
Strand(s): Number Sense and Algebra
Overall Expectations: NAV.03.
Specific Expectations: NA3.03, NA3.05.

Prior Knowledge Required

- solving simple equations, up to the level $ax + b = c$
  e.g., equations of the form:

  $x + 4 = 10$ $\quad$ $-5 - a = 11$ $\quad$ $-6y = -18$ $\quad$ $\frac{k}{5} = -4$ $\quad$ $3 - 4x = 11$

- rearranging formulas in the context of measurement problems.

Planning Notes

Students have solved simple equations in Units 1 and 2 of this course and in solving for one variable in a measurement formula. If the balance method has already been demonstrated, skip the first step described below. If not, borrow a two-pan balance from the Science department.

Teaching/Learning Strategies

- Demonstrate the use of a two-pan balance – to weigh an item, put it on one side, then add weights to the other side until the pans come in balance.
  The equals sign (=) in an equation is like the balance: to keep the equation in balance, whatever you do to one side, you must do to the other.

  Two Basic Rules for Solving Equations

  - To keep an equation in balance (=), if you do something to one side, you must do the same to the other.
  - To remove a term from an equation, perform the *opposite* or *inverse* operation:

OPERATION	INVERSE
+	-
-	+
x	÷
÷	x

- Use the balance method to model the solutions to some simple equations, such as:

  a) $-3x = 21$ $\quad$ b) $\frac{k}{-3} = 2$ $\quad$ c) $12 = k - 5$ $\quad$ d) $4a + 1 = 9$

  Note that in solving equations in which fractions are involved, it is important to maintain the balance method rather than "cross-multiplying".
- Discuss with students which steps are required and which may be omitted. Provide opportunities for practice as appropriate to the needs of students.

- During the first four hours allotted to this activity, extend the students' experience in using the balance method to include equations at each stage shown below:
  - $5 + 9x = 29 - 3x$
  - $5x - 3 + 6x - 7x = 6 + 8 + 9x - 2$
  - $3 + 5(x - 1) = 2(x - 6) + 1$

  At each stage, discuss which steps may be done mentally and which should be written down. Encourage students to leave steps out as they are ready. Some students may find it necessary to include all steps.
- Provide ample opportunities for practice at each stage.

Assessment/Evaluation

- Hold periodic small quizzes as necessary during the teaching.
- During the last couple of hours of allotted time, conduct a review and pencil and paper test.

Unit 3B: Optimization of Measurement

Time: 6.25 hours

Unit Description

Students design and construct boxboard containers to serve as gift boxes for toys to be donated as Christmas presents. The containers will be in the shapes of rectangular prisms and cylinders. Certain volumes are designated for the containers and students build a variety of samples to provide that volume. Students will seek to identify the dimensions that require the minimum amount of material for each given volume and each shape by constructing and examining tables and graphs. Students discuss other applications in which it is important to know the minimum surface area for a given volume.
Students explore the relationships between the perimeter and the area of a rectangle and identify examples of situations in which these relationships are important.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE3b, CGE4a, CGE7j.
Strand(s): Measurement and Geometry
Overall Expectations: MGV.01.
Specific Expectations: MG1.01, MG1.02, MG1.03, MG1.04.

Activity Titles (Time and Sequence)

Activity 1	Building Gift Boxes to Explore Minimum Surface Area	225 minutes
Activity 2	Exploring the Relationship Between the Perimeter and the Area of a Rectangle	150 minutes

Unit Planning Notes

The first activity in the unit gives students an opportunity to visualize the relationships between surface area and volume of an object, and between the perimeter and area of a rectangle. Students explore the physical and numerical relationship between volume and surface area by constructing rectangular prisms and cylinders, by calculation, and by graphing surface area versus height for a given volume. This unit provides an opportunity for students to revisit the expectations relating to graphing, finite differences, and rates of change within the context of optimization of measurement.

Students work co-operatively, either in pairs or larger groups, throughout the unit. An assessment rubric is provided that tracks observation of student characteristics during group work. To make the tracking possible, it is important to consider the structure of the groups at the beginning of the unit, and keep them constant throughout.

Provide a variety of materials including paper, scissors, tape, glue, rulers, protractors, and compass sets.

Prior Knowledge Required

- formulas for perimeter and area of a rectangle
- construction of prisms and cylinders
- formulas for the surface area and volume of prisms and cylinders

Teaching/Learning Strategies

- whole group presentations and class discussions
- students working in pairs and groups
- students carrying out investigations

Assessment/Evaluation

- Assess students by observation throughout the unit, using the Student/Teacher Worksheet Assessment of Work in a Group. Space is left on the chart for the addition of other characteristics.
- Assess written reports of the volume/surface area investigations using the criteria provided on the worksheets
- Observation and rating by the teacher
  During the time in which students are working in class on this unit the teacher will observe and rate students on some or all of the characteristics.
- Rating by other students at the end of the activity
  Each student chooses two characteristics on which he/she wishes to be rated by the other people in the group.

Student/teacher Worksheet: Assessment of Work in a Group

CHARACTERISTIC	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
The student:				
LISTENING	- is easily distracted	- listens intermittently to others	- listens attentively to others	- listens actively and focuses full attention on the speaker
RECEIVING AND USING FEEDBACK	- makes limited use of the suggestions	- accepts feedback from others	- uses feedback as a basis for improvement	- builds new ideas from the feedback of others
PROVIDING FEEDBACK TO OTHERS	- provides limited feedback to others	- provides relevant but sometimes fragmented feedback to others	- provides constructive, relevant feedback to others	- provides detailed feedback and creative strategies for improvement
COMMITMENT TO TASK	- pays limited attention to the task	- has occasional lapses in attention to task	- remains on task throughout the activity	- remains on task throughout the activity and effectively encourages others to do so

NAME OF STUDENT BEING RATED: ____

RATING DONE BY: ____

Activity 1: Building Gift Boxes to Explore Minimum Surface Area

Time: 225 minutes

Description

Students design and construct boxboard containers to serve as gift boxes for toys to be donated as Christmas presents. The containers will be in the shapes of rectangular prisms and cylinders. Certain volumes will be designated for the containers and students build a variety of samples to provide that volume. Students seek to identify the dimensions that require the minimum amount of material for each given volume and each shape, by constructing and examining tables and graphs. Students discuss other applications in which it is important to know the minimum surface area for a given volume.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE3b, CGE4a, CGE7j.
Strand(s): Measurement and Geometry
Overall Expectations: MGV.01.
Specific Expectations: MG1.02, MG1.03, MG1.04.

Planning Notes

Students need space and flat surfaces to work on for this activity. For constructing sample rectangular prisms and cylinders, materials include blank paper, scissors, tape, glue, rulers, protractors, and compass sets.

For calculations and graphing, students may work by hand or use spreadsheets or graphing calculators. A printed record is required of all tables, calculations, and graphs, however.

Prior Knowledge Required

- construction of prisms and cylinders
- formulas for the surface area and volume of prisms and cylinders

Teaching/Learning Strategies

- Students work in groups of four to complete Student Worksheet – Building Gift Boxes to Explore Minimum Surface Area. Students work together to do constructions and calculations, and to discuss results and conclusions. Students complete the written report as individuals, however.
- The teacher facilitates whole class discussions to highlight results and conclusions.

Assessment/Evaluation

- Begin the assessment by observation, using the Student/Teacher Worksheet – Assessment of Work in a Group found at the beginning of the unit. Continue using the rubric to assess by observation each day. Provide students with the rubric on the first day. They will do a peer assessment during the next activity.
- Assess the written report, using the criteria listed on the worksheet.

Accommodations

- Modify the task as necessary for students who may have difficulty in the constructions.

Student Worksheet: Building Gift Boxes to Explore Minimum Surface Area

In this activity, you will work as a member of a group of four to do constructions, make calculations, and discuss results and conclusions. On your own, you will complete and hand in the work for questions 1 to 11.

Setting the Context

A Grade 9 class is constructing containers in which to wrap toys that have been donated for Christmas. The containers will be in the shape of rectangular prisms and cylinders, and the material used will be a colourfully decorated "boxboard". After considering the problem carefully, students decided to construct containers in a sequence of volumes, intended to hold toys of different sizes. The volume they chose were: 150 cm^3, 1000 cm^3, 3000 cm^3, 8000 cm^3, and 15000 cm^3.

Having chosen the volumes, students then began to explore the possible dimensions for each volume.

Square-based Rectangular Prisms

To simplify the exploration, students considered only boxes having square bases.

1. Working in a group of four students, select *one box* for which a table has been provided below.
   a) Complete the labelled columns in the table of values and use paper to construct each box that it would be reasonable to construct (e.g., Ask yourself, what toy would likely come in that size and shape?)

Box #1: Volume = 150 cm3						
Volume (cm^3)	**Length (cm)**	**Width (cm)**	**Height (cm)**		**Total Surface Area (cm^2)**	
150	1	1				
150	2	2				
150	3	3				
150	4	4				
150	5	5				
150	6	6				
150	7	7				
150	8	8				
150	9	9				
150	10	10				
150	11	11				

Box #2: Volume = 1000 cm3						
Volume (cm^3)	**Length (cm)**	**Width (cm)**	**Height (cm)**		**Total Surface Area (cm^2)**	
1000	2	2				
1000	4	4				
1000	6	6				
1000	8	8				
1000	10	10				
1000	12	12				
1000	14	14				
1000	16	16				
1000	18	18				
1000	20	20				
1000	22	22				
1000	24	24				

Box #3: Volume = 3000 cm3						
Volume (cm^3)	**Length (cm)**	**Width (cm)**	**Height (cm)**		**Total Surface Area (cm^2)**	
3000	2					
3000	4					
3000	6					
3000	10					
3000	15					
3000	20					
3000	25					
3000	30					
3000	35					

Box #4: Volume = 8000 cm3						
Volume (cm^3)	Length (cm)	Width (cm)	Height (cm)		Total Surface Area (cm^2)	
8000	3	3				
8000	5	5				
8000	10	10				
8000	15	15				
8000	20	20				
8000	25	25				
8000	30	30				
8000	35	35				
8000	40	40				

Box #5: Volume = 15000 cm3						
Volume (cm^3)	Length (cm)	Width (cm)	Height (cm)		Total Surface Area (cm^2)	
15000	5	5				
15000	10	10				
15000	15	15				
15000	20	20				
15000	25	25				
15000	30	30				
15000	40	40				
15000	50	50				
15000	60	60				

b) If you were to draw a graph of surface area versus height for the box volume that you are investigating, would you expect it to be linear or non-linear? Why?

c) Calculate the finite differences in the table of values. Use the empty columns in the table and label them “Change in *h*” and “Change in *TSA*”.
Is the relation linear or non-linear? How can you tell this from the finite differences?
What does this tell you about the rate of change of surface area with respect to height?

d) Construct a graph of surface area versus height. Were your hypotheses regarding shape and rate of change correct?

2. Examine the boxes you have constructed. Which shape (i.e., which set of dimensions) would likely be most useful as a gift for toys? Explain your choice.
3. The decorative boxboard being used is quite expensive, and so it is important to *minimize* the amount of material used. Examine your table and graph. Which shape would require the minimum amount of material? Identify the dimensions and the amount of material.
4. Compare your answer to Questions 2 and 3. Comment.

Cylinders

The next task is to construct a set of cylindrical gift boxes, again using the decorative boxboard. The chosen volumes of 150 cm^3, 1000 cm^3, 3000 cm^3, 8000 cm^3, and 15000 cm^3 will be used again.

5. Working in a group of four students, select *one cylinder* for which a table has been provided below.
   a) Complete the labelled columns in the table of values and use paper to construct each cylinder that it would be reasonable to construct (e g., Ask yourself, what toy would likely come in that size and shape?).

CYLINDER #1: Volume = 150 cm^3					
Volume (cm^3)	**Radius (cm)**	**Height (cm)**		**Total Surface Area (cm^2)**	
150	1				
150	2				
150	3				
150	4				
150	5				
150	6				

CYLINDER #1: Volume = 1000 cm^3					
Volume (cm^3)	**Radius (cm)**	**Height (cm)**		**Total Surface Area (cm^2)**	
1000	2				
1000	4				
1000	6				
1000	8				
1000	10				
1000	12				
1000	14				
1000	16				
1000	18				

CYLINDER #1: Volume = 3000 cm^3					
Volume (cm^3)	**Radius (cm)**	**Height (cm)**		**Total Surface Area (cm^2)**	
3000	1				
3000	3				
3000	5				
3000	10				
3000	15				
3000	20				
3000	25				
3000	30				

CYLINDER #1: Volume = 8000 cm^3					
Volume (cm^3)	**Radius (cm)**	**Height (cm)**		**Total Surface Area (cm^2)**	
8000	1				
8000	3				
8000	5				
8000	10				
8000	15				
8000	20				
8000	25				
8000	30				
8000	35				

CYLINDER #1: Volume = 15000 cm^3					
Volume (cm^3)	**Radius (cm)**	**Height (cm)**		**Total Surface Area (cm^2)**	
15000	2				
15000	5				
15000	7				
15000	10				
15000	15				
15000	20				
15000	30				
15000	40				
15000	50				

b) If you were to draw a graph of surface area versus height for the cylinder volume that you are investigating, would you expect it to be linear or non-linear? Why?

c) Calculate the finite differences in the table of values. Use the empty columns in the table. Label them "Change in *h*" and "Change in *SA*".
Is the relation linear or non-linear? How can you tell this from the finite differences?
What does this tell you about the rate of change of surface area with respect to height?

d) Construct a graph of surface area versus height. Were your hypotheses regarding shape and rate of change correct?

6. Examine the cylinders that you have constructed. Which shape (i.e., which set of dimensions) would likely be most useful as a gift for toys? Explain your choice.
7. Examine your table and graph. Which shape would require the minimum amount of material? Identify the dimensions and the amount of material.
8. Compare your answer to Questions 6 and 7. Comment.

Comparison

9. a) In questions 2 and 6, you selected the most likely containers to use as gift boxes. Compare your choices. Consider shape, utility, amount of material, and any other factors you consider important.

   b) In questions 3 and 7, you identified the containers that would yield minimum surface area. Which has the smaller surface area – the square-based rectangular prism or the cylinder? Why do you think this is so?

10. a) Compare your results to question 9 to those of other students in the class. Describe any common conclusions or trends that occur.

    b) Compare across the class the dimensions of the square-based prism that gave minimum surface area. Do you notice anything about the dimensions?

    c) Compare across the class the dimensions of the cylinder that gave minimum surface area. Do you notice anything about the dimensions?

Other Applications

11. In this activity, you have explored the possible shapes of square-based rectangular prisms and cylinders for given volumes. You considered the needs in packaging of minimizing the surface area in order to minimize the amount of material used. Describe other applications in which it would be important to minimize the surface area of a given volume.

Assessment of the Written Submission

The assessment of the written submission will be based on the following criteria and marking scheme:

Marks	Criterion
Square-Based Rectangular Prism	
✔✔	(1a) The calculations in the table are complete and correct.
✔	(1b) The hypothesis regarding linearity/non-linearity is correct.
✔	(1c) The description of the rate of change is correct.
✔✔✔	(1d) The graph is complete, correct, and represents proper form.
✔✔	(2) The choice of the most likely prism is reasonable and supported.
✔✔	(3) A reasonable minimum surface area is identified. The dimensions of the prism are stated.
✔✔	(4) The comparison of the most likely prism and the prism having minimum surface area demonstrates logical thought and is supported.
Cylinder	
✔✔	(5a) The calculations in the table are complete and correct.
✔	(5b) The hypothesis regarding linearity/non-linearity is correct.
✔	(5c) The description of the rate of change is correct.
✔✔✔	(5d) The graph is complete, correct, and represents proper form.
✔✔	(6) The choice of the most likely cylinder is reasonable and supported.
✔✔	(7) A reasonable minimum surface area is identified. The dimensions of the cylinder are stated.
✔✔	(8) The comparison of the most likely cylinder and the cylinder having minimum surface area demonstrates logical thought and is supported.
Comparison	
✔	(9a) The comparison of likely choices is logically supported.
✔✔	(9b) The comparison of containers having minimum surface area is clearly stated and makes reference to the characteristics of a rectangular prism and of a cylinder.
✔✔	(10 a, b) The trends identified across the class are clearly explained.
✔✔	(10c) A relationship is identified among the dimensions of square-based rectangular prisms having minimum surface area for a given volume. A relationship is identified between the dimensions of a cylinder having minimum surface area for a given volume.
Application	
✔✔	At least two other reasonable applications are clearly described.

Activity 2: Comparing Perimeter and Area of a Rectangle

Time: 150 minutes

Description

Students work in pairs to explore the relationships between the perimeter and the area of a rectangle. Students identify examples of situations in which these relationships are important.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE3b, CGE4a, CGE7j.
Strand(s): Measurement and Geometry
Overall Expectations: MGV.01.
Specific Expectations: MG1.01.

Prior Knowledge Required

- formulas for the perimeter and area of a rectangle
- experience of fixing one measure of an object while varying the other

Teaching/Learning Strategies

- Hand out Student Worksheet – The Relationship between Perimeter and Area. Monitor student activity while the activities are being completed.
- Facilitate whole class discussions of results of investigations at appropriate points.

Assessment/Evaluation

- Continue the assessment of students by observation, using the rubric Assessment of Work in a Group given at the beginning of the unit.
- Have students complete Student/Teacher Worksheet – Assessment of Work in a Group, rating group members from the previous activity.

Student Worksheet – The Relationship Between Perimeter and Area

Consider the relationship between the perimeter and area of a rectangle.

Suppose perimeter were fixed, how would area vary?

Suppose area were fixed, how would perimeter vary?

1. A man has 200 m of material available to fence in a rectangular exercise run for his dog. He wishes to maximize the area inside the fence. What dimensions should he choose?

   a) Hypothesize the dimensions that would yield maximum area. Explain your answer.
   b) Consider the previous activity. When we had a fixed volume, how did we go about examining the dimensions of various possible shapes?
   Follow that logic here. Select appropriate values and then perform the needed calculations to complete the table below. Add additional rows, if you think they are necessary.
   You will need the formulas for the perimeter and the area of a rectangle.

Perimeter (cm)	**Length (cm)**	**Width (cm)**	**Area (cm^2)**

   c) Predict the shape of the graph of area versus length.
   d) Construct the graph of area versus length.
   e) Using the graph and the table, identify the dimensions of the rectangle that will give maximum area. Identify the maximum area.
   f) Complete this conclusion: The rectangle that has maximum area for a given perimeter is

2. A woman has the same amount of fencing, but decides to build the rectangular run for her dog with one side against the house.
   a) Set up a table to explore the possible dimensions of the dog run and the possible areas. Construct a graph of area versus length.
   b) Identify the maximum area and the dimensions of the rectangle that yield it.
   c) Compare your results to those of question 1. Comment.

3. a) Describe a situation in which it might be important to know the minimum perimeter of a rectangle having a given area.
   b) A rectangle has an area of 450 cm^2. Hypothesize the dimensions that would yield minimum perimeter. Explain your reasoning.
   c) Design an investigation to determine the dimensions that would yield minimum perimeter for a rectangle having an area of 450 cm^2. Carry out the investigation.
   d) Compare the result of your investigation to your hypothesis in part b). Comment.

Unit 3C: Exploring Geometric Properties of Plane Figures

Time: 10 hours

Unit Description

Students review and apply the angle properties of triangles, quadrilaterals, and parallel lines. They investigate the properties of the medians, angle bisectors, and altitudes in various types of triangles. Students explore the properties of the sides and diagonals of quadrilaterals.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE 3c, CGE 5e.
Overall Expectations: MGV.03.
Specific Expectations: MG 3.01, MG3.02, MG3.03, MG3.04.

Activity Titles (Time and Sequence)

Activity 1	Quadrilaterals Involving Parallel Lines	150 minutes
Activity 2	Investigating and Applying the Properties of Diagonals in Quadrilaterals	150 minutes
Activity 3	Investigating Interior and Exterior Angles: Applications to Patterns	75 minutes
Activity 4	Investigating Geometric Relationships – Properties of Angle Bisectors, Medians, Altitudes, and Perpendicular Bisectors	150 minutes
Activity 5	Summative Assessment Activity	75 minutes

Unit Planning Notes

Dynamic geometry software will be used to identify properties of plane figures and explore relationships among them. In most activities, alternative approaches are provided that do not include dynamic geometry software.

The resource, *Exploring Geometry with the Geometer's Sketchpad*, is referenced throughout. Published by Key Curriculum Press, this resource is licensed to the Ontario Ministry of Education and is available in ".pdf" form on *The Geometer's Sketchpad*™ CD-ROM being sent to each secondary school.

Prior Knowledge Required

- basic vocabulary of geometry (e.g., types of angles, types of triangles, types of quadrilaterals)
- geometric properties of angles in triangles and in parallel lines

Teaching Learning Strategies

- Students work as individuals, in pairs, and in groups.
- Teacher facilitates independent student work.
- Teacher leads whole class discussions.

Assessment/Evaluation

- assessment by observation
- written explanations, journal entries
- diagnostic test
- pencil/paper tests and tasks

Resources

Exploring Geometry with the Geometer's Sketchpad. Key Curriculum Press (licensed to the Ontario Ministry of Education)

Activity 1: Quadrilaterals involving parallel lines

Time: 150 minutes

Description

Students re-examine the properties of corresponding, co-interior, alternate and opposite angles as they are related to parallel lines. This provides an opportunity for teachers to introduce the use of dynamic geometry software in a familiar setting. They extend their knowledge to examine quadrilaterals formed by two transversals crossing a pair of parallel lines.

Strand(s) and Expectations

Catholic Graduate Expectations: CGE 3c, CGE 5e.
Strand(s): Measurement and Geometry
Overall Expectations: MGV.03.
Specific Expectations: MG3.01.

Planning Notes

STEP	TITLE	TIME
1	Students complete diagnostic test on Grade 8 skills.	10 minutes
2	Activity to re-examine angles and parallel lines either with dynamic software or using hands-on investigations.	45 minutes
3	Students complete additional practice in the textbook.	20 minutes
4	Activity to explore properties of the angles and sides related to quadrilaterals formed by two transversals crossing a pair of parallel lines.	30 minutes
5	Class discussion on the properties of sides and angles in special quadrilaterals.	30 minutes
6	Students write a summary in their journals of the properties of sides and angles of special quadrilaterals.	15 minutes

- Gather the following materials: grid paper, protractor, ruler, dynamic geometry software, if available. If teachers choose to use software for all or part of this activity students must have facility with geometry software.

Prior Knowledge Required

Geometry and Spatial Sense (Grade 8): identify and investigate the relationships of angles; identify angle properties of parallel and perpendicular lines (interior, corresponding, opposite, alternate); describe the relationship between pairs of angles within parallel lines and transversals.

Teaching/Learning Strategies

Diagnostic Assessment: Teachers prepare a brief diagnostic assessment based on the grade 8 expectations. The results allow the teacher to make adjustments to this activity.

Introduction: Teacher facilitates a class discussion to generate examples of parallel lines within the classroom (e.g., edges of window frames, walls,) and in building and decorating (e.g., bricks, rooflines, wallpaper borders). Teacher poses several questions such as:

- How can you tell if lines in a room are parallel – for example, the top and bottom edges of the blackboard? Most students will suggest measuring to see if the distance is constant.
- When would this method be difficult? (e.g. lines that are very far apart, lines that are slanted.)
- How can we be sure that the edges of the ceiling and floor are parallel if we don't measure the distance between them?
- Consider a jungle gym. How can the installers be sure that two climbing bars are parallel if they meet the same slanted pole?

Student Activity:

A. If The Geometer's Sketchpad™ is available:

1. Students examine the angles formed by a transversal by working through Properties of Parallel Lines, p. 17 in *Exploring Geometry with The Geometer's Sketchpad*.
2. Students examine the properties of quadrilaterals formed by two transversals:
   - Students construct a pair of parallel lines as in the previous work. They then construct a transversal to intersect these lines at A and B respectively and a second transversal to intersect the lines at D and C respectively.
   - Students measure the angles at the four vertices of the quadrilateral ABCD.
   - Students try to drag a vertex until ABCD is:
     - a parallelogram
     - ·a trapezoid
     - a rectangle
   - Students share their reasons for deciding when ABCD is a particular shape and refer to angle measurements to justify their reasoning.
   - Students are asked to name any other quadrilateral they can create (some may offer square or rhombus) and describe how they could be sure they had constructed one of these (they would need to measure lengths of sides.)
   - Students are asked to name any quadrilateral they cannot create by dragging ABCD (all are possible except the general quadrilateral.)

B. If dynamic software is not available:

1. Students may do the following:
   - Students draw a slanted line across a ruled sheet of paper or fold the paper on an angle.
   - Students measure all angles at the intersection of this transversal and any two of the ruled lines.
   - Students are asked to share their results and describe orally any patterns they notice.
   - Students draw a slanted line across two lines that are almost parallel. They check whether there are any patterns amongst the angles.
   - Students share their results and formulate a test for parallel lines.
   - Students use their test by checking whether lines in patterns around them are parallel.
   - Students write a summary statement about the angles formed by a transversal intersecting parallel lines and draw their own diagram using different colours or symbols to illustrate alternate, corresponding and co-interior angles.

2. Students then:
   - Draw a pair of parallel lines on lined paper. They draw two transversals (non-parallel) and label the intersections with the pair of parallel lines so that the quadrilateral formed is ABCD.
   - Students measure sides of ABCD and the angles generated at the vertices.
   - Students repeat the above procedures to draw a parallelogram, rhombus, square, and rectangle by choosing appropriate transversals.

Concluding the Activity
The teacher leads a class discussion on the properties of the angles and sides in these special quadrilaterals and students write a summary of the angle and side properties of special quadrilaterals in their journals.

Possible Extensions
Discuss what is meant by "parallel".
- Look at art and examine how parallel lines are displayed to show perspective.
- How can we extend the word parallel to planes? (e.g., What does it mean to say "the desk top is parallel to the floor.")

Assessment/Evaluation
Formative
- Diagnostic Assessment is used at the beginning of the activity.
- Assess journal entries, using the Communication section of Appendix B: Written Report (from Units 1 and 2).
- Teacher could follow-up with short knowledge/understanding formative assessment at the start of the next class to check that students are able to calculate angle measures in questions involving using parallel lines.

Resources
Exploring Geometry with The Geometer's Sketchpad. Key Curriculum Press (licensed to the Ontario Ministry of Education)

Accommodations
- Students could be paired with another student for the activity.

Activity 2: Investigating and Applying the properties of diagonals in quadrilaterals and polygons
Total Time: 150 minutes

Description
In this activity, students use dynamic geometry software or geometric instruments to examine some properties of sides and diagonals in quadrilaterals and other polygons. They use their skills to design a colourful kite (or ornament) as a community project.

Strand(s) and Expectations
Catholic Graduate Expectations: CGE2b, CGE3c, CGE5a, CGE7j.
Strand(s): Measurement and Geometry, Relationships
Overall Expectations: MGV.03.
Specific Expectations: MG3.03, MG3.04.

Planning Notes

- Gather the following materials: Student Worksheet - Diagonals of a Quadrilateral, photocopies of assorted quadrilaterals as described in Part A, protractor, ruler, dynamic geometry software, if available.
- If teachers choose to use software for all or part of this activity, students require facility with dynamic geometry software.

Prior Knowledge Required

- Geometry and Spatial Sense (Grade 8): identify, describe, compare and classify geometric figures; construct and solve problems involving lines and angles;
- Measurement and Geometry: illustrate and explain the properties of …angles related to parallel lines.

Teaching/Learning Strategies

<u>Part A</u>

Prior to starting each of these investigations, the teacher may need to do some skill review (either using dynamic geometry software or geometric instruments).

Step 1

If dynamic geometry software is available:

- Students investigate properties of the diagonals of quadrilaterals using the following: *Exploring Geometry with The Geometer's Sketchpad*, pp. 89-92, 95-96.

If dynamic geometry software is not available:

- The students work in groups of four.
- The teacher prepares photocopies of assorted large scale quadrilaterals. Each group requires one each of a general quadrilateral, parallelogram, rhombus, square, rectangle and trapezoid. Note: To facilitate discussion there should be several different versions of each figure. · Students draw diagonals for each of the above shapes and measure and discuss the following:
  - the lengths of the sides
  - the measurements of the angles formed at the vertices of the quadrilateral
  - the lengths of the diagonals
  - the manner in which the diagonals divide the interior angles
  - the measurements of the angles formed at the intersection of the diagonals
  - whether the diagonals bisect one another

Step 2

All students individually complete Student Worksheet – Diagonals of a Quadrilateral. This involves a summary of the results of their investigation of the various quadrilaterals.

Step 3

If dynamic software is available,

- Students proceed with the activities on pp. 102 and 103, *Exploring Geometry with The Geometer's Sketchpad*.

If software is not available, students proceed as follows:

- Students draw six different quadrilaterals, join the midpoints of their sides to form another quadrilateral.
- Students examine the new figures and make a prediction about the shapes. Students then test their conjecture and write a report in their journals to summarize their findings.

Suggestion: After a prediction has been made for the first quadrilateral, discuss with students how to test the prediction. Students might complete a table, such as:

Original Quadrilateral		**Quadrilateral Formed by Joining the Midpoints of the Sides**		
Quadrilateral	**Side Lengths**	**Prediction Regarding Shape**	**Results of test of Prediction**	**Conclusion**
1				
2				
3				
4				
5				
6				

Part B

A class of students volunteers to make colourful kites (or polygonal Christmas tree ornaments) to present to a local daycare, seniors centre or preschool. If the item is a kite it should be designed to include all diagonals as braces and be finished in an artistic fashion.

Note: A kite is a formal geometric term describing a quadrilateral with two distinct pairs of consecutive equal sides. It is expected that students will think of a "kite" simply as a lightweight object that flies while pulled by a string. Teachers should clarify this concept at the start of this activity.

This portion of the project would be an excellent opportunity for students to work collaboratively in groups of two or three to create a design. The teacher may choose to hold a friendly competition for the best design and display all of the designs around the classroom or in a prominent place in the school. Teachers should review criteria of the assessment rubric to ensure that students understand expectations of a level 3 or 4 project.

This project will be started in class but it should be completed on the students' own time.

Project Guidelines

The Kite (or ornament) design must be polygonal in shape and include diagonals as braces.
The finished product must include:

- a scale diagram of the kite design on grid paper.
- a list of material used to complete the design.
- one to two paragraphs highlighting aspects of the design.

Possible Extensions

Students can investigate the star formed by the diagonals in a non-regular pentagon. They can be encouraged to find an explanation for the sum of the angles at the points.

Assessment/Evaluation

- Use the observation rubric for group work, Appendix C (from Units 1 and 2).
- Assess paper and pencil tasks for accuracy and completeness.
- Assess journal entries, using the Communication section of Appendix B: Sample Rubric- Written Report (from Units 1 and 2).
- The teacher may use the summative assessment rubric for the design activity included below for application and communication skills during the kite design project.

Resources

Exploring Geometry with The Geometer's Sketchpad. Key Curriculum Press (licensed to the Ontario Ministry of Education)

Internet web sites on kites such as the hila science campsite:
http://fox.nstn.ca/~hila/projects/Kite/kite.html

Accommodations

- Students who have difficulty using the toolbar in geometry software could be paired with another student for the activity.
- For enrichment, students could research and then create a more sophisticated, complex design such as the box kite. Internet web sites are an excellent resource.
- Students work in pairs.

Student Worksheet – Diagonals of a Quadrilateral

Shape and Marked Diagram	Side Lengths	Measures of Interior Angles	Diagonal Lengths	Division of Interior Angles by Diagonals	Angles at inter-section of Diagonals	Division of Diagonals
Quadrilateral						
Square						
Rectangle						
Parallelogram						
Rhombus						
Trapezoid						

Summative Assessment Rubric for the Design Activity

SCALE CATEGORY	LEVEL ONE	LEVEL TWO	LEVEL THREE	LEVEL FOUR
Application • Chooses appropriate materials and tools for task and recognizes limitations	- chooses appropriate materials with considerable teacher assistance	- able to choose appropriate materials with some teacher assistance	- chooses appropriate materials independently and with confidence	- demonstrates innovation, creativity and confidence in their choice of materials
• Applies concepts of area, diagonals in a quadrilateral or regular polygon to task	- applies concepts only with considerable assistance - makes several significant errors in calculations	- applies concepts with some assistance - makes some significant errors in calculations	- applies concepts accurately and consistently, without assistance	- applies concepts accurately and with ease; assists others with applying concepts
• Applies logical, efficient, co-operative procedure to the task and recognizes limitations of design process	- infrequently applies efficient procedure to task and requires considerable assistance in following sequence· - frequent lack of co-operation - rarely recognizes limits in process without considerable assistance	- applies efficient procedure to task some of the time, and requires limited assistance in following sequence - usually co-operative, may need some prompting - able to recognize some limits in process with assistance/prompting	-· applies efficient procedure to task most of the time, with minimal clarifying questions - generally co-operative approach to task - able to recognize and communicate limits in process (e.g., limits of given material)	- routinely applies efficient and innovative procedure to task - consistently co-operative and encourages co-operation and creativity of others·· - recognizes limits in process and suggests alternative solutions
Communication • Communicates their reasoning, in writing and with a diagram for their kite design	- incomplete diagram with limited detail (e.g., few or no measurements)· - explanation unclear and incomplete	- complete diagram with some detail· - explanation attempts to justify some decisions but does not address at least one key area in design (e.g. Diagonal braces)	- complete diagram with all important details included· - explanation is clear, complete, logical and provides considerable justification for elements of the design	- complete diagram with sophisticated, creative detail (e.g. consideration of aerodynamics in design)· - explanation is complete, clear and easy to read with elaborate justification for elements of design

Activity 3: Investigating Interior and Exterior angles: Applications to Patterns

Time: 75 minutes

Description

Students explore and analyse the properties of interior and exterior angles of triangles and quadrilaterals, then use these to examine possibilities for design. They find the sum of the interior angles of a triangle, the sum of the angles of a quadrilateral and the sum of exterior angles for triangles and quadrilaterals. Dynamic geometry software may be used.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: 2c, 3c, 4b.
Strand(s): Measurement and Geometry
Overall expectations: MGV. 03.
Specific Expectations: MG3.01.

Planning Notes

This activity explores the properties of interior and exterior angles.

Step	Activity	Time (approx.)
1	Examine patterns in fabric design and flooring.	5 minutes
2	Investigation of the interior and exterior angles of triangles and quadrilaterals.	45 minutes
3	Apply the knowledge obtained to re-examine the patterns.	10 minutes
4	Do activities from the textbook to backup concepts.	15 minutes

- Have students work in pairs.
- Provide each group scissors, protractors.

Prior Knowledge Required

- Geometry and Spatial Sense (Grade 8): identify the angle properties of intersecting, parallel, and perpendicular lines; understand the sum of the interior angles of a triangle.

Teaching/Learning Strategies

1. Teacher displays patterns in fabric and flooring materials that use triangles and quadrilaterals to tessellate (tile the floor, leaving no gaps). Sources may include flooring brochures, pictures in catalogues, actual fabric. Teacher leads a discussion of the various shapes (e.g. some slate tiles may be general quadrilaterals, some ceramic tiles may be diamond shaped or triangular). Teacher asks the students whether you could make a tile in the shape of a triangle or quadrilateral that would not work to create a floor or fabric design without gaps.
2. The teacher encourages students to explore the sum of the interior angles in a triangle and in a quadrilateral through a hands-on approach.
   - If *The Geometer's Sketchpad*™ is available, students may investigate the sum through the worksheet Triangle Sum, p. 65 in *Exploring Geometry with The Geometer's Sketchpad* (Key Curriculum Press), licensed to the Ontario Ministry of Education. They can then extend the method used to explore the sum of the interior angles of a quadrilateral.

- If dynamic geometry software is not available, students can explore the ideas in the following way:
    - Each student cuts out a large triangle, measures the angles and writes these angle measurements on the triangle.
    - Students rip off the vertices of the triangles and line them up. Teacher leads a discussion of why this shows that the sum of the angles is 180°.
    - This activity is repeated for a general quadrilateral. Relate the sum of the angles to the fact that a quadrilateral can be cut into two triangles.

3. To investigate the exterior angles:
    - Students may complete the activities Exterior Angles in a Triangle, p. 66 in *Exploring Geometry with The Geometer's Sketchpad.* The activity Exterior Angles in a Polygon, p. 109 can be done to investigate the sum of the exterior angles of quadrilaterals; however, it leads students to explore the results for polygons in general. An alternative is to have students extend the method on p. 66 to explore the sum of the exterior angles in quadrilaterals.
    - If dynamic software is not available, students may complete the following explorations.
        - Students construct a triangle with pencils so that the sides extend past the vertices. Students discuss the relationship between each interior angle and its associated exterior angle.
        - Students make a very small triangle with the pencils. The teacher asks them to predict the sum of the exterior angles. Students check their conjecture by drawing a triangle, measuring the exterior angles and finding the sum.
        - The teacher asks students to predict the sum of the exterior angles in a quadrilateral. Each pair of students constructs a quadrilateral and checks the conjecture.
        - The teacher creates a chart on the board and students enter the results for their quadrilateral. The teacher leads a discussion of how the pencil model followed above helps explain why the sum of the exterior angles is 360°.
4. Re-examine the flooring and fabric patterns as a whole class. Students write an answer to the question posed in part 1, justifying their answers by referring to the results obtained.
5. Students practice further by using the material in the textbook.
6. The teacher asks probing questions of each pair while circulating through the class in order to ensure that students have a grasp of the concepts. The teacher verifies that each student has contributed to the best of his/her ability.

Follow-Up: The teacher checks that the students have a solid grasp of the new concepts as they work on exercises in the textbook.

Assessment/Evaluation

- Use the observational rubric for group work Appendix C.
- Journal entry – Use criteria for Communication in Appendix B: Sample Rubric-Written Report
- Paper and pencil tasks.

Resources

Dynamic geometry software

Exploring Geometry with The Geometer's Sketchpad. Key Curriculum Press (licensed to the Ministry of Education)

Mathematics Teacher, "Problem Solvers", October 1994, Volume 87, Number 7, pp. 490-495.

Accommodations

- Students may work in pairs.

Extensions
- Students hypothesize and then investigate the sum of the interior angles in other polygons by dividing them into triangles.
- Students can investigate the size of the angle in each of the regular polygons: (e.g., regular pentagon, regular hexagon.)
- Students could extend their knowledge of interior and exterior angles to higher order polygons through further investigation with or without dynamic geometry software.

Activity 4: Investigating Geometric Relationships – Properties of Angle Bisectors, Medians, Altitudes, and Perpendicular Bisectors

Time: 150 minutes

Description

Students investigate the medians, altitudes, angle bisectors and the perpendicular bisectors of sides of triangles. Students construct four cardboard triangles and find one of the special intersection points for each. They will then consider several problem scenarios and use their constructions to find the solutions.

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE 4b, 2c.
Strand(s): Measurement and Geometry
Overall expectations: MGV.03.
Specific Expectations: MG3.02, MG3.04.

Planning Notes

This activity is divided into four parts over 150 minutes.

Students:

Step	Activity	Time (approx.)
1	Construct and cut out four cardboard scalene triangles with special intersection points constructed.	60 minutes
2	Work on additional questions from the textbook.	15 minutes
3	Using the constructions, solve two problem scenarios and communicate the solutions in writing.	50 minutes
4	Work on additional questions from the textbook.	25 minutes

An alternative approach to the explorations with the cardboard triangles is to use the blackline masters pp. 71 to 77 in *Exploring Geometry with The Geometer's Sketchpad* (Key Curriculum Press), licensed to the Ontario Ministry of Education.

Prior Learning Required

Geometry and Spatial Sense (Grade 8):
- construct and solve problems involving lines and angles;
- investigate geometric mathematical theories to solve problems;
- use mathematical language effectively to describe geometric concepts, reasoning and investigations.

Teaching/Learning Strategies

1. Students construct four triangles using regular paper. Teachers emphasize that the students draw large, scalene triangles. After constructing one of the special sets of lines on each triangle students glue their triangles to cardboard and cut them out. It is important that teachers demonstrate appropriate construction techniques using ruler and compasses, paper folding or MIRA®s. Teachers circulate around the classroom, assisting students in completing their activities.
2. Students (in pairs) investigate the solutions to the following scenarios.

   A. Pose the challenge:

   If equal weights are attached to each vertex of a triangle, find the point at which a support must be placed so that the triangle will balance.
   - The students attach 1 large paper clip (or tape a penny) to each vertex of each constructed triangle. They try to balance each triangle on the tip of a sharp pencil.
   - Pairs share their results and identify the special point.
   - The teacher leads a brief discussion of balance:
     - Demonstrate that a rod with equal weights on each end will balance at the midpoint.
     - If the weight is doubled at one end the balance point divides the rod length in the ratio 2:1.
   - Pairs use their constructed triangles and the information about balance to explain and justify the location of the centroid and communicate this in their journals.

   B Pose the challenge:

   The provincial government is planning to locate a communications tower to serve three communities in the north. Three straight roads link these communities in a triangular shape. If the tower is to be placed at the same perpendicular distance from each road, find its location.
   - Students use the constructed triangles to model this scenario. The sides represent the three roads.
   - Students examine the four intersection points to discover which has the required property.
   - Students explain in writing their reasons for making their choice.

Extensions

- Consider the question in B with the following change. The tower is to be located equidistant from each community.
- Ask students to find the balance point of a pentagon. They can explore this using the weight ideas above.

Assessment/Evaluation

- Use the observational rubric for group work Appendix C (from Units 1 and 2).
- Journal entry – use criteria for Communication in Appendix B: Written Report (from Units 1 and 2)
- Paper and pencil tasks.

Resources

Exploring Geometry with The Geometer's Sketchpad. Key Curriculum Press (licensed to the Ontario Ministry of Education)

Resources for use of MIRAs (available from suppliers of mathematical resources)

Accommodations

- All written instructions can be given to students orally.
- Students work in pairs

Activity 5: Summative Assessment Activity

Time: 75 minutes

Description

Sample Questions for Paper and Pencil Test

Strand(s) and Expectations

Ontario Catholic School Graduate Expectations: CGE2b, CGE2c, CGE3c.
Overall Expectations: MGV.03.
Specific Expectations: MG3.01, MG3.02, MG3.03, MG3.04.

Planning Notes

These sample questions are designed to allow students to demonstrate the full range of their learning in this unit. It is important to include a representative collection of these questions, in addition to questions that demonstrate knowledge and understanding, in any summative test prepared for this

Sample Questions for Pencil and Paper Test

1. A sports engineering firm has designed a very innovative and challenging triangular trampoline with sides measuring 3m each. They wish to mark the balance point of the trampoline with their logo. Where should they place the logo? Justify the method you chose to find this location.

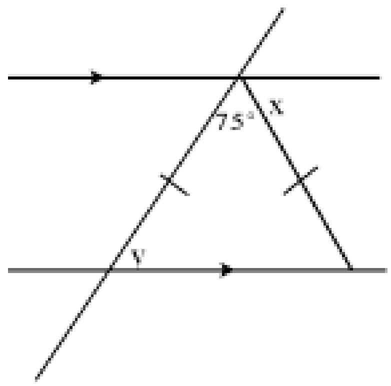
2. a) In which type of triangle (scalene, isosceles, etc.) are the centroid, orthocentre, and incenter the same point? Explain your reasoning.
   b) In which type of triangle do an angle bisector, a median, and an altitude coincide? Explain your reasoning.

3. a) Construct a large isosceles triangle. Construct its medians, altitudes, and angle bisectors.
   b) What special property does the centroid, orthocentre, and incenter share?

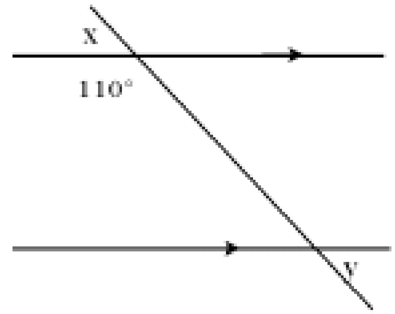
4. What am I?
   a) I have two pairs of equal sides and one of my angles is a right angle. I am not a square.
   b) I have four sides and my angles are not all equal but my diagonals intersect at right angles.
   c) I have 9 diagonals.
   d) When you draw my diagonals you get a smaller copy of me.

5. State whether the following statements are true or false:
   a) A square is a rectangle.
   b) A rectangle is a square.
   c) A pentagon has 8 diagonals.
   d) The exterior angles of a pentagon have a sum of 360°.

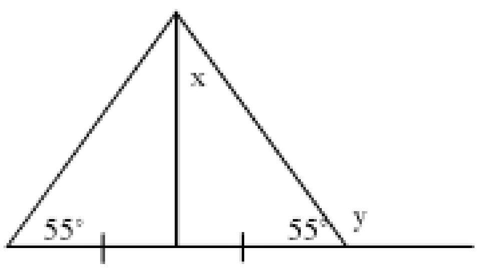
6. Calculate the value of x and y in each of the following diagrams.



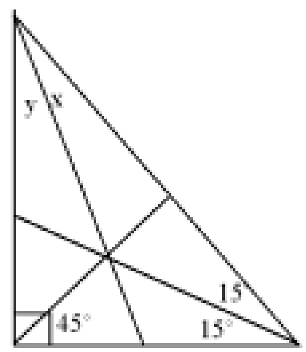
a)



b)



c)



d)

Unit 4: Making Connections

Time: 10 hours

Unit Description

Students engage in activities that reflect the content and procedures of the course, in preparation for final assessment activities which may include a performance assessment and a final exam.

Strand(s) and Expectations

Ontario Catholic School Graduation Expectations: CGE 2b, CGE 5a, CGE 5b.
Overall Expectations: all
Specific Expectations: all

Unit Planning Notes

Ten hours are allotted for preparation and carrying out of the final assessment activities. It is recommended that these activities include both a performance assessment and a final examination.

Four sample activities are included that might be used as part of the preparation for the exam or performance assessment. It is recommended that teachers supplement these activities with material drawn from the textbook or other sources.

The activities are embedded within the following context:

> *A local organization has donated a piece of land to be used as a community park. The land and a small wading pool have been donated, and the community has come together to do the preparation work necessary. Students from the local schools are very involved.*

The activities are found on the following worksheets.

Student Worksheet – The Paper Chase

You have been put in charge of Advertising for the grand opening of the Community Park. You have contacted two local newspapers to inquire about their rates for placing an ad. *The Lake Graphit Gazette* would charge $15 per day for the ad that you want to place, regardless of how many days you wish to place the ad. Another local paper, *The Mount Slope Reporter* would charge a flat rate of $100 plus $5 per day for the same ad.

1. Write an equation to represent the relationship between total cost (C) and number of days (n) for:
   a) *The Lake Graphit Gazette*
   b) *The Mount Slope Reporter*

2. a) Construct a table of values for each relation. Pay special attention to which variable is the independent variable.

   b) Construct the graph of each relation on the same set of axes.

   c) Which line is steeper? Why?

   d) State the slope of each relation.

   e) Explain the meaning of the slope within this application.

   f) What useful information is provided by graphing the two lies on the same axes?

3. You discover a third local publication, which would charge $137.50 for 5 days and $162.50 for 15 days. Assume that there is a linear relation between the cost of the ad and the number of days that it would run.

   a) Determine the cost per day for the ad in this publication.

   b) Determine the flat fee charged.

   c) Write the equation of the linear relation.

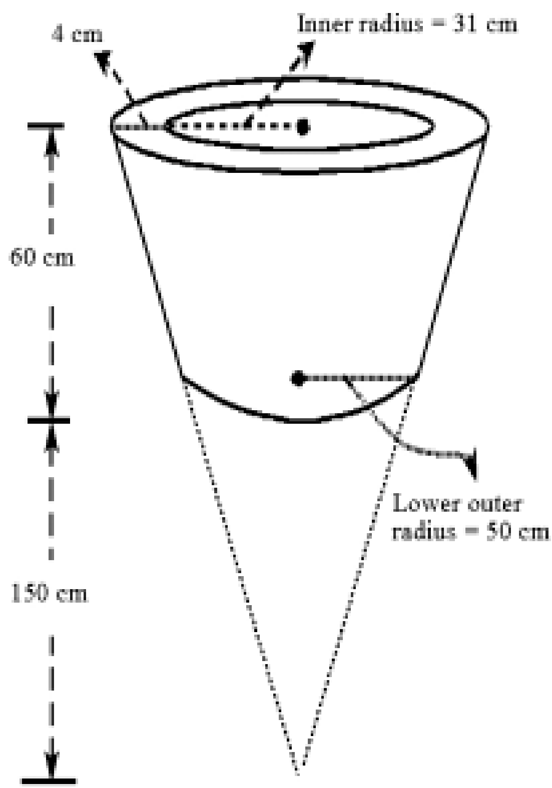
   d) Add this line to your graph. How does it compare with the other two lines?

Student Worksheet – a Pool Is Cool!

In one corner of the park, a section has been fenced off to enclose an in-ground wading pool for younger children. The section is in the shape of a trapezoid, having height 14 m and base lengths 25 m and 18 m. The wading pool is cylindrical in shape, having a diameter of 8 m and a height of 50 cm. The area outside the pool, but inside the fence, is planted in lawn.

A group of students has assumed responsibility for preparing the play area.

1. Draw a diagram to represent the situation.
2. One part of the preparation is the paining of the fence, which is 1.5 m high and constructed of closely packed boards. A local merchant is donating paint that requires 4 L for every 65 m^2 of coverage. To put two coats of paint on *both* sides of the fence, how many 4L-cans will be needed?
3. The wading pool is to be filled to a depth of 30 cm. A hose is available that flows water at a rate of 9 L/min. How long will it take to fill the pool to the required height?
4. Before opening the area, the lawn is to be topped with top soil and then re-seeded. If the top soil is to be applied at a constant depth of 2 cm, how many m^3 of top soil will be required?
5. There is a planter in the shape of a frustum on each side of the entrance to the play area. As shown in the diagram, a frustum is a cone that has been truncated (cut) parallel to its base. The dimensions of the planter are shown in the diagram. If each planter is filled to the brim with potting soil, how much soil will be needed?



Student Worksheet: Something For The Gardeners

Another group of students has been assigned the task of designing a garden area for the park. It is to be surrounded by a low hedging plant and enough plants are available to create a perimeter of 60 m.

The group has been asked to identify the figure (e.g., triangle, rectangle, pentagon, ...) that would provide the maximum enclosed area for a perimeter of 60 m.

Design an investigation to identify the required shape. For each different figure, make calculations to determine the dimension that would yield maximum area for a perimeter of 60 m. Then compare the maximum areas of the different figures to select the overall maximum.

a) Set up a table of values to examine possible areas for a field that is a rectangle. Include a graph of area versus length. Identify the maximum area and the dimensions of the rectangle that yield it.\

b) Set up a table to examine possible areas for a field that is an isosceles triangle.

Perimeter (m)	**Base (m)**	**Equal Sides (m)**	**Height* (m)**	**Area (m^2)**

-
-
-

*__Note:__ You have to use your knowledge of the properties of isosceles triangles to calculate the height from the base and the length of one of the equal sides.

c) Construct a graph of area versus height.
Identify the maximum area and the dimensions that yield it.

d) Select one other figure to examine (other than a triangle or a quadrilateral). Set up a table to examine possible areas. Construct a graph of area versus height. Identify the maximum area and the dimensions that yield it.

e) Examine the maximum areas that resulted from your three investigations. If you continued to examine other figures, which would yield maximum area for a fixed perimeter? Explain your reasoning.

Student Worksheet: Water, Water, Everywhere!

All of the students brought along water because the day was warm and the work quite strenuous. One student had a sealed plastic bottle filled with water. A leak opened in the bottle and the water drained out at a constant rate.

The table below identifies the height (in cm) of water in the bottle at time (*t*) seconds after the draining began.

Time (*t*) (seconds)	**Height (*h*) of water in bottle(cm)**
0	25
10	22
20	19
30	16
40	14
50	12
60	8
70	6
80	4
90	2
100	0

a) Recopy the table and extend to calculate the finite differences.

b) Is this relation linear or non-linear? Explain.

c) Construct a graph for the data.

d) Using the shape of the graph and values of the finite differences, sketch a possible shape for the bottle. Explain your reasoning.