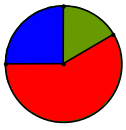


BIG PICTURE

Students will:

- develop teamwork skills through cooperative learning;
- take risks when carrying out an investigation and demonstrate perseverance;
- apply a variety of problem-solving strategies;
- apply a number of estimation strategies during problem solving;
- justify their solutions and choice of strategies;
- make connections between prior and new knowledge to draw conclusions;
- represent their thinking in a variety of ways, reflect on their learning, and communicate effectively.

Day	Lesson Title	Description	Expectations
1	Encouraging Others	<ul style="list-style-type: none"> • Practise the social skill of encouraging others. • Identify strategies involving estimation problems. • Set the stage for using estimation as a problem-solving strategy. 	8m7, 8m28 CGE 5a
2	Solving a Fermi Birdseed Problem	<ul style="list-style-type: none"> • Find a solution to a problem involving estimation. 	8m6, 8m31, 8m112 CGE 3c
3	Taking Turns	<ul style="list-style-type: none"> • Practise the social skill of taking turns. • Find a solution to a problem involving estimation. 	8m6, 8m9, 8m32 CGE 5a
4	Paraphrasing and Summarizing	<ul style="list-style-type: none"> • Practise the social skill of active listening and paraphrasing. • Practise developing good problem-solving strategies. 	8m6, 8m9, 8m32, 8m112 CGE 2a
5	Including All Participants and Recording Mathematics	<ul style="list-style-type: none"> • Practise the social skill of including all participants. • Develop a method for effective recording of mathematics learning. • Find a solution to a problem involving estimation. 	8m6, 8m9, 8m14, 8m31 CGE 5a
6	Disagreeing in an Agreeable Way While Analysing Good Math Records	<ul style="list-style-type: none"> • Practise the social skill of disagreeing in an agreeable way. • Examine math recordings, suggest how to improve them, and articulate what good writing looks like in mathematics. • Create a concept map to help consolidate their thinking over the last few days. 	8m6, 8m9, 8m14, 8m31, 8m32, 8m35, 8m39 CGE 5e



Description

- Practise the social skill of encouraging others.
- Identify strategies involving estimation problems.
- Set the stage for using estimation as a problem-solving strategy.

Materials

- BLM 1.1
- birdseed

Assessment Opportunities

Minds On ...

Whole Group → Brainstorm

Explain why it is important to encourage others. Explicitly teach the social skill, Encouraging Others, through a group brainstorm. Create an anchor chart using the criteria: What does encouragement look like? What does it sound like? (TIP 2).

Action!

Think/Pair/Share → Gather Data

Use an overhead of the Think/Pair/Share process (TIP 8) and student copies of BLM 1.1. Students gather data.

Learning Skill/Observation/Mental Note: Circulate, observing social skills and listening to students.

Share with students some of the positive words and actions observed during the activity and invite students to make additions to the anchor chart on Encouraging Others.

Whole Class → Sharing

Based on 'teachable topics' during the Think/Pair/Share Activity, e.g., a particularly effective phrase/statement expressed by a student, clarification of the cooperative learning strategy, an interesting result on BLM 1.1, ask representatives of groups to share their results or report on their process.

Whole Class → Setting Stage

Explain that during the first week, the class will solve a number of different kinds of Fermi problems to sharpen their problem-solving and estimation skills (Poster: Teaching Through Problem Solving). Discuss the concept of Fermi problems (TIP 3).

Show a large bag of birdseed and ask, How many seeds do you think are in the bag? Tell the class that tomorrow they will work on solving this problem.

Consolidate Debrief

Whole Class → Discussion

Use the posters Teaching Through Problem Solving, Problem-Solving Strategies, and Understand the Problem. Discuss how these posters will be of assistance over the next few days as well as during the whole math program. Point out that when students encourage others, it makes it safe for them to try new things and contribute to group activities.

Home Activity or Further Classroom Consolidation

Interview one or more adults about estimation using the following guiding questions and record your responses in a math journal. Summarize what you notice about the responses. You may be asked to share this math journal entry with the class.

- When do you use estimation and how often?
- When are accurate calculations necessary?

Answer the following questions in the math journal to assess your growth in social skills.

- The social skill focus of this activity was _____.
- Something I said or did to demonstrate the social skill was _____.
- I helped the group work in a positive way by _____.
- An area I want to work on is _____.

"Learning is socially constructed; we seldom learn isolated from others."
- Bennett & Rolheiser

Consider using stickers as recognition for examples of the social skill being applied by a group.

Solving Fermi problems is a way to collect diagnostic assessment data about social skills, academic understandings, and attitudes towards mathematics (TIP 3).




Social Skill Practice Reflection

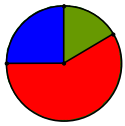
1.1: What Do We Have in Common?

Name:

Date:

Fill in the following table with your partner.

Name:		
Which TV shows do you like to watch?		
 What kind of music do you like to listen to?		
What do you enjoy doing in your spare time?		
What movies have you seen lately?		
 What sports do you like to watch or participate in?		
Do you have brothers and sisters?		
What are some activities you participate in during the summer?		
 What school subjects do you enjoy?		
What did you find the most interesting in your comparison?		



Description

- Find a solution to a problem involving estimation.

Materials

- large bag of birdseed
- chart paper
- *Impact Math – Number Sense*

Assessment Opportunities

Minds On...

Whole Class → Introduce the Problem

Learning Skills/Journal/Anecdotal: Ask selected students to share their math journal responses.

Read one of the books listed as a jumping-off point for the lesson. This makes an important link between mathematical and language literacy. A King/Rajah starts by placing one grain of rice on a chessboard and then doubles the value for each consecutive square, i.e., 1, 2, 4, 8, 16, until the last square. Stop the story before it gets too far along in order to not give away the strategy used for measuring the rice, i.e., volume or mass. Introduce the problem: How many seeds do you think are in this large bag of birdseed?

Stories to set a context: *The King's Chessboard* by David Birch, ISBN 0140548807 or *The Rajah's Rice* by David Barry, ISBN 0716765683
An on-line version of the King's Chessboard – <http://www.2.bc.edu/~grout/chessboard/html/pg01.htm>

Action!

Think/Pair/Share → Guided Cooperative Problem Solving

Guide students through the Think/Pair/Share process as it pertains to the birdseed problem. (TIP 8) Tell students that you are looking for a variety of creative strategies for solving the problem.

Curriculum Expectations/Question & Answer/Anecdotal: Circulate to look for evidence of the social skill, strategies used, and students who are having difficulty. If there are a number of students who need help:

- scaffold through questioning: How can we measure the birdseed? How can these measurements help us solve the problem? What tools might help us? How? How might a smaller container help? How would a scale help?
- model your thinking process through a think aloud (See Section 2 – Research, Scaffolding) pausing to allow students to contribute their own ideas and strategies as the group solves the problem together.

Set ground rules for sharing:

- Everyone has a perspective that should be considered.
- We need everyone's ideas for the best result.
- We could miss an important point/perspective if we do not share our thinking.
- Suggestions of all students will be listened to and used.

Consolidate Debrief

Whole Class → Sharing Strategies

Share some of the positive words and actions observed during the activity and invite students to make additions to the anchor chart on Encouraging Others (TIP 2). Pose the question: How did you decide on your estimate?

Select one person from three or four groups to share their group strategies and estimations. Pick groups with different approaches to help students realize there are many ways to solve this problem. Have the class compare group estimations and decide on reasons why they may vary and whether some are more valid.

Clarify, summarize, and record student responses on chart paper.

Possible strategies include:

- Using mass comparison
- Using volume comparisons
- Counting seeds in a small sample
- Using measuring cups
- Using measuring tapes

Application Exploration

Home Activity or Further Classroom Consolidation

Discuss the concept of a Fermi problem with a friend or someone at home, then generate an example of a Fermi problem that deals with something from home. Write up your problem and its solution in your math journal.



Description

- Practise the social skill of taking turns.
- Find a solution to a problem involving estimation.

Materials

- BLM 3.1, 3.2
- phone books
- calculators

Assessment Opportunities

Minds On...

Whole Class → Sharing

Curriculum Expectations/Observation/Anecdotal: Listen to students and provide immediate feedback as Inside Outside Circle (TIP 13) are used to have students share the Fermi problem they developed.

Rotate the circles and have students share their Fermi problem with a new partner. Review with the class any elements that have been misunderstood.

Whole Class → Brainstorm

Brainstorm to create an anchor chart for Taking Turns (TIP 2).

Groups of 4 → Cooperative Group Problem Solving

Use BLM 3.1 for a Placemat cooperative activity (TIP 9) Ask each group: Why was it difficult to solve the birdseed problem? What information did you need to know? Based on the class list of questions, each group generates a list of questions to guide them in making their estimates. Model for the class how to create one or two of these questions.

Whole Class → Discussion

One person from each group is selected at random to share one question from the Placemat activity. Record the questions on chart paper or on a transparency. Using a transparency of BLM 3.2, the class orders the questions from the class chart from broadest at the top to more narrow information at the bottom so that the combined answers give an appropriate estimation. How did you decide on your questions?

Action!

Groups of 4 → Cooperative Problem Solving

Learning Skills & Curriculum Expectations/Observation/Anecdotal: Circulate while groups are working.

Using a Placemat activity, students choose a strategy to solve the problem, How many names are there in the phone book? After solving the problem, students use the Ranking Ladder (BLM 3.2) to sequence the questions they used to arrive at an accurate estimation.

Consolidate Debrief

Whole Class → Discussion

Curriculum Expectations/Exhibition/Checklist: Select one person from two or three of the groups to present their problem-solving strategies to the class. Choose groups with different methods for solving the problem. Encourage students to show how each strategy follows the estimation model. Record strategies on a transparency or chart paper.

Summarize strategies with the class, modelling the selection of important information. Tell the class that they will build the summarizing skills used today during next class. Reaffirm how estimation skills improve with practice.

Home Activity or Further Classroom Consolidation

In your math journal, identify a situation where estimation is needed, then describe a strategy that could be used to establish a reasonably accurate estimate.

Reflection



Inside/Outside Circles help develop a positive classroom climate and a community of learners.



During cooperative learning, use a 2-colour disk as a barometer. Show the white side when the group is demonstrating the social skill. Show the red side when they are not using the social skill.



Fermi solved his legendary problems by developing a series of questions and estimating the answers.

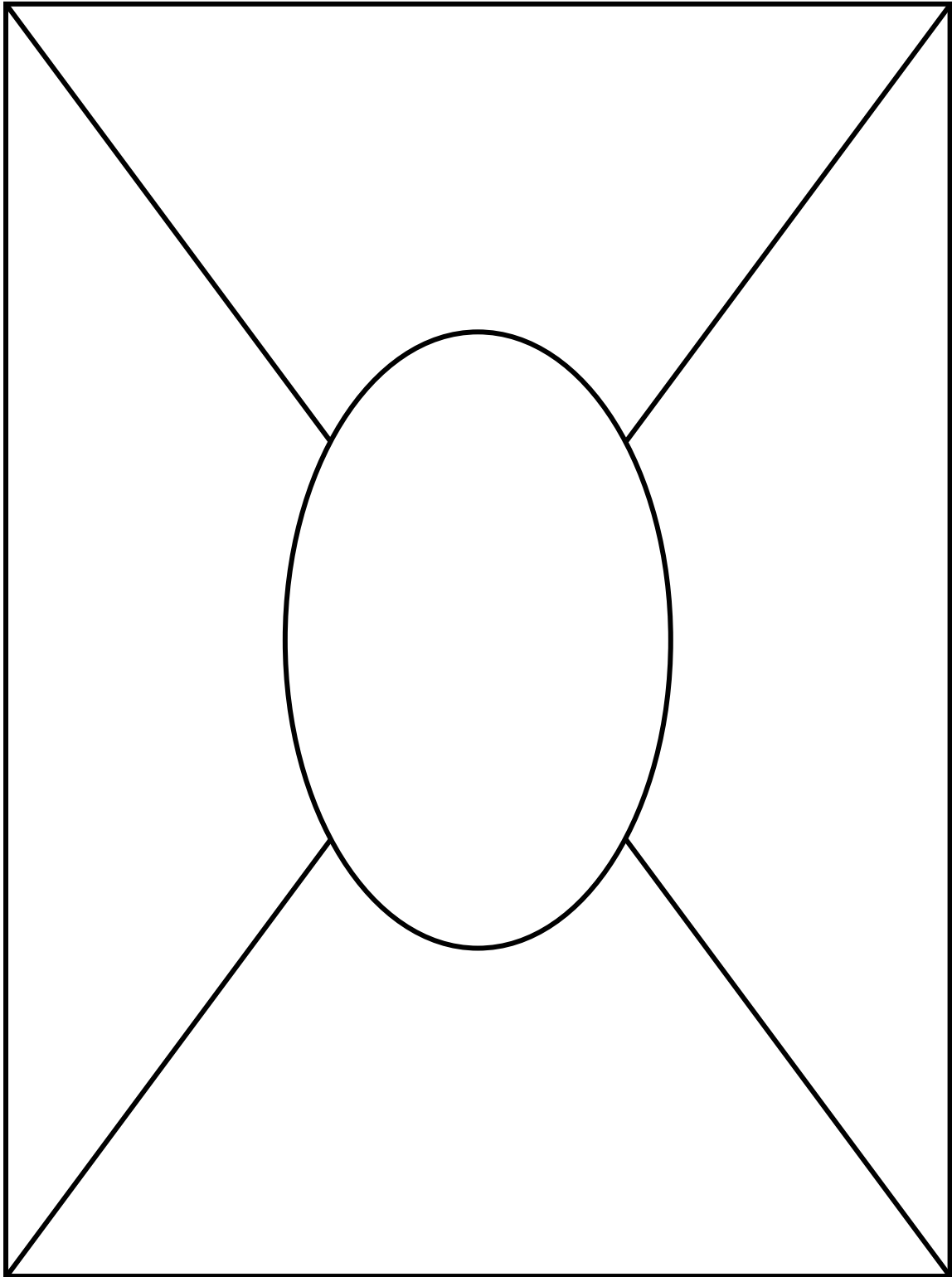


Give examples of some of the positive things (Taking Turns and Encouraging Others) and add them to the class anchor charts. See TIP 15, Questioning, for suggestions on how to elicit mathematical thinking.

3.1: Placemat

Names:

Date:



3.2: Ranking Ladder

Name:

Date:

Use the ranking ladder to organize the questions you used to arrive at an accurate estimation. List the first question you would ask yourself to solve the problem at the top of the ladder. List the last question at the bottom of the ladder, and use the middle rungs to put the other questions in order.

The diagram is a ranking ladder, which is a trapezoidal shape formed by two parallel slanted lines on the left and right sides, connected by five horizontal rungs. The rungs are evenly spaced and decrease in length from top to bottom, creating a series of five rectangular boxes for writing questions. The top rung is the shortest, and the bottom rung is the longest.



Description

- Practise the social skill of active listening and paraphrasing.
- Practise developing good problem-solving strategies.

Materials

- BLM 3.2

Assessment Opportunities

Minds On...

Whole Group → Active Listening and Summarizing

Explain the meaning of Active Listening and Paraphrasing and through brainstorming develop a class anchor chart for this social skill (TIP 2).

Groups of 3 → Cooperative Problem Solving

Introduce the Fermi problem: How many times does the wheel of your bicycle turn on a trip from the school to the Sky Dome in Toronto? (Change the destination as needed.) Have the class think of some questions the interviewer may ask:

- What strategies will you use?
- What questions do you need answered to estimate the solution?
- What information do you need to know?
- What confuses you?

In small groups students quietly think for a few minutes before starting the interviews.

Whole Class → Sharing Session

Select one or two groups who were successful in paraphrasing and have them model for the class. On the anchor chart, make any additions that emerge from the sharing.

Action!

Whole Group → Discussion

Guide a class reflection on problem-solving steps and strategies that students effectively used to solve Fermi problems. Students brainstorm criteria for good problem solving using the focus question: What does a good problem solver do? Students briefly discuss with a partner, then draw out and record their ideas. Refer to classroom posters on Problem Solving.

Groups of 4 → Cooperative Problem Solving

Each group appoints a recorder and tracks the steps and strategies the group follows to solve the Fermi problem. Another member tracks the questions asked to arrive at a reasonable estimate. Each group records its hierarchy of questions on BLM 3.2.

Curriculum Expectations/Performance Task/Anecdotal: Circulate and look for groups to share their problem solving process, strategies and questions.

Learning Skills/Observation/Tracking Sheets: Encourage the groups as necessary using prompting questions (TIP 17 Learning Skills Tracking Sheet).

Consolidate Debrief

Whole Class → Sharing

Select one student from two or three groups to present their problem solving strategies, and ranking ladder questions. Choose groups with different methods for solving the problem, demonstrating that there are many good ways to solve the same problem. Use think aloud to model, paraphrase, and record each group's ideas on chart paper or overhead.

Learning Skills/Exhibition/Mental Note: Assess students as they present their strategies.

Whole Class → Consolidate

Debrief the steps for being a problem solver. Compare all the strategies used throughout Lessons 1-4 to the ones listed on the poster, Problem-Solving Strategies. Use the strategy, Logical Reasoning, when developing and ordering questions.

Home Activity or Further Classroom Consolidation

Reflect on the steps and strategies you use to problem solve and write about them in your math journal.

The interview process provides an opportunity to apply the skill of active listening and paraphrasing to collectively determine an appropriate strategy for solving the problem.

Provide road or computer maps.

Prompting questions:
 How do you make sense of the problem?
 How do you get started?
 How do you know what to do?
 How do you organize and communicate your thinking?
 How do you pick a strategy?
 How do you solve problems?

Reflection



Description

- Practise the social skill of including all participants.
- Develop a method for effective recording of mathematics learning.
- Find a solution to a problem involving estimation.

Materials

- BLM 1.2, 3.2, 5.1
- Math posters
- colour markers

Assessment Opportunities

Minds On...

Whole Class → Reflection

Introduce the social skill: Including All Participants. Students discuss why the skill is important and what it looks like and sounds like (TIP 2).

Learning Skill/Self-Assessment/Anecdotal: Using journal question at the end of Day 1, have students self-assess their social skills development to date.

Groups of 4 → Graffiti Board

Use the following questions to help students begin their graffiti board:

- Why record in math?
- What should the written explanation of your records include?
- In what other ways besides words can you organize and show your thinking?
- If you were trying to understand someone else’s thinking, what information and organizational formats (diagrams, tables, charts, etc.) would help you?

Circulate and use prompting questions, as necessary (TIP 15).

Whole Class → Sharing

Students consolidate their thinking and develop a list of criteria for good math records. Create and post a class anchor chart listing the criteria for good math records.

Action!

Whole Class → Introduce the Fermi problem

Introduce today’s problem: How many hours do students in Grade 7 and 8 in Ontario talk on the telephone in one year?

Groups of 4 → Solve the Problem

Students discuss and record their questions on the Ranking Ladder (BLM 3.2). When they have found a satisfactory solution, the group discusses and creates their best record using the criteria developed during the Graffiti Board exercise. Provide markers and chart paper. Use BLM 5.1 on a transparency to guide students’ thinking.

Learning Skills/Observation/Checklist: During the problem-solving process, look for students who are recording their series of questions in sequence and groups that are using different strategies. Call on these groups during consolidation.

Consolidate Debrief

Whole Class → Discussion

Curriculum Expectations/Performance Task/Rating Scale: Select groups to display their recordings and explain their estimation/problem-solving process. Clarify, if necessary, having students turn to a partner to paraphrase what was explained. Guide the discussion, as necessary.

Ask students to reflect on whether they are becoming more accomplished estimators. Discuss estimation strategies that you have observed throughout the class. Tell students they will discuss records in more detail during the next class.

*Application
Concept Practice*

Home Activity or Further Classroom Consolidation

Explain to someone how you would solve today’s Fermi problem.

Note: Display the Math posters prominently: Teaching Through Problem Solving, Representations Make Our Thinking Visible, Understand the Problem, and Problem-Solving Strategies.

Link to writing for different audiences.

Display the teacher-made charts of student strategies for Fermi problems

Remind students to use the social skills they have learned to date.

Be sure that students note that good records should include one or more representations of thinking: diagrams, words, numbers or symbols, tables, etc.

5.1: Thinking to Solve Problems



Name:

Date:

What do you predict? Why?

What question will you use to begin estimating?

How will you decide how many students there are in Grades 7 and 8?

What surprises you? Why?

What do you find interesting? Explain.

Describe any trends you see in the data?

Why do you think these trends are happening?



Description

- Practise the social skill of disagreeing in an agreeable way.
- Examine math records, suggest how to improve them, and articulate what good writing looks like in mathematics.
- Create a concept map to help consolidate their thinking over the last few days.

Materials

- BLM 6.1

Assessment Opportunities

Minds On...

Whole Class → Discovery

Present both positive and negative examples of the social skill shown on BLM 6.1. With a partner, students compare the two scenarios. When the class agrees on the social skill, help them decide on an appropriate name for it. Discuss why this skill is important to their learning and cooperative group work. Create an anchor chart for Disagreeing in an Agreeable Way (TIP 2).

Explain that today the class will analyse some examples from the previous day’s student records, specifically looking for evidence of good mathematics communication. Select two examples of group records from Day 5’s work (remove student names). Remind them that students worked hard to make these the best records possible. It is important to respect their effort by noting the strengths of the recordings, making positive suggestions for improvement. Remind students of all the positive social skills they have developed to date.

Action!

Think/Pair/Share → Peer Assessment

Students examine the examples and jot down at least three things that demonstrate the criteria the class established and one or two positive ways the authors can improve their records. They pair and share their findings. Look for students who have found evidence of the established criteria and for examples of students disagreeing positively. Use prompting questions to encourage students, as needed.

Students self-assess their group work, using the questions from Day 1.

Whole Class → Sharing Session

Select one person from each group to make thoughtful positive comments and suggestions for improvement. Record each of the group’s suggestions on chart paper and summarize their findings. Point out any evidence students may have missed of representations, thinking, strategies, and noticing patterns.

Whole Class → Brainstorm

Ask what students have done during the first five math classes. Record responses on chart paper or transparency (social skills, cooperative group work, Fermi problems, estimation, setting the criteria for good mathematical recordings). Discuss a concept map (poster) with students.

Groups of 4 → Concept Map Activity

Students make a concept map to summarize what they have learned so far in math class. Remind them to use the social skills they have learned.

Learning Skills/Observation/Checklist: Circulate and observe, noting the symbols and other features students use on their concept maps. Listen for good use of social skills. Assess each student’s contribution to the group as they work on their concept maps.

Consolidate Debrief

Whole Class → Sharing

Learning Skills/Exhibition/Checklist: Post concept maps. Students name some symbols that help them to remember the past week.

Choose one student from each group to tell what the Fermi problems/estimating taught them.

Home Activity or Further Classroom Consolidation

Reflect on your group records in your math journal looking for strengths and improvements. Write a letter to the teacher to explain what you learned this week in math class and your goals for the term. Explain how you learn mathematics best.

Reflection

Be sensitive to the fact that in some cultures it is considered disrespectful to maintain direct eye contact with another person.

Post TIP 2 or place it on a transparency.

Social skills should be left posted to remind students of the expectations when working in groups.



Students can be more successful making concept maps if they have time to talk about and process their memories.

Concept maps may be collected and commented on for group work and effort.

6.1: What is the Social Skill?

Name:

Date:

Look at the examples below and decide which social skill is being demonstrated by the positive examples:

Positive Examples	Negative Examples
<i>Looks like ...</i> <ul style="list-style-type: none">◆ Eye contact with a slight shake of the head◆ Listening to someone's entire idea before speaking◆ Smiling at the speaker◆ Puzzled or questioning look	<i>Looks like ...</i> <ul style="list-style-type: none">◆ Listener interrupts the speaker◆ Shaking the head rapidly back and forth◆ Impatiently challenging the speaker◆ Rapidly tapping the fingers◆ Angry challenging look
<i>Sounds like ...</i> <ul style="list-style-type: none">◆ I understand what you are thinking but have you ever considered?◆ Your idea is important but have you thought about ...?◆ I think I understand what you are saying but have you thought about ...?◆ Calm, quiet, controlled voices	<i>Sounds like ...</i> <ul style="list-style-type: none">◆ No way! I disagree, my idea is much better than that.◆ So what. Who cares? I have a different idea.◆ I totally disagree with everything you just said.◆ Loud, angry, or aggressive voices

Look at the examples below and decide whether each is a positive or negative example for the social skill. Discuss why this skill is important for successful learning and for getting along in your teams:

Examples

1. That's what you think...My idea is much better!
2. Something else to consider is _____, which is a little different than your idea.
3. My idea is fine. I'm not changing anything.
4. Is there anything we can add to _____'s idea?
5. You think only your ideas are important. What about mine?